# University Physics 1

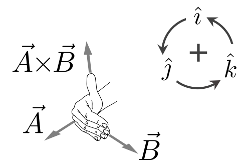
MADISON AREA | TECHNICAL COLLEGE

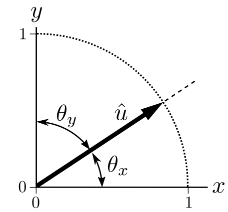
## Vectors

$$\vec{A} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} = A\hat{u} \qquad \hat{u} = \frac{\vec{A}}{A} \qquad c\vec{A} + d\vec{B} = \begin{bmatrix} cA_x + dB_x \\ cA_y + dB_y \\ cA_z + dB_z \end{bmatrix}$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z = AB\cos\theta \qquad \vec{A}\times\vec{B} = (AB\sin\theta)\,\hat{u}_\perp$$

$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\,\hat{\imath} + (A_zB_x - A_xB_z)\,\hat{\jmath} + (A_xB_y - A_yB_x)\,\hat{k}$$




$$\sin\theta_x = \cos\theta_y$$
$$\hat{u} = \hat{\imath}\cos\theta_x + \hat{\jmath}\sin\theta_x$$

## Kinematics

$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k} \qquad \vec{v} = \frac{d}{dt}\vec{r}(t) \qquad \vec{a} = \frac{d}{dt}\vec{v}(t)$$

$$\vec{r}(t) = \vec{r}_0 + \int_{t_0}^{t} \vec{v}(t')\,dt' \qquad \vec{v}(t) = \vec{v}_0 + \int_{t_0}^{t} \vec{a}(t')\,dt' \qquad \text{where } \begin{cases} \vec{v}_0 = \vec{v}(t_0) \\ \vec{r}_0 = \vec{r}(t_0) \end{cases}$$

**relativity (Galilean)**

$\vec{r}_{AB}$ = location of $A$ relative to $B$

$\vec{v}_{AB}$ = velocity of $A$ relative to $B$

$$\vec{r}_{BA} = -\vec{r}_{AB} \qquad \vec{r}_{AB} = \vec{r}_{AC} + \vec{r}_{CB}$$
$$\vec{v}_{AB} = -\vec{v}_{BA} \qquad \vec{v}_{AB} = \vec{v}_{AC} + \vec{v}_{CB}$$

**constant acceleration**

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2 \qquad \vec{v}(t) = \vec{v}_0 + \vec{a}t \qquad v_s^2 = v_{0s}^2 + 2a_s\Delta s$$

projectile motion:

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + t\begin{bmatrix} v_{0x} \\ v_{0y} \end{bmatrix} + \tfrac{1}{2}t^2\begin{bmatrix} 0 \\ -g \end{bmatrix} \qquad \begin{bmatrix} v_x(t) \\ v_y(t) \end{bmatrix} = \begin{bmatrix} v_{0x} \\ v_{0y} \end{bmatrix} + t\begin{bmatrix} 0 \\ -g \end{bmatrix}$$

**circular motion**

$$\vec{r}(t) = r\left[\hat{\imath}\cos\theta + \hat{\jmath}\sin\theta\right]$$

$$\omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d\omega}{dt} = \frac{a_t}{r}$$

$$a_c = \frac{v^2}{r} = \omega^2 r \qquad a_t = \frac{d\,|\vec{v}|}{dt}$$

$$\theta(t) = \theta_0 + \int_{t_0}^{t}\omega(t')dt' \qquad \omega(t) = \omega_0 + \int_{t_0}^{t}\alpha(t')dt' \qquad \text{where } \begin{cases} \theta_0 = \theta(t_0) \\ \omega_0 = \omega(t_0) \end{cases} \qquad \vec{a} = \vec{a}_c + \vec{a}_t \qquad v = \frac{2\pi r}{T} \qquad \theta = \frac{s}{r}$$

## Dynamics

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{\text{net}} = \textstyle\sum_i \vec{F}_i \qquad \vec{F}_G = m\vec{g} \qquad F_{\text{Sp}} = -k(x - x_0)$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_c = m\vec{a}_c$$

**friction**

$$f_s \le f_{s,\max} = \mu_s n \qquad f_k = \mu_k n \qquad F_D = \tfrac{1}{2}C\rho A v^2$$

1. identify system boundary
2. draw FBD w/ only external forces (direct contact or field force)
3. draw $\vec{F}_{\text{net}}$ vector below the FBD
4. construct equation $F_{\text{net}} = ma$ for each coordinate direction
5. repeat as needed for another system

## Work & Energy

$$W = \int_A^B \vec{F}\cdot d\vec{s} = F\Delta s\cos\theta \quad \text{(if } \vec{F} \text{ and } d\vec{s} \text{ are constant)}$$

$$E_{\text{mech}} = K + U \qquad K = \tfrac{1}{2}mv^2 \qquad \Delta U = -W_{\text{int}} = -\int_A^B \vec{F}_{\text{int}}\cdot d\vec{s}$$

$$F_s = -\frac{dU}{ds} \qquad U_G = mgy \qquad U_{\text{Sp}} = \tfrac{1}{2}ks^2 \qquad P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$$

$$W_{\text{ext}} = \Delta E_{\text{sys}} = \Delta(K + U + E_{\text{th}})$$

if $W_{\text{ext}} = 0$ and no friction loss then $\Delta K + \Delta U = 0$

## Constants

| | | |
|---|---|---|
| gravity field on Earth | $g$ | 9.81 N/kg |
| universal gravity const. | $G$ | $6.67\times10^{-11}$ N·m²/kg² |

## Center of Mass

$$x_{\text{cm}} = \frac{\sum_i m_i x_i}{M} = \frac{1}{M}\int x\,dm \qquad M = \textstyle\sum_i m_i$$

$$\vec{r}_{\text{cm}} = \frac{1}{M}\int \vec{r}\,dm \qquad \vec{v}_{\text{cm}} = \frac{d}{dt}\vec{r}_{\text{cm}}$$

## Momentum

$\vec{p} = m\vec{v}$ $\quad \vec{P} = \sum_i \vec{p}_i$ $\quad \vec{P} = M\vec{v}_{\text{cm}}$ $\quad \vec{F}_{\text{net}} = \frac{d\vec{P}}{dt}$ $\quad K = \frac{p^2}{2m}$

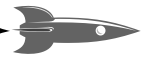
$\Delta\vec{P} = \vec{J} = \int \vec{F}_{\text{ext}}\, dt = \vec{F}_{\text{ave}}\Delta t$ $\quad$ if $\vec{F}_{\text{ext}} = 0$ then $\Delta\vec{P} = 0$

elastic 1D collisions, with $v_{2i} = 0$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

rockets

$\left| u\frac{dm}{dt} \right|$ →



$$\Delta v = u \ln\left(\frac{m_i}{m_f}\right)$$

## Rotational Dynamics

$\vec{\tau} = \vec{r} \times \vec{F}$ $\quad \tau = Fr\sin\phi = \pm Fd$ $\quad \vec{\tau} = \frac{d\vec{L}}{dt}$ $\quad \vec{\tau} = I\vec{\alpha}$

$\vec{L} = \vec{r} \times \vec{p}$ $\quad \vec{L} = I\vec{\omega}$ $\quad K_{\text{rot}} = \frac{1}{2}I\omega^2$ $\quad K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}}$

$I_{\text{particle}} = mr^2$ $\quad I_{\text{disk}} = \frac{1}{2}mr^2$ $\quad I_{\text{para}} = I_{\text{cm}} + md^2$

$\Omega = \frac{mgd}{I\omega}$ $\quad$ (+) ↺ $\quad$ (−) ↻



## Statics

for static equilibrium: $\vec{F}_{\text{net}} = 0$ and $\vec{\tau}_{\text{net}} = 0$ (about any point)

## Gravitation

$F_G = G\frac{m_1 m_2}{r^2}$ $\quad g = \frac{GM}{r^2}$ $\quad E_{\text{mech}} = K + U_G$

bound orbit: $E_{\text{mech}} < 0$
unbound orbit: $E_{\text{mech}} > 0$

$U_G = -G\frac{m_1 m_2}{r}$

$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$

circular orbits

$\frac{GM}{r^2} = \frac{v^2}{r}$ $\quad T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$ $\quad E_{\text{mech}} = \frac{1}{2}U_G$

## Oscillations

$\ddot{s}(t) = -\omega^2 s(t)$ $\quad f = \frac{1}{T}$ $\quad \omega = 2\pi f = \frac{2\pi}{T}$

$x(t) = A\cos(\omega t + \phi)$ $\quad \omega = \sqrt{\frac{k}{m}}$ $\quad \omega = \sqrt{\frac{g}{L}}$ $\quad \omega = \sqrt{\frac{mgL}{I}}$

$v(t) = -\omega A\sin(\omega t + \phi)$

$a(t) = -\omega^2 A\cos(\omega t + \phi)$ $\quad E = K + U = \frac{1}{2}kA^2 = \frac{1}{2}mv_{\text{max}}^2$

damped: $E(t) = E_0 e^{-bt/m} = E_0 e^{-t/\tau}$

## Waves

$v^2 \frac{\partial^2 y(x,t)}{\partial x^2} = \frac{\partial^2 y(x,t)}{\partial t^2}$ $\quad y(x,t) = A\sin(kx \mp \omega t + \phi)$

$k = \frac{2\pi}{\lambda}$ $\quad \omega = \frac{2\pi}{T}$ $\quad v = f\lambda = \frac{\omega}{k}$ $\quad v = \sqrt{\frac{F_T}{\mu}}$ $\quad n\frac{\lambda}{2} = L$

## Symbols and Units

| | | |
|---|---|---|
| acceleration | $a$ | m/s$^2$ |
| area | $A$ | m$^2$ |
| amplitude | $A$ | m |
| bulk modulus | $B$ | N/m$^2$ |
| damping constant | $b$ | kg/s |
| moment arm | $d$ | m |
| distance | $d$ | m |
| energy | $E$ | J |
| force | $f, F$ | N |
| frequency | $f$ | Hz, s$^{-1}$ |
| gravity field strength | $g$ | N/kg |
| moment of inertia | $I$ | kg·m$^2$ |
| impulse | $J$ | N·s |
| kinetic energy | $K$ | J |
| spring constant | $k$ | N/m |
| wave number | $k$ | rad/m |
| angular momentum | $L$ | kg·m$^2$/s |
| length | $L$ | m |
| mass | $m, M$ | kg |
| normal force | $n$ | N |
| mode of standing wave | $n$ | (none) |
| pressure | $p$ | Pa |
| power | $P$ | W, J/s |
| momentum | $p, P$ | kg·m/s |
| position vector | $\vec{r}$ | m |
| radius, distance | $r, R$ | m |
| path length | $s$ | m |
| time | $t$ | s |
| period | $T$ | s |
| unit vector | $\hat{u}$ | (none) |
| rocket exhaust speed | $u$ | m/s |
| potential energy | $U$ | J |
| velocity | $v$ | m/s |
| work | $W$ | J |
| $x$-position | $x$ | m |
| $y$-position | $y$ | m |
| $z$-position | $z$ | m |
| angular acceleration | $\alpha$ | rad/s$^2$ |
| angle | $\theta$ | radians |
| wavelength | $\lambda$ | m |
| linear mass density | $\mu$ | kg/m |
| kinetic friction coeff | $\mu_k$ | (none) |
| static friction coeff | $\mu_s$ | (none) |
| density | $\rho$ | kg/m$^3$ |
| torque | $\tau$ | N·m |
| time constant | $\tau$ | s |
| angle | $\phi$ | radians |
| angular velocity | $\omega$ | rad/s |
| angular frequency | $\omega$ | rad/s |
| gyro precession freq | $\Omega$ | rad/s |