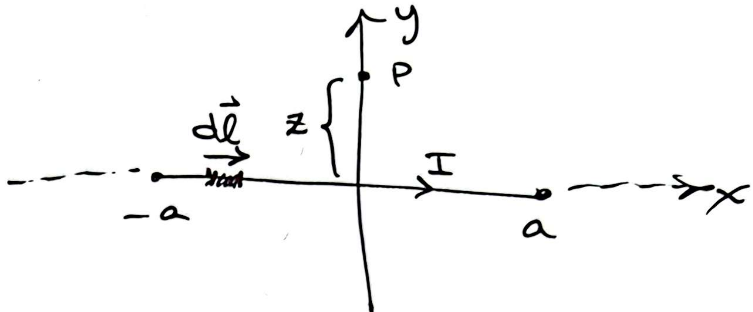
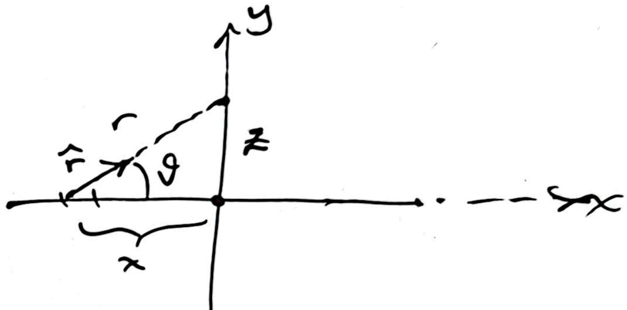
Use the Biot-Savart Law to find the field a distance z away from a long straight wire carrying a current I.



$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l}\times\hat{r}}{r^2}$$

$$d\vec{l} = dx\,\hat{i}$$



$$r^2 = z^2 + x^2$$

$$d\vec{l} = dx\,\hat{i}$$

$$d\vec{l}\times\hat{r} = dx\,\hat{k}\cdot\sin\theta$$

$$\sin\theta = \frac{z}{\sqrt{z^2+x^2}}$$

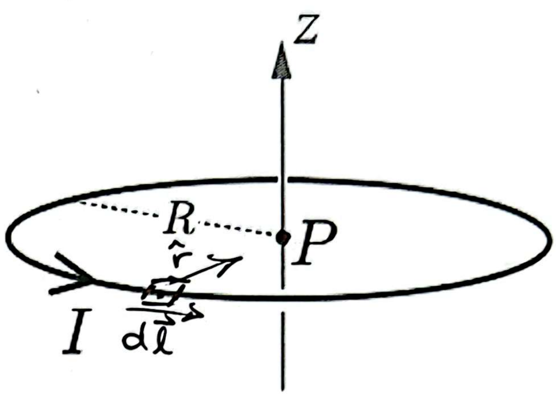
$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0 I}{4\pi}\int\left(\frac{z\,dx}{\sqrt{z^2+x^2}}\cdot\hat{k}\cdot\frac{1}{z^2+x^2}\right)$$

$$= \frac{\mu_0 I z}{4\pi}\hat{k}\int_{-a}^{a}\left(z^2+x^2\right)^{-3/2}dx = \frac{\mu_0 I z}{4\pi}\hat{k}\left[\frac{x}{z^2\sqrt{x^2+z^2}}\right]_{-a}^{a}$$

$$= \frac{\mu_0 I}{4\pi}\hat{k}\cdot\frac{1}{z}\;\frac{2a}{\sqrt{a^2+z^2}} \qquad \text{if } a \gg z$$

$$= \frac{\mu_0 I}{2\pi}\cdot\frac{1}{z}\,\hat{k}$$

Find the magnetic field at the center of a loop of current I.



$$d\vec{l} \times \hat{r} = dl \cdot \hat{k}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi} \hat{k} \int \frac{dl}{R^2} = \frac{\mu_0 I}{4\pi} \frac{\hat{k}}{R^2} \underbrace{\int dl}_{2\pi R}$$

$$= \frac{\mu_0}{2} \frac{I}{R} \hat{k}$$