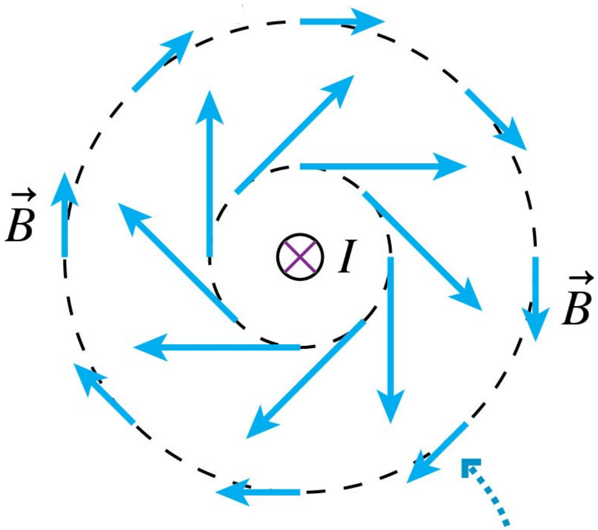
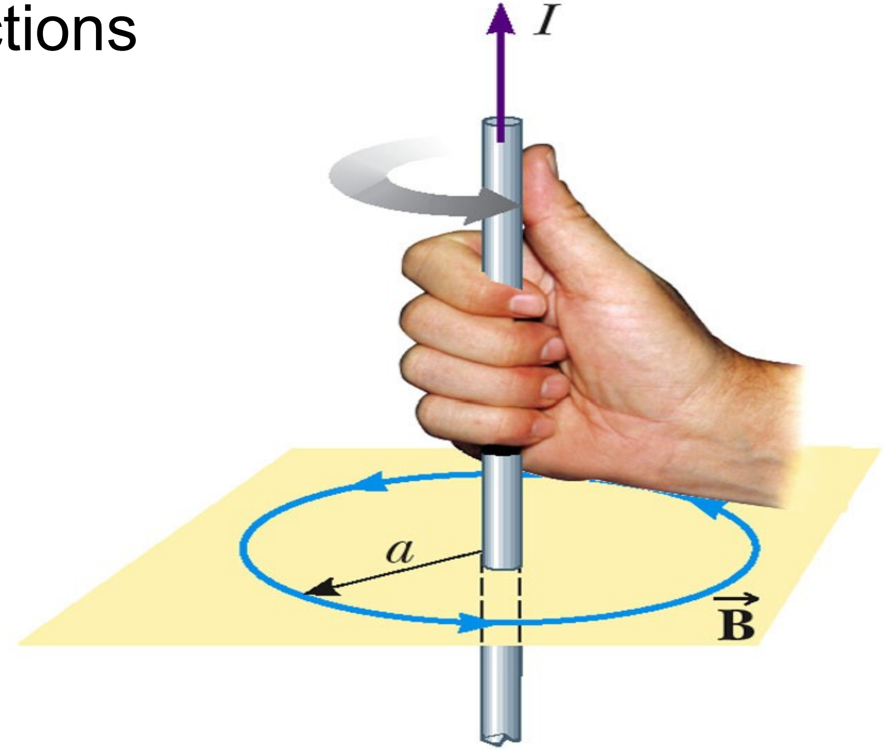
Sources of Magnetic Fields

- A current-carrying wire produces a magnetic field
- The compass needle deflects in directions tangent to the circle

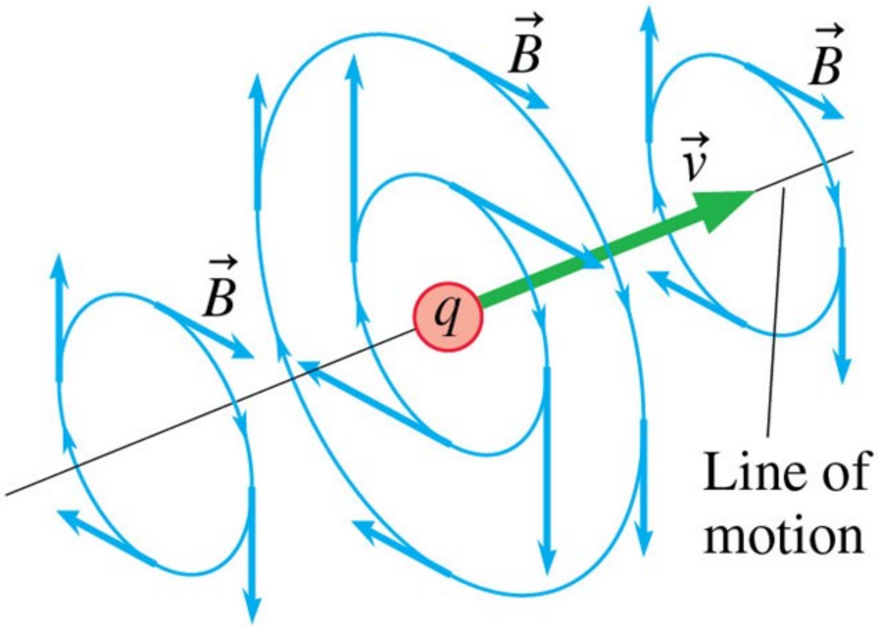


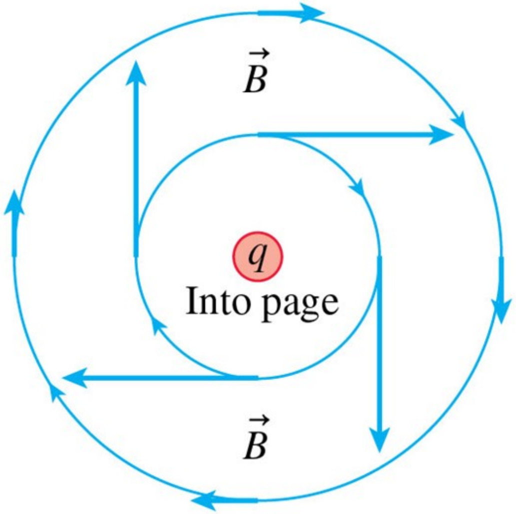
The field is weaker farther from the wire.



Sources of Magnetic Fields

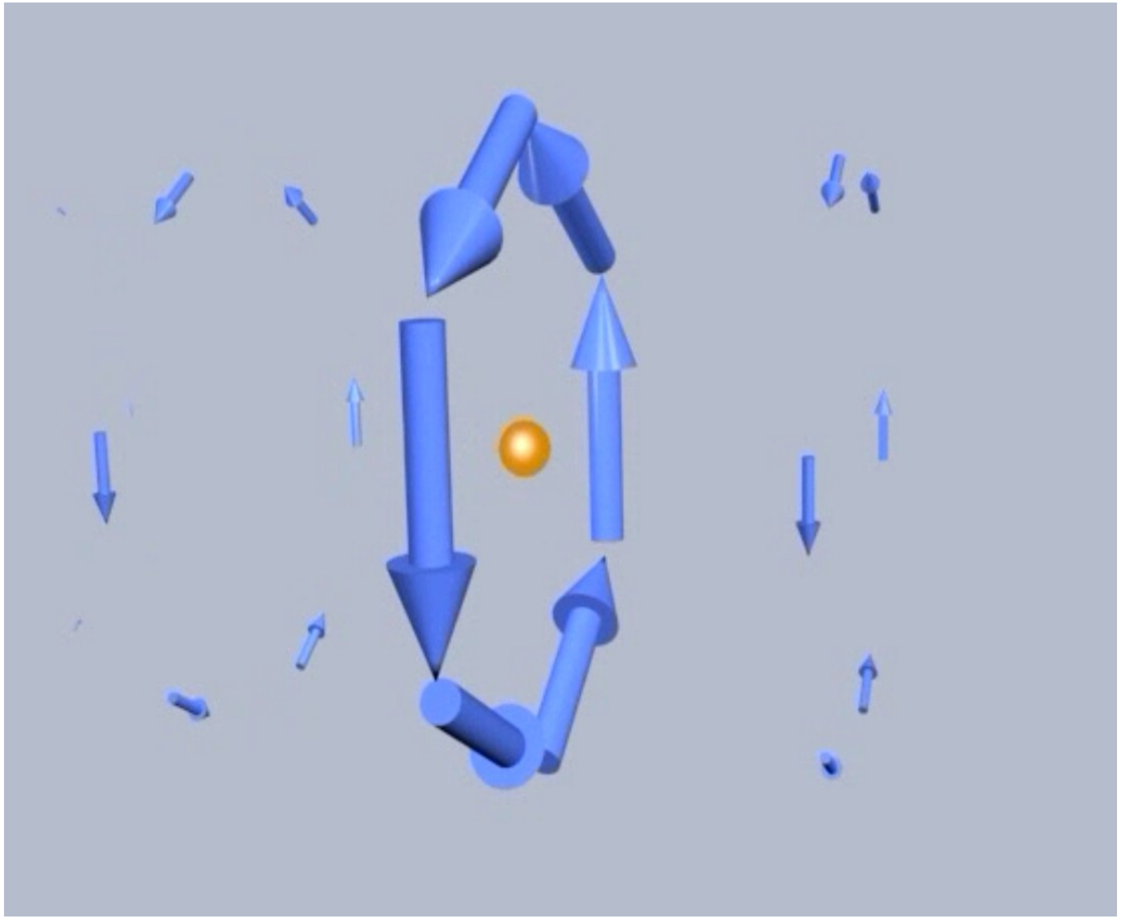
field of a single moving charge





Sources of Magnetic Fields

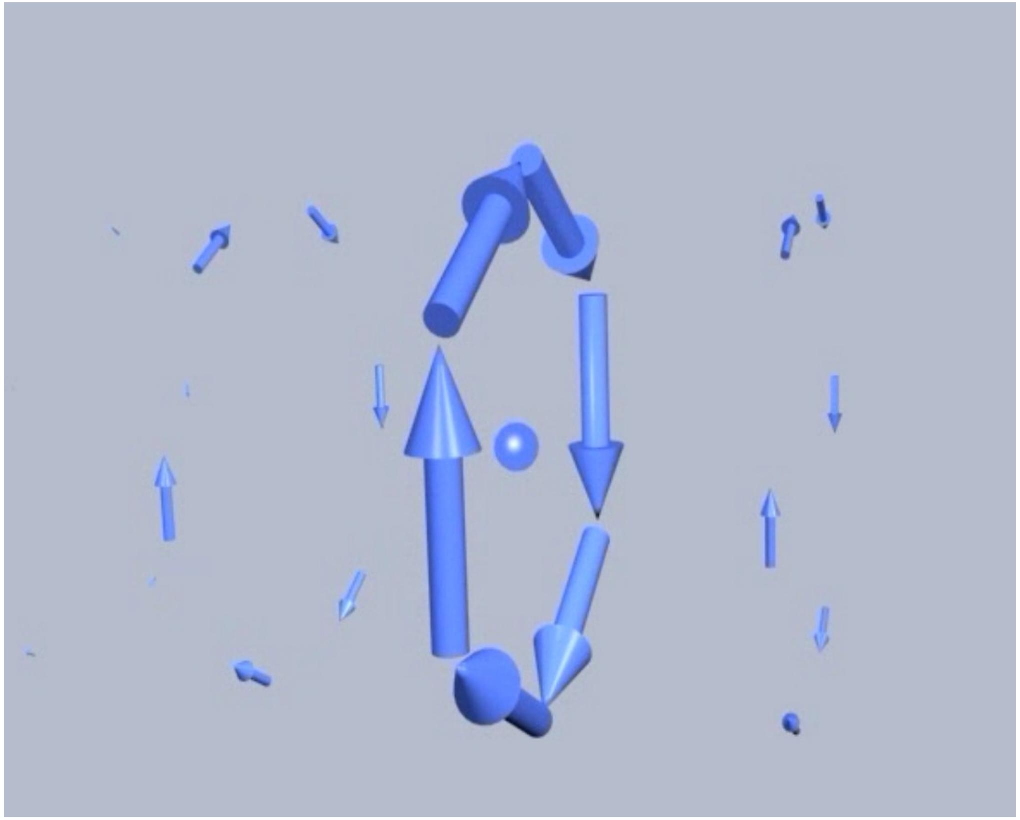
positive charge
moving right



https://web.mit.edu/8.02t/www/802TEAL3D/visualizations/magnetostatics/index.htm

Sources of Magnetic Fields

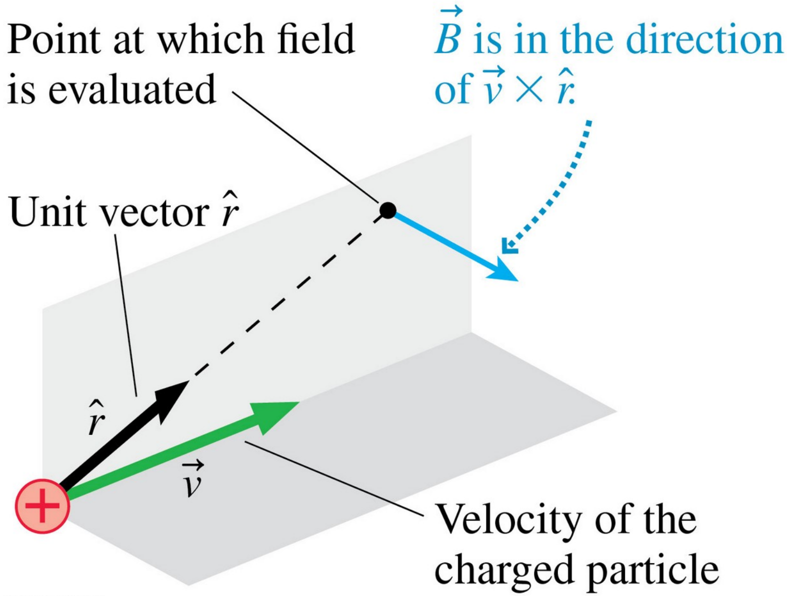
negative charge
moving right



Sources of Magnetic Fields

- The magnetic field of a charged particle q moving with velocity v is given by the **Biot-Savart law**:

$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$



Sources of Magnetic Fields

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

The constant μ_0 in the Biot-Savart law is called the **permeability constant**:

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\ m/A} = 1.257 \times 10^{-6}\ \mathrm{T\ m/A}$$

The constant μ_0 is often found in the fraction $\frac{\mu_0}{4\pi}$

$$\frac{\mu_0}{4\pi} = 10^{-7}\ \ \mathrm{T\ m/A}$$

Sources of Magnetic Fields

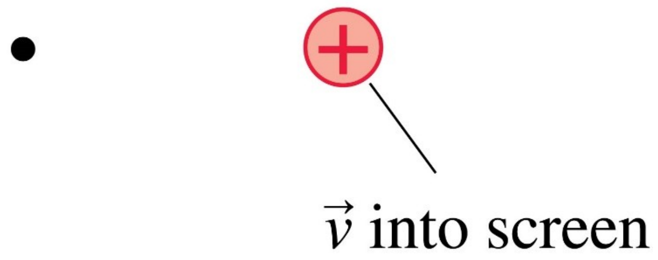
- Magnetic fields obey the principle of superposition.
- If there are *n* moving point charges, the net magnetic field is given by the vector sum:

$$\vec{B}_{\text{total}} = \vec{B}_1 + \vec{B}_2 + \cdots + \vec{B}_n$$

Sources of Magnetic Fields

What is the direction of the magnetic field at the position of the dot?

A. Into the screen
B. Out of the screen
C. Up
D. Down
E. Left



Sources of Magnetic Fields

What is the direction of the magnetic field at the position of the dot?

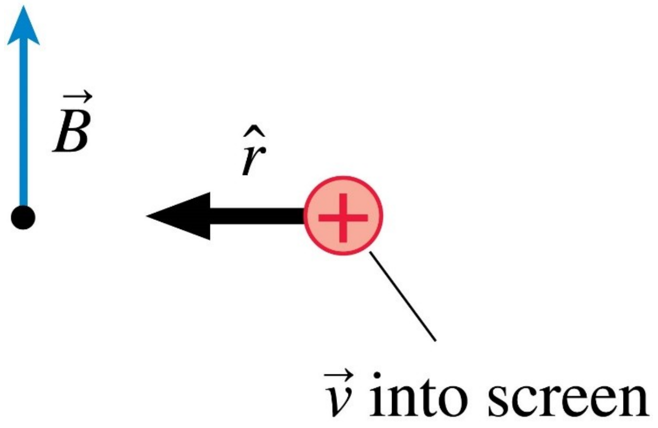
A. Into the screen

B. Out of the screen

✔ C. **Up** Direction of $\vec{v} \times \hat{r}$

D. Down

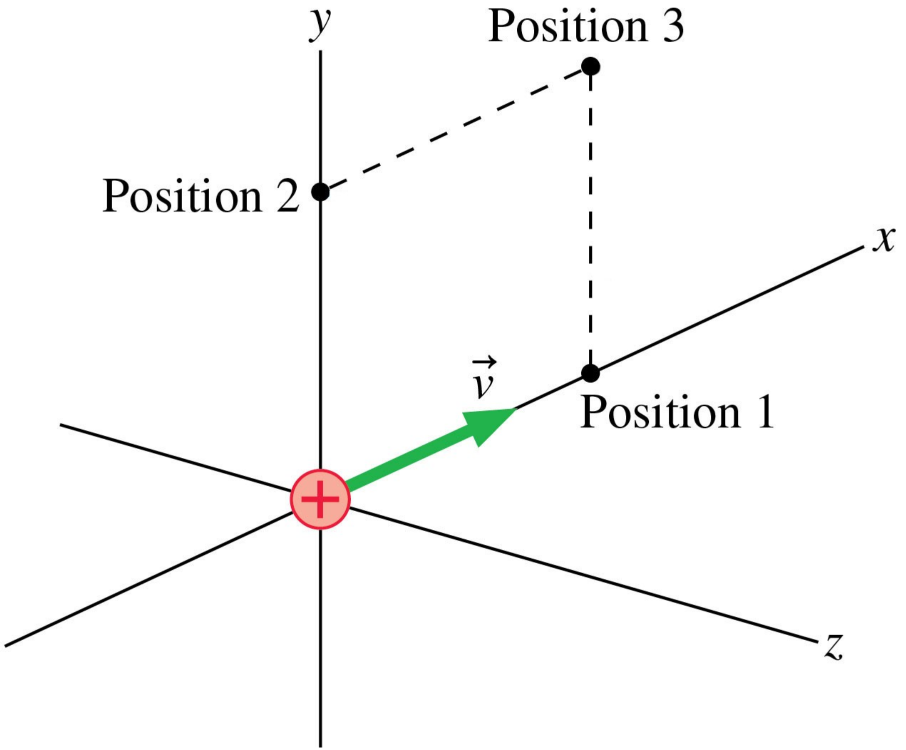
E. Left



Sources of Magnetic Fields

A proton moves with velocity $\vec{v} = 1.0 \times 10^7\ \hat{\imath}$ m/s. As it passes the origin, what is the magnetic field at the (x, y, z) positions (1 mm, 0 mm, 0 mm), (0 mm, 1 mm, 0 mm), and (1 mm, 1 mm, 0 mm)?

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$$



At position 1:

$\vec{B}_1 = 0$

Sources of Magnetic Fields

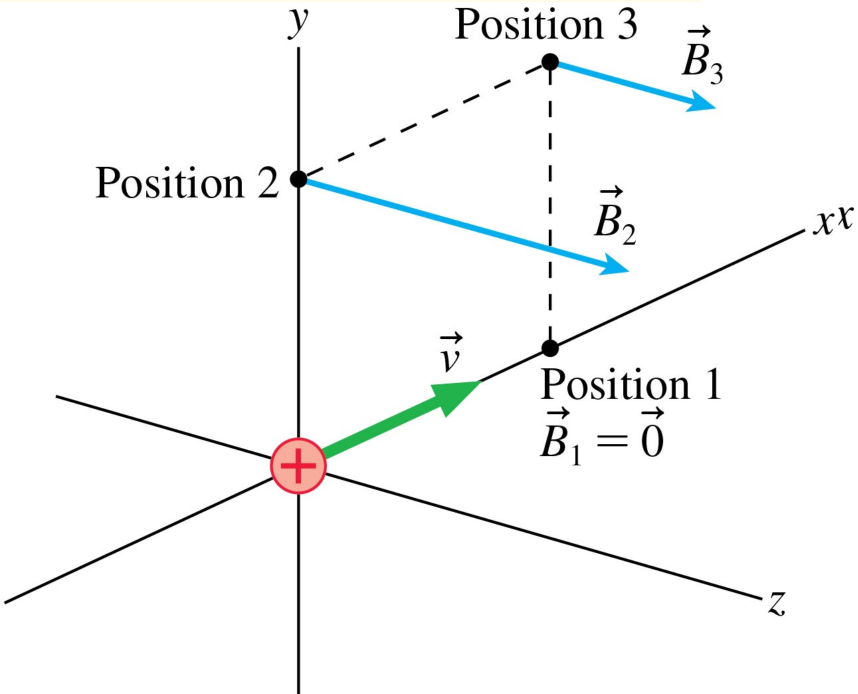
At position 2:

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\theta_2}{r_2{}^2}$$

$$= \frac{4\pi \times 10^{-7}\ \mathrm{T\,m/A}}{4\pi}\frac{(1.60 \times 10^{-19}\ \mathrm{C})(1.0 \times 10^{7}\ \mathrm{m/s})\sin 90^\circ}{(0.0010\ \mathrm{m})^2}$$

$$= 1.60 \times 10^{-13}\ \mathrm{T}$$

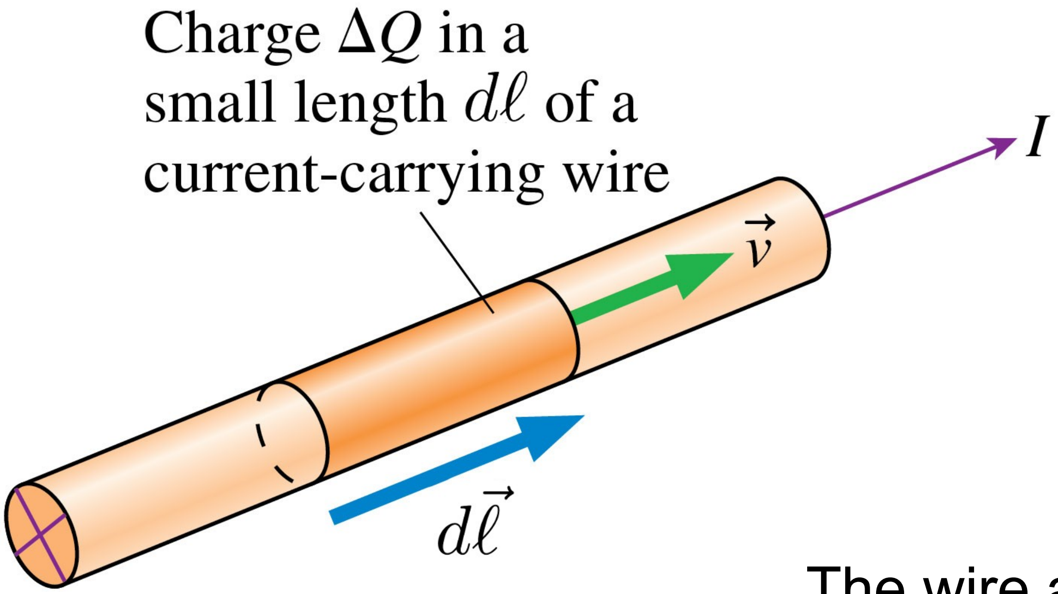
$$\vec{B}_2 = 1.60 \times 10^{-13}\ \hat{k}\ \mathrm{T}$$



At position 3:

$$\vec{B}_3 = 0.57 \times 10^{-13}\ \hat{k}\ \mathrm{T}$$

Sources of Magnetic Fields

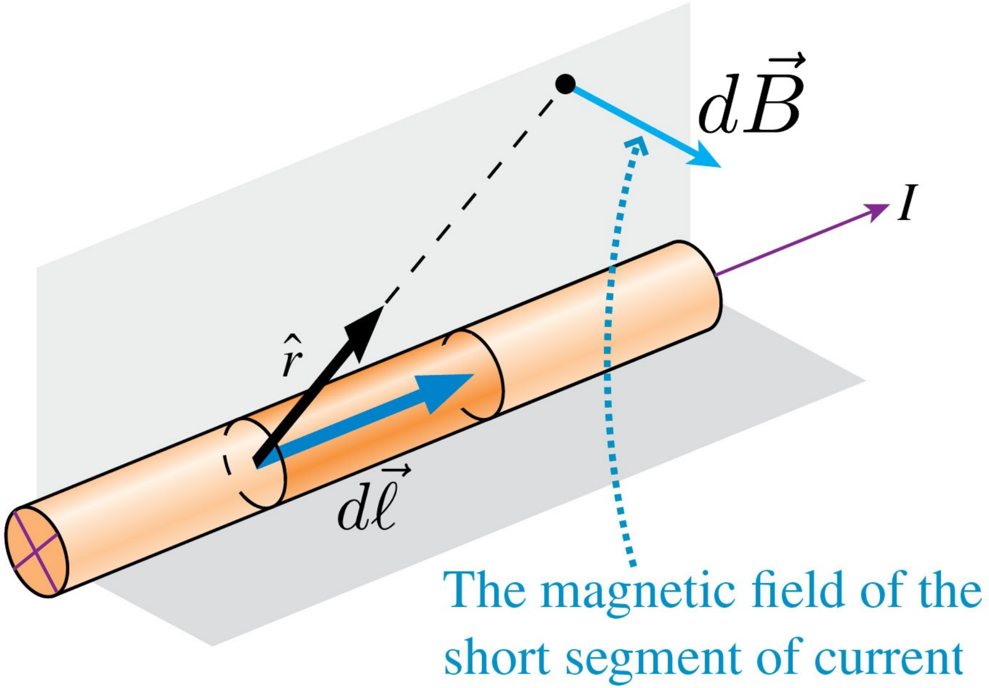


The wire as a whole is electrically neutral, but current I represents the motion of positive charge carriers through the wire:

$$(dQ)\vec{v} = dQ\frac{d\vec{\ell}}{dt} = \frac{dQ}{dt}d\vec{\ell} = I\,d\vec{\ell}$$

Sources of Magnetic Fields

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{\ell} \times \hat{r}}{r^2}$$

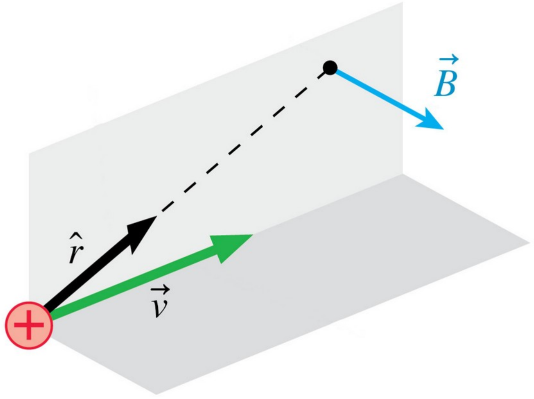


Sources of Magnetic Fields

Biot-Savart Law

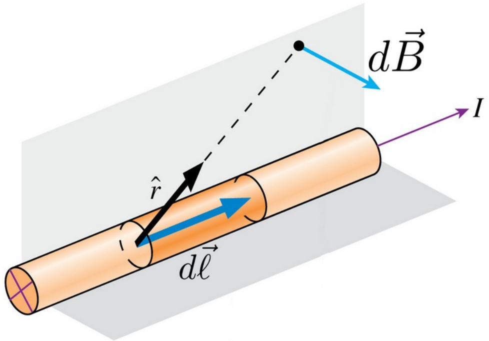
for a moving point charge

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$$



for a short segment of current

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{\ell} \times \hat{r}}{r^2}$$



Sources of Magnetic Fields

Use the Biot-Savart Law to find the field a distance z away from a long straight wire carrying a current I.

To solve integrals of the form $\int (x^2 + a^2)^{-3/2} dx$ you should consult an integral table or check Wolfram Alpha.

answer: $\vec{B} = \frac{\mu_0 I}{2\pi z}\hat{\theta}$

where $\hat{\theta}$ is the cylindrical unit vector

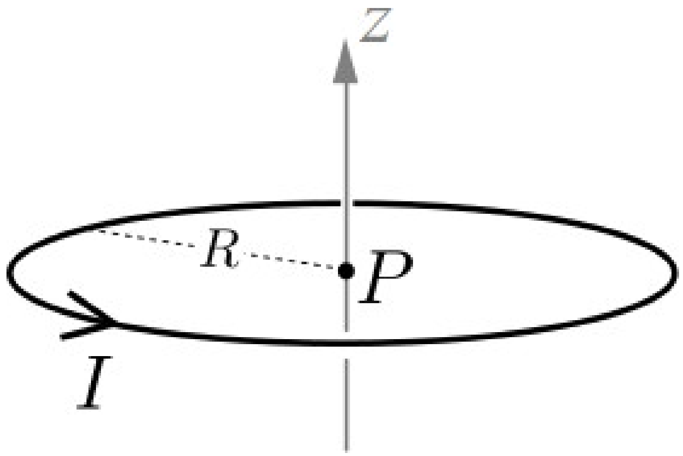
Sources of Magnetic Fields

- The magnetic field of a long, straight wire carrying current I at a distance r from the wire is

$$B_{\text{wire}} = \frac{\mu_0}{2\pi}\frac{I}{r} \quad \text{(long, straight wire)}$$

Sources of Magnetic Fields

Find the magnetic field at the center of a loop of current I.

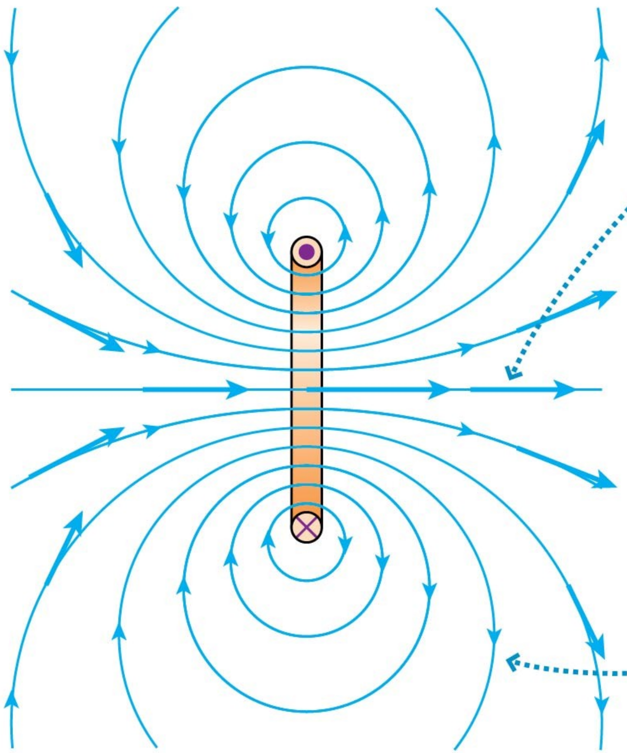


$$\vec{B}(P) = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$$

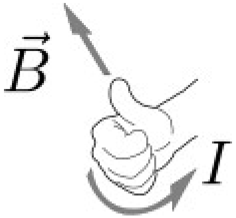
$$\vec{B}(P) = \frac{\mu_0 I}{2R} \hat{k}$$

Sources of Magnetic Fields

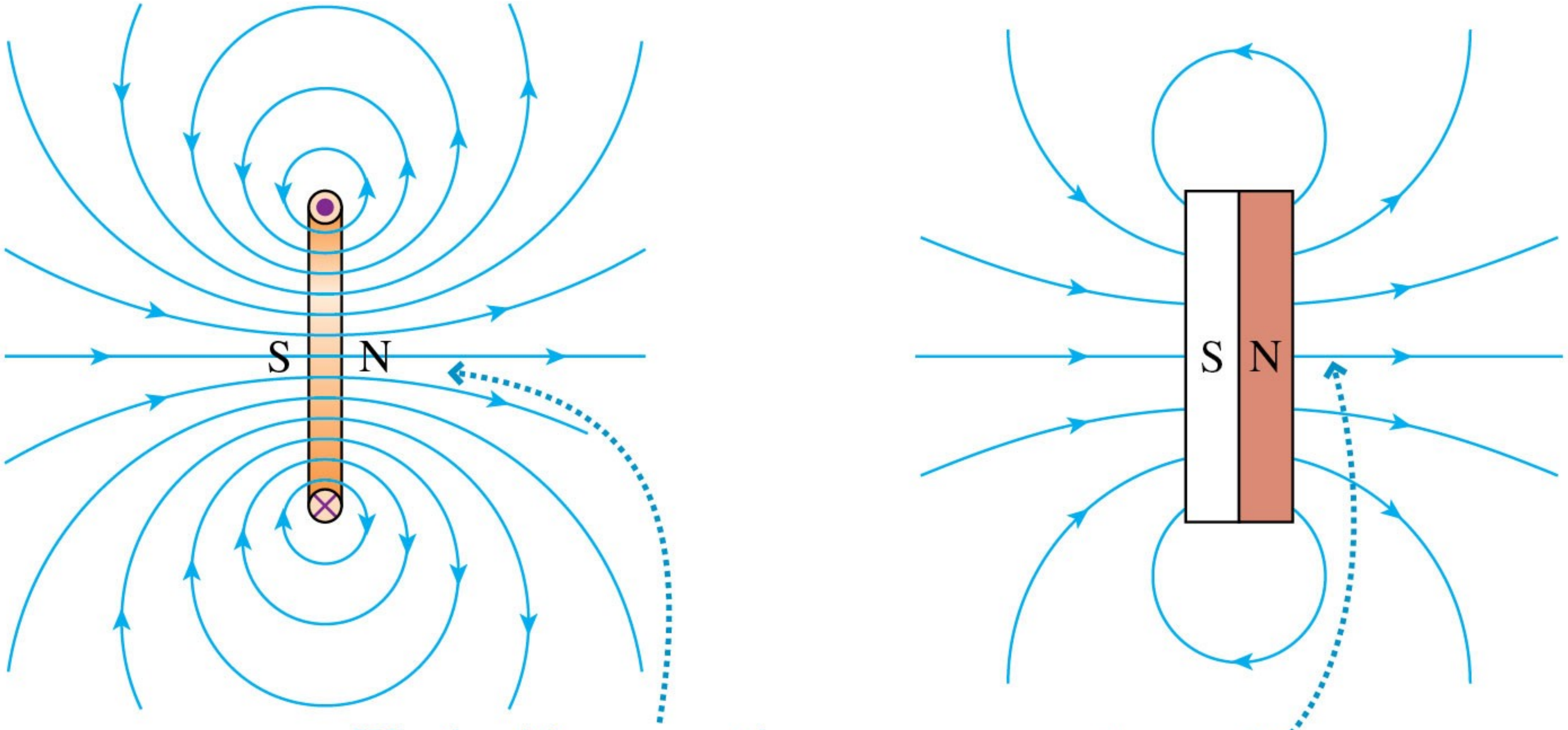
Cross section through the current loop



right-hand rule for loops



Sources of Magnetic Fields

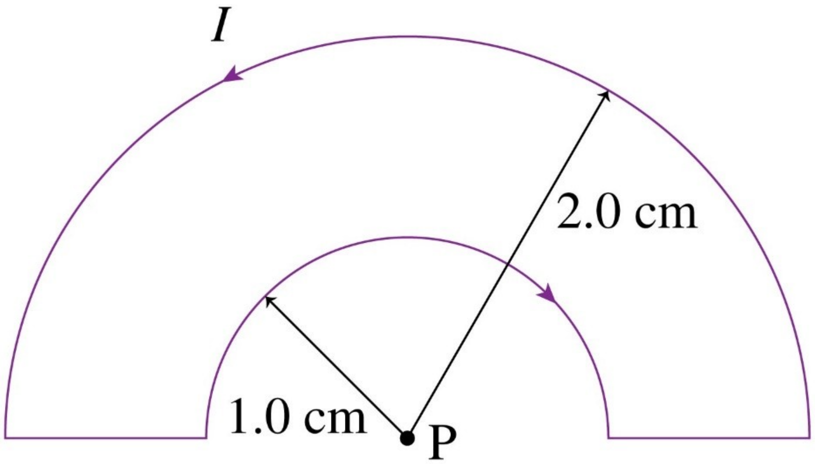


Whether it's a current loop or a permanent magnet, the magnetic field emerges from the north pole.

Sources of Magnetic Fields

The magnet field at point P is

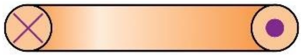
✔ A. Into the screen.
B. Out of the screen.
C. Zero.



Sources of Magnetic Fields

Where is the north magnetic pole of this current loop?

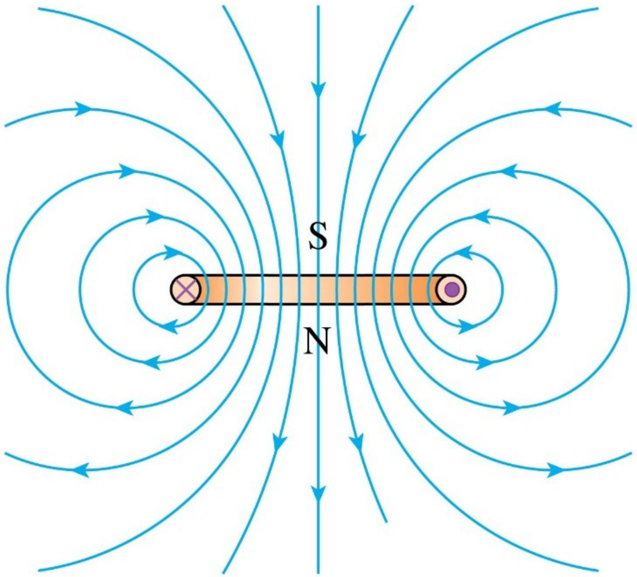
A. Top side.
B. Bottom side.
C. Right side.
D. Left side.
E. Current loops don't have north poles.



Sources of Magnetic Fields

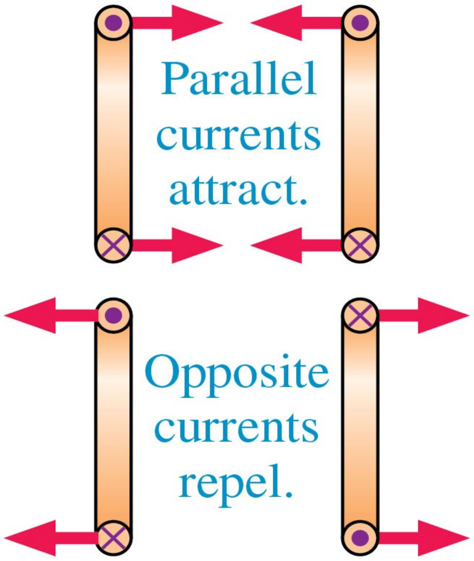
Where is the north magnetic pole of this current loop?

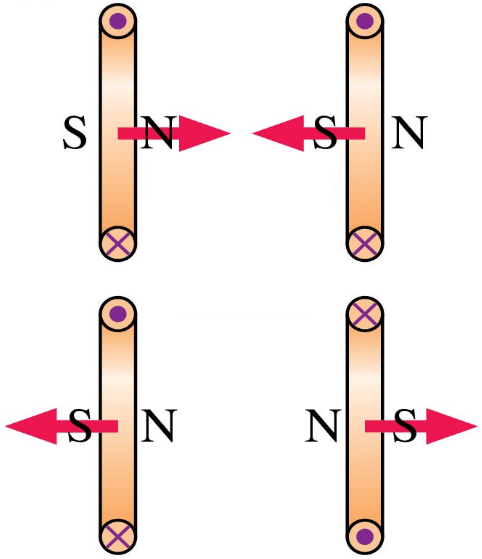
A. Top side.
✔ B. Bottom side.
C. Right side.
D. Left side.
E. Current loops don't have north poles.



Forces Between Current-Carrying Wires

two equivalent ways to view magnetic forces between two current loops.





Parallel currents attract, opposite currents repel.

Opposite poles attract, like poles repel.

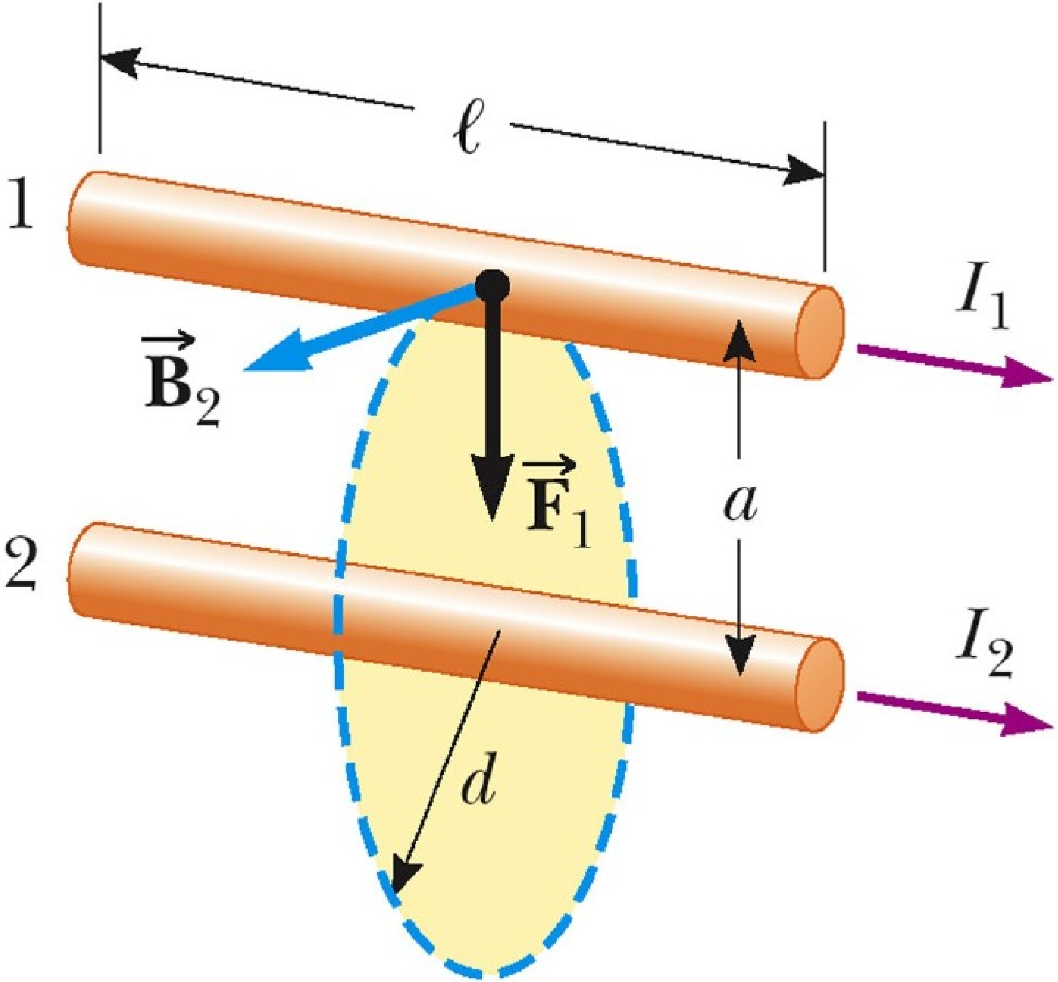
Forces Between Current-Carrying Wires

- A current I_2 creates a magnetic field.

$$B_2 = \frac{\mu_0 I_2}{2\pi r}$$

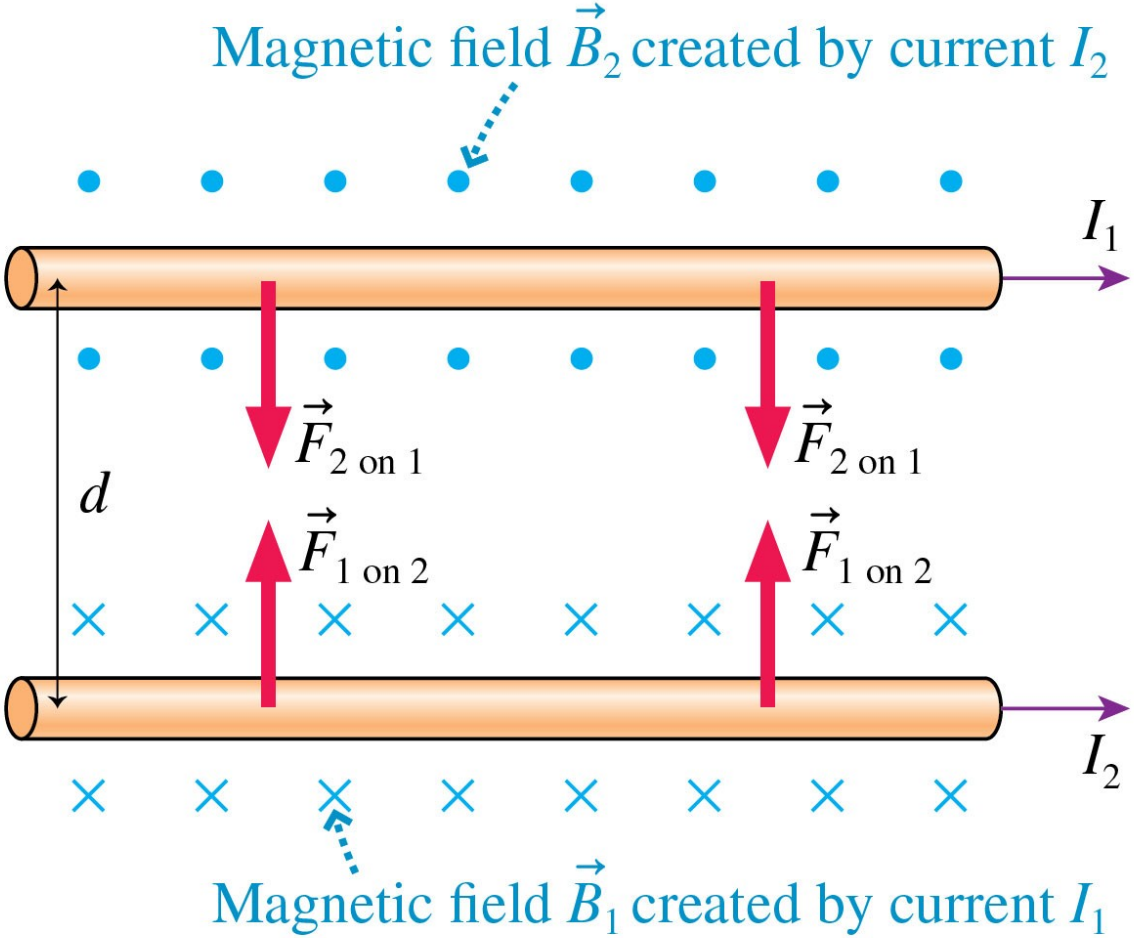
- A current I_1 in that magnetic field feels a force.

$$F_1 = B_2 I_1 \ell$$



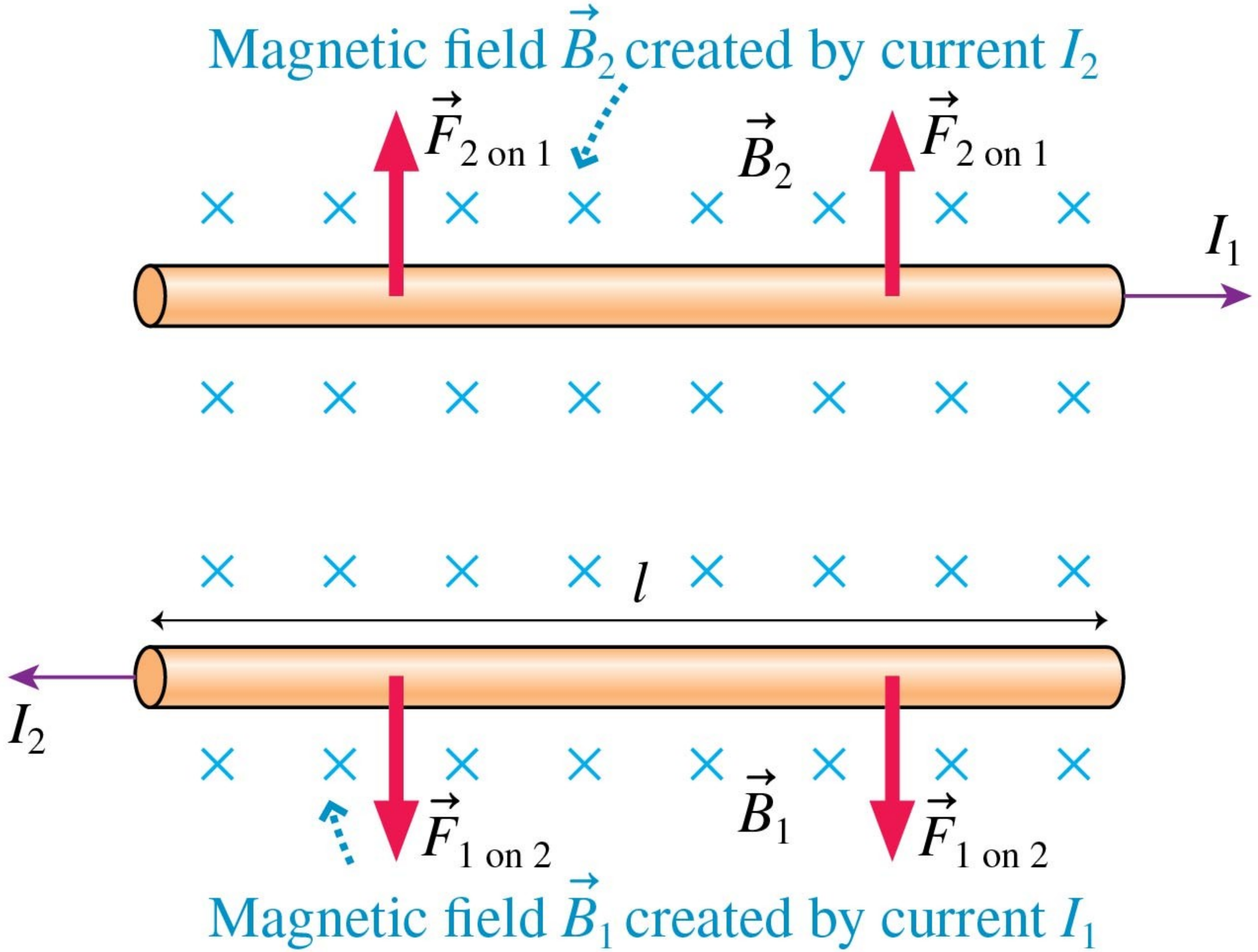
Forces Between Current-Carrying Wires

$$F = \frac{\mu_0 \ell I_1 I_2}{2\pi d}$$

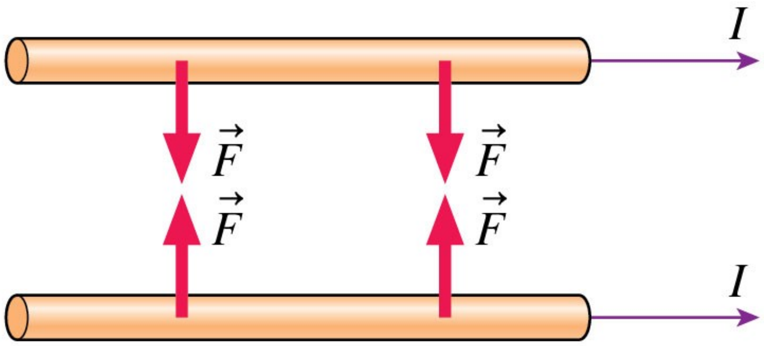


Forces Between Current-Carrying Wires

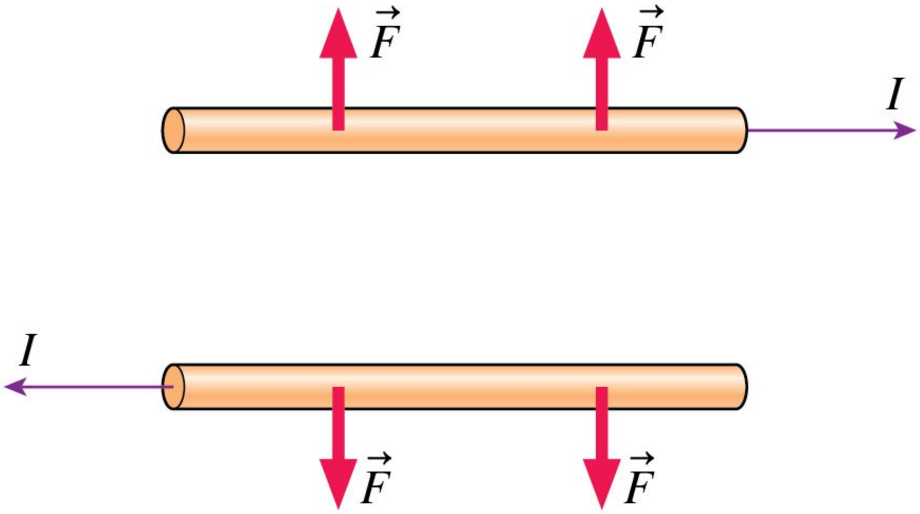
$$F = \frac{\mu_0 \ell I_1 I_2}{2\pi d}$$



Forces Between Current-Carrying Wires

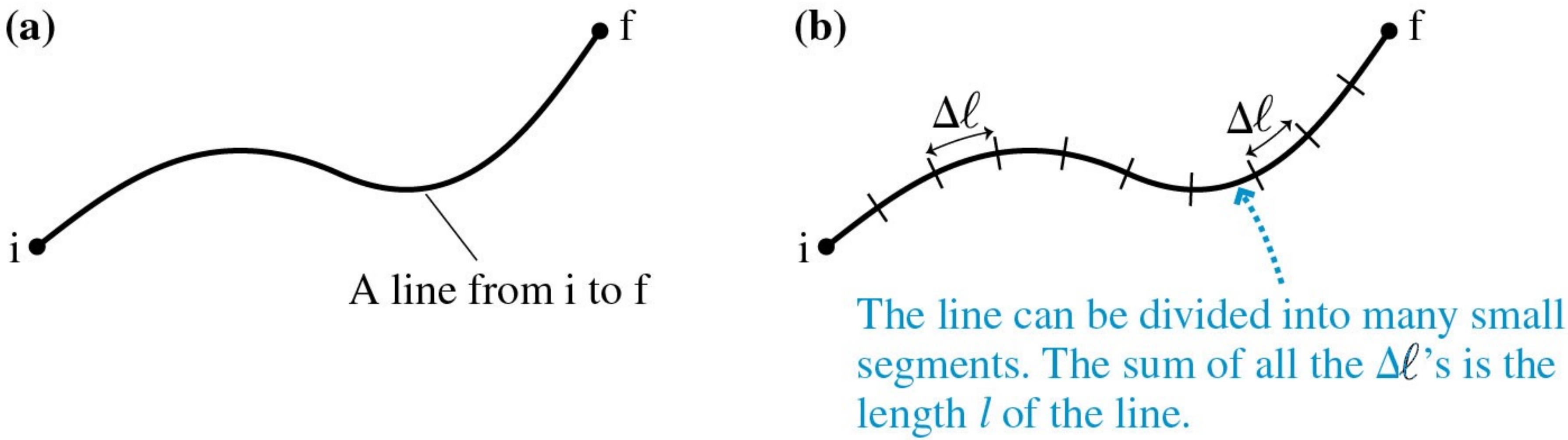


“Like” currents attract.



“Opposite” currents repel.

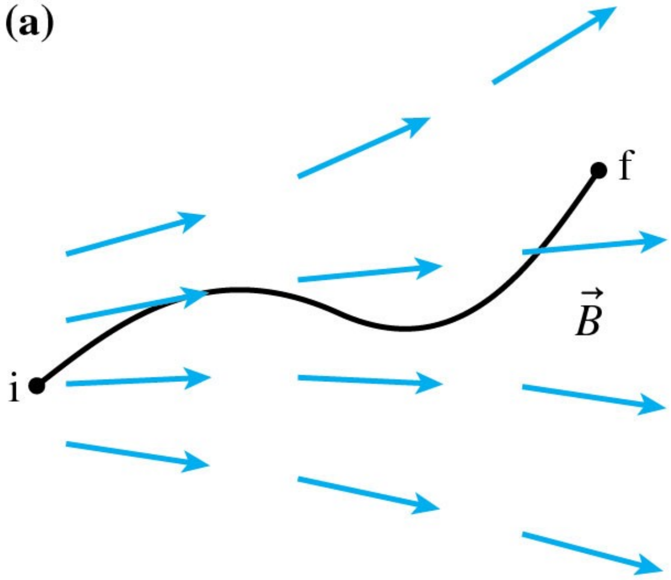
Ampère’s Law

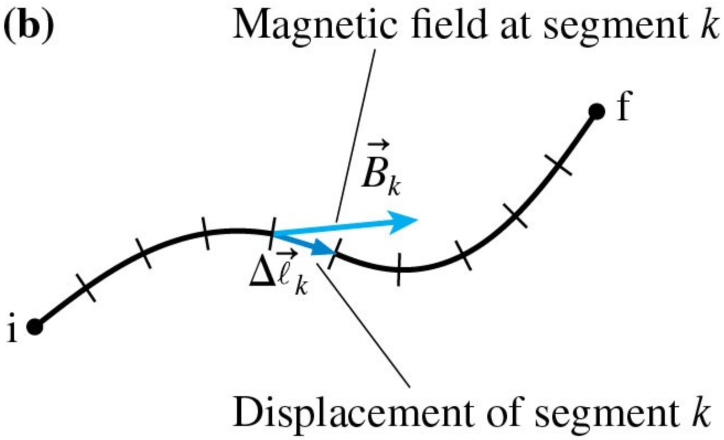


- a curved line from i to f.
- The length L of this line can be found by doing a path integral:

$$L = \sum_k \Delta\ell_k \to \int_i^f d\ell$$

Ampère’s Law





- Suppose the line passes through a B field
- we will need the path integral

$$\sum_k \vec{B}_k \cdot \Delta\vec{\ell}_k \rightarrow \int_i^f \vec{B} \cdot d\vec{\ell}$$

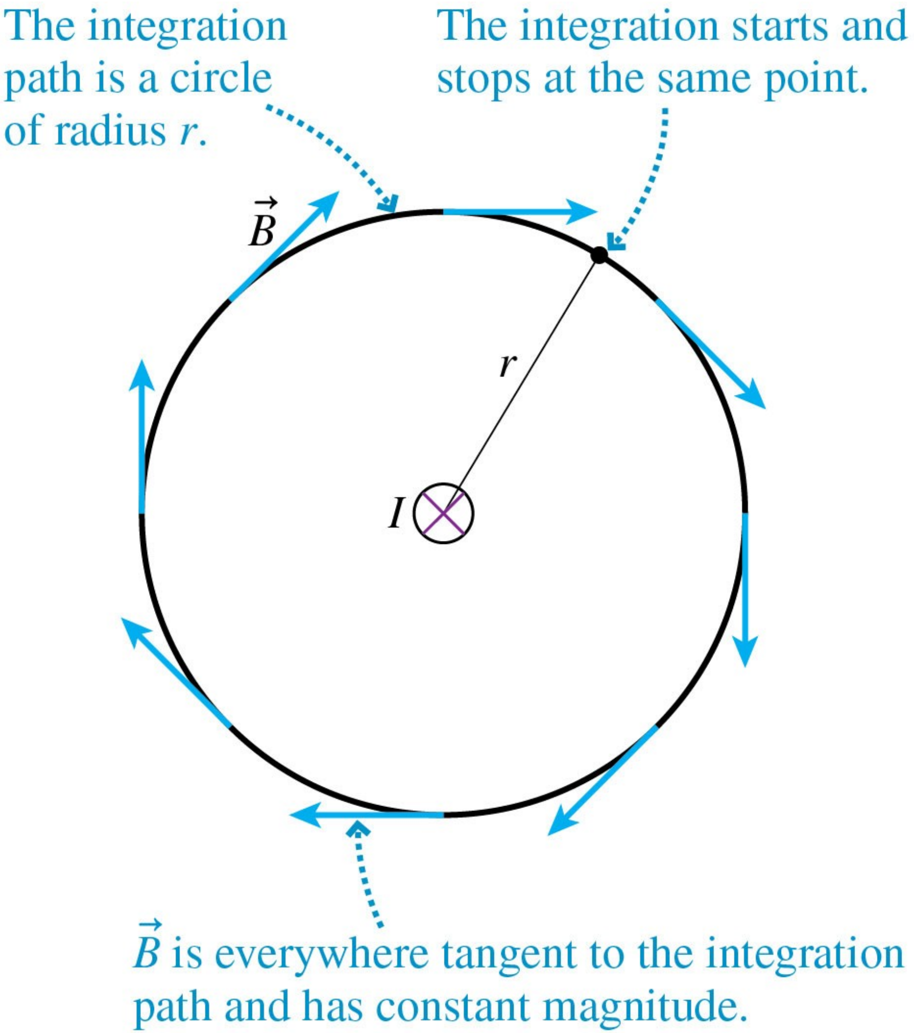
Ampère's Law

If the field is constant along the path and tangent to it:

$$\int_i^f \vec{B} \cdot d\vec{\ell} = B \int d\ell = BL$$

If the path is a closed loop:

$$\int_i^f \vec{B} \cdot d\vec{\ell} = \oint \vec{B} \cdot d\vec{\ell}$$



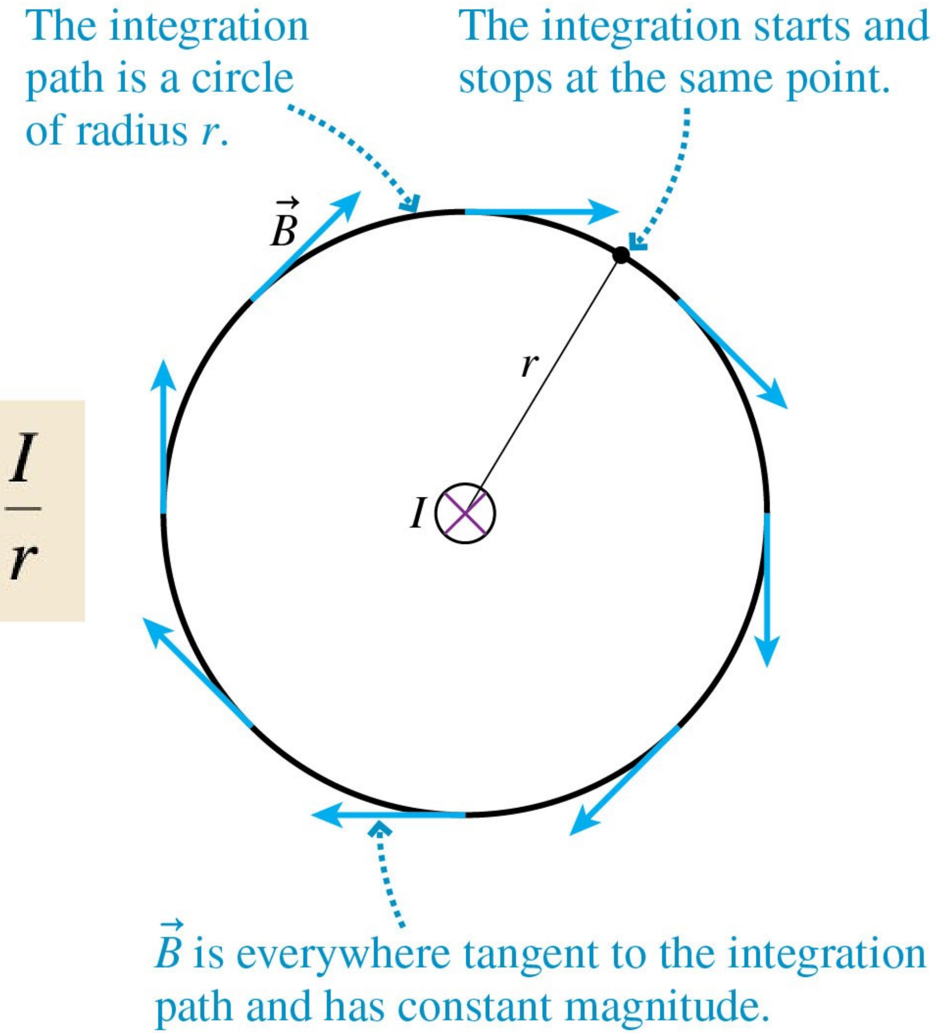
Ampère's Law

For the circle around current I

$$\oint \vec{B} \cdot d\vec{\ell} = B(2\pi r)$$

We know the field here is $B_{\text{wire}} = \frac{\mu_0}{2\pi} \frac{I}{r}$

$$\oint \vec{B} \cdot d\vec{\ell} = \frac{2\pi r \mu_0 I}{2\pi r} = \mu_0 I$$



Ampère's Law

This turns out to be true for any closed path.

It's called Ampère's law:

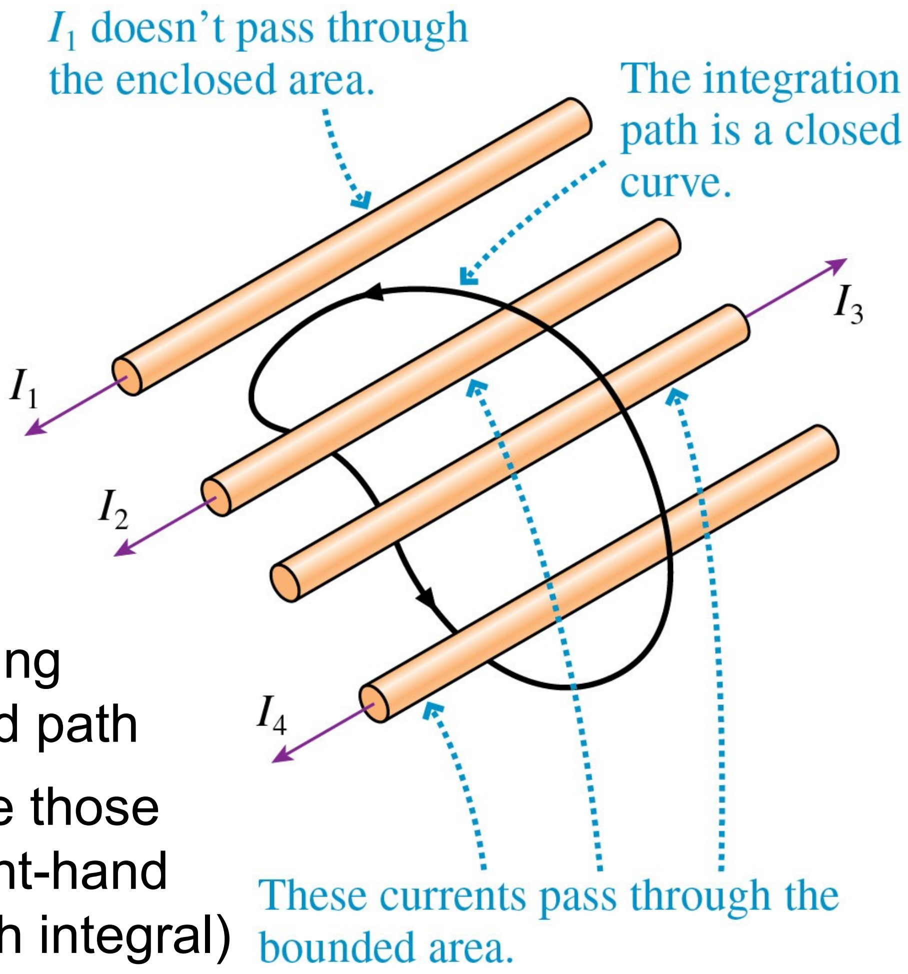
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{through}}$$

closed path integral

permeability constant

total current passing through the closed path

(positive currents are those corresponding to right-hand circulation of the path integral)



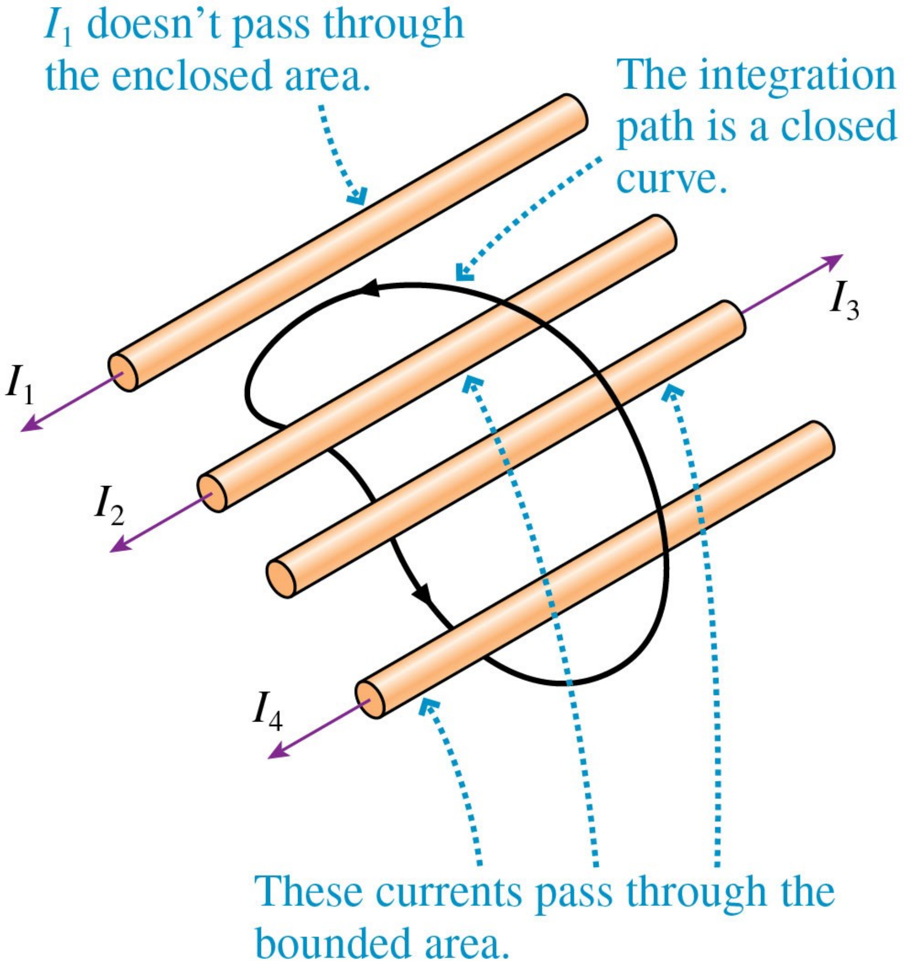
Ampère's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{thru}}$$

For I_{thru} we define positive currents as those corresponding to right-hand circulation of the path integral.

For the figure shown,

$$I_{\text{thru}} = I_2 - I_3 + I_4$$

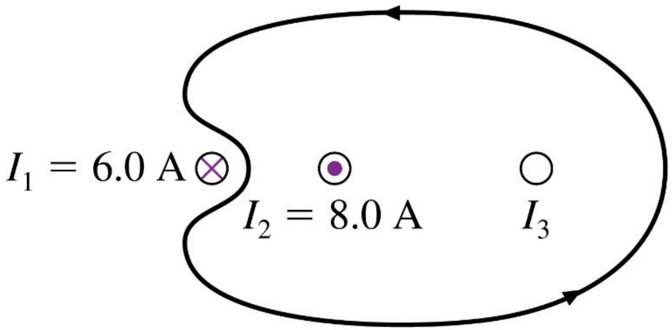


Ampère's Law

The line integral of B around the loop is $\mu_0 \cdot 7.0$ A.

Current I_3 is

A. 0 A.
B. 1 A out of the screen.
C. 1 A into the screen.
D. 5 A out of the screen.
E. 5 A into the screen.



$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{thru}}$$

Ampère's Law

The line integral of B around the loop is $\mu_0 \cdot 7.0$ A.

Current I_3 is

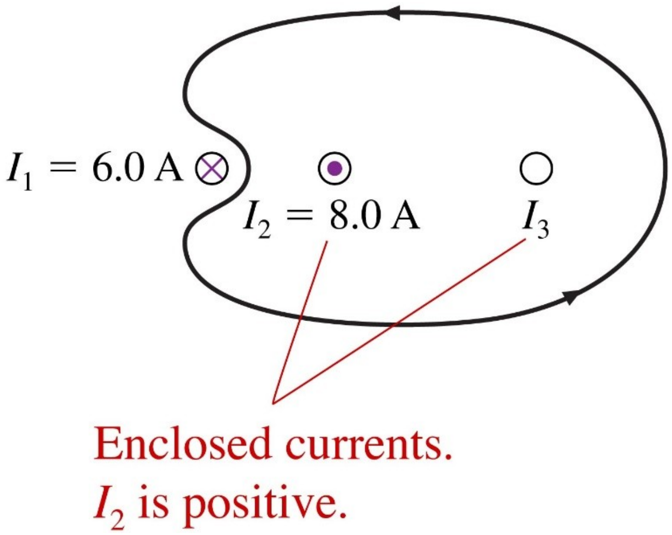
A. 0 A.

B. 1 A out of the screen.

✔ **C. 1 A into the screen.**

D. 5 A out of the screen.

E. 5 A into the screen.



$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{thru}}$$

Ampère's Law

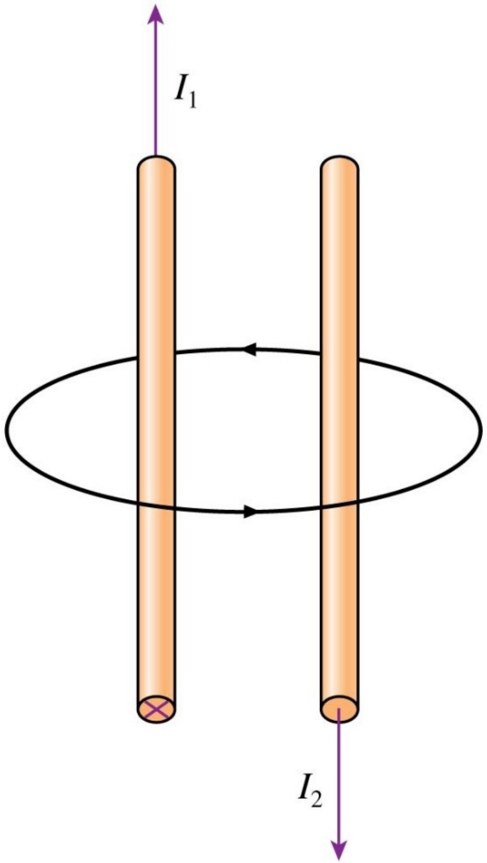
For the path shown, $\oint \vec{B} \cdot d\vec{\ell} =$

A. 0

✔ B. $\mu_0(I_1 - I_2)$

C. $\mu_0(I_2 - I_1)$

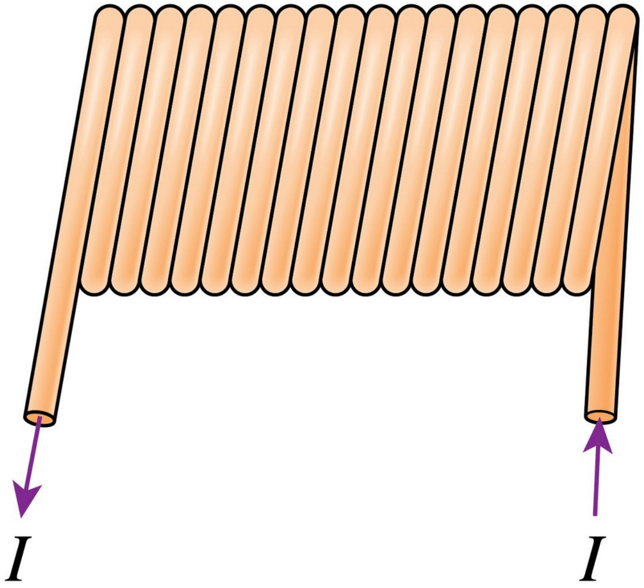
D. $\mu_0(I_1 + I_2)$

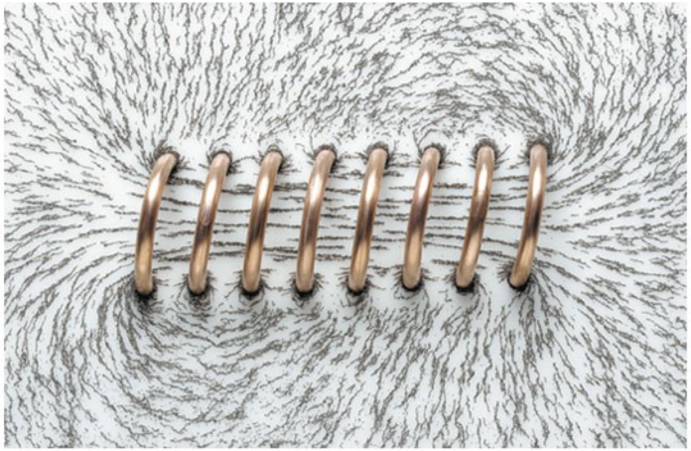


$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{thru}}$$

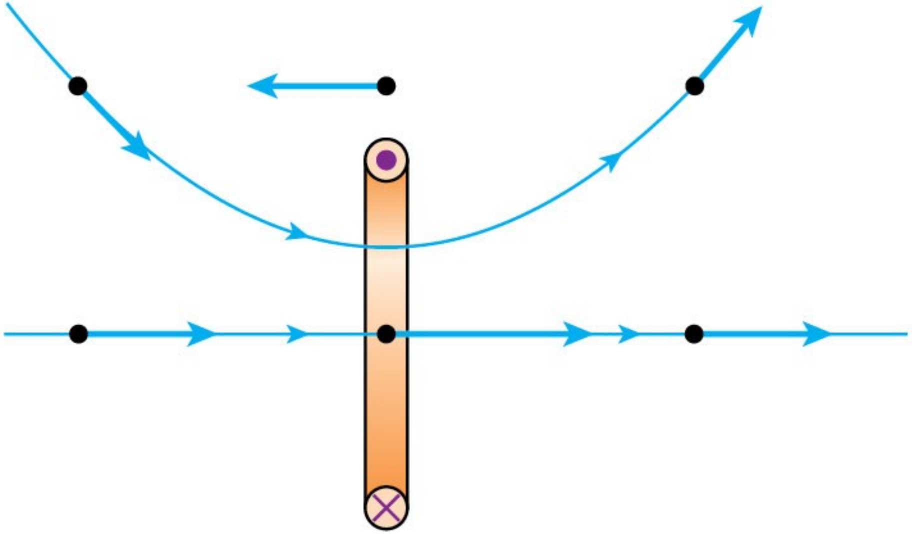
Magnetic Field of a Solenoid

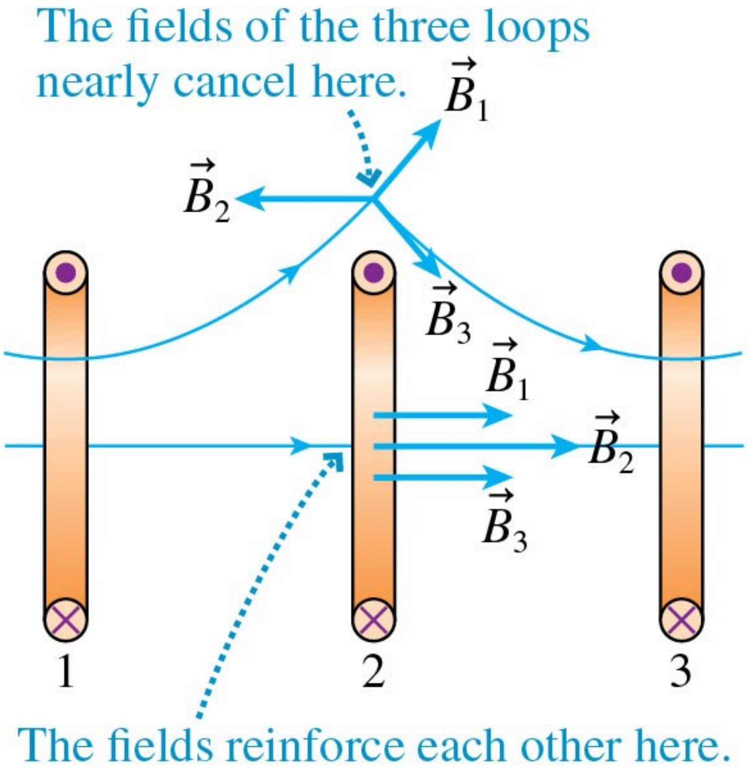
A **uniform magnetic field** can be generated with a **solenoid**.





Magnetic Field of a Solenoid

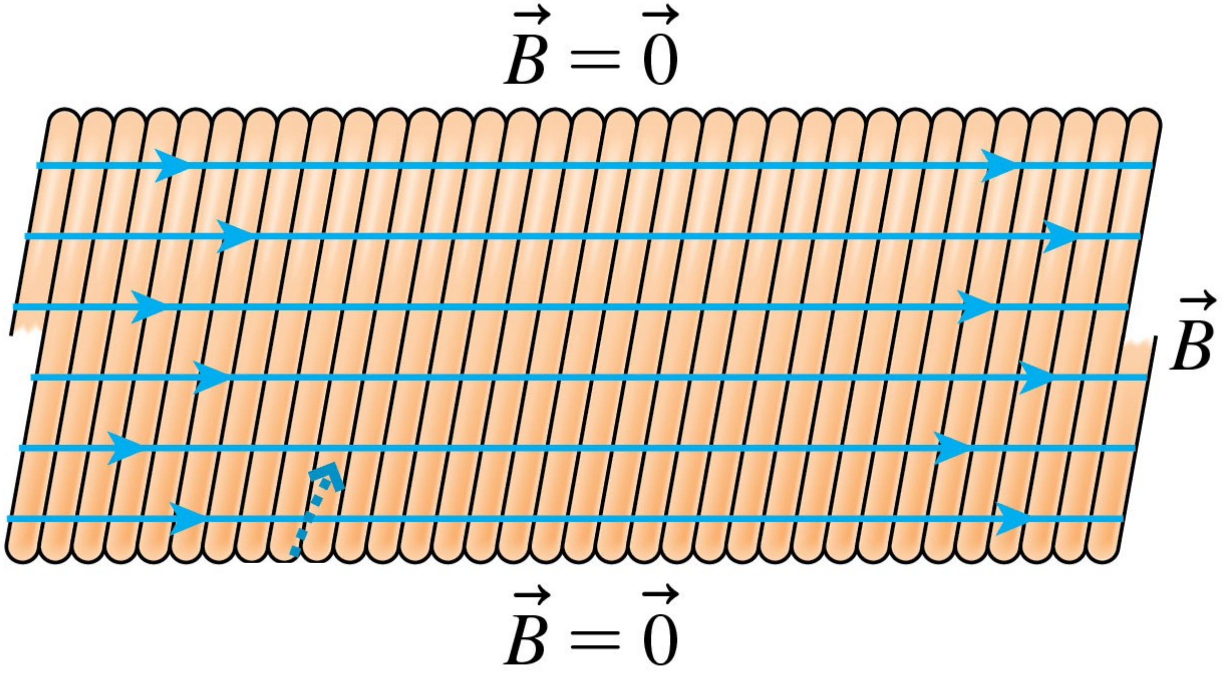




Magnetic Field of a Solenoid

For an "ideal solenoid"

- the field inside is strong, parallel to the axis, and uniform
- the field outside is close to zero.

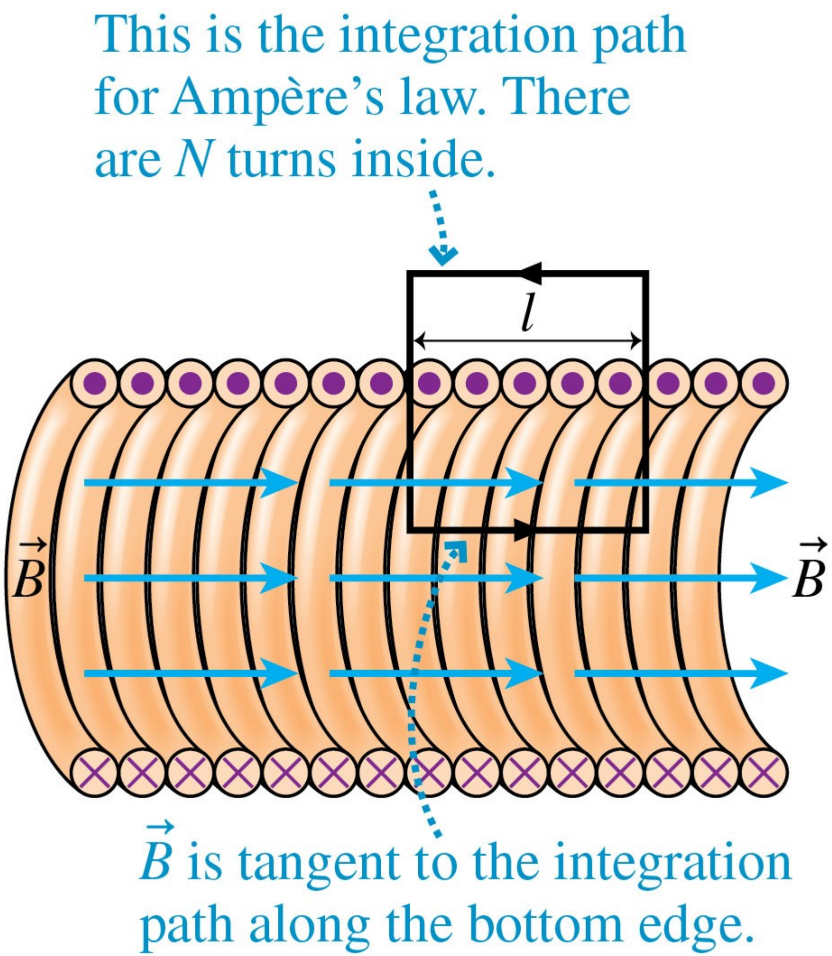


Magnetic Field of a Solenoid

The magnetic field strength can be found with Ampère's Law.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{thru}}$$

- The current passing through this rectangle is $I_{\text{thru}} = NI$.



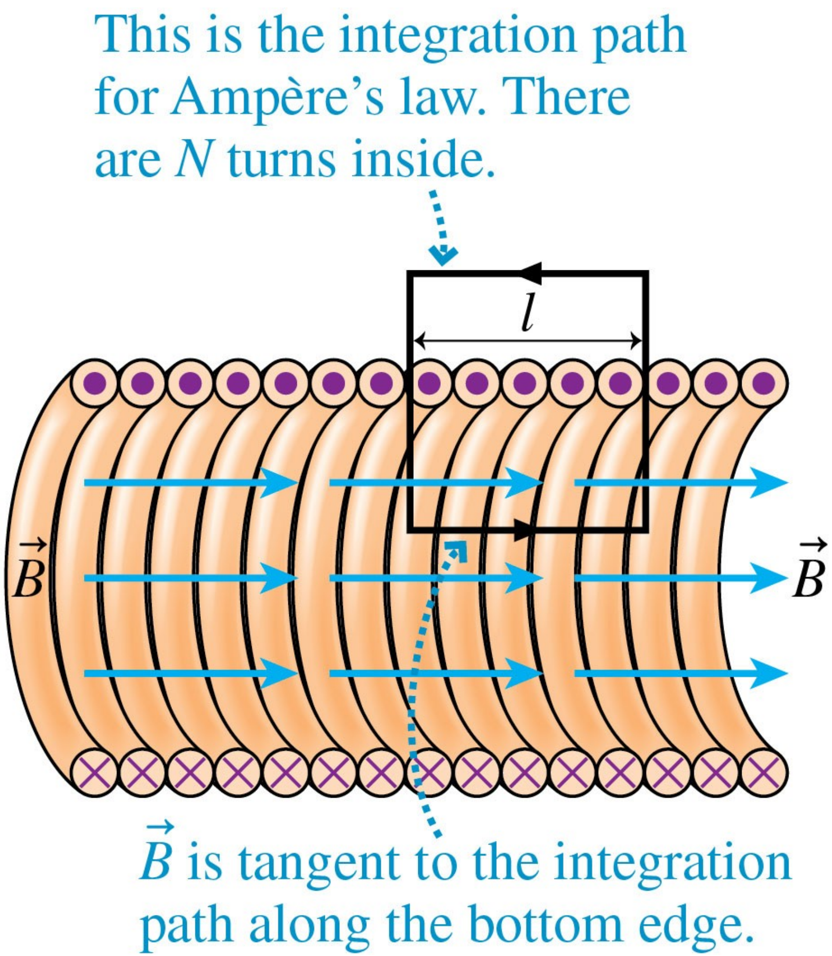
Magnetic Field of a Solenoid

- The line integral is

$$\oint \vec{B} \cdot d\vec{\ell} = B\ell$$

- Ampère's Law gives

$$B\ell = \mu_0 N I$$



Magnetic Field of a Solenoid

The uniform field in a solenoid is

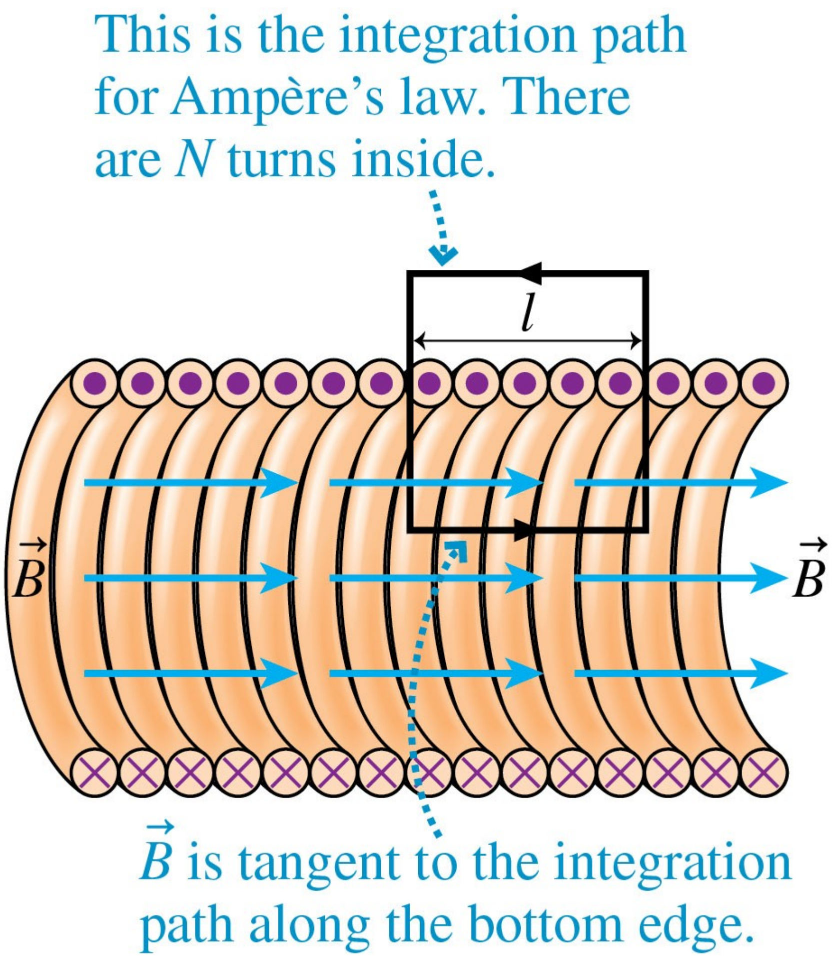
$$B_{\text{sol}} = \frac{\mu_0 N I}{\ell}$$

Sometimes written as

$$B_{\text{sol}} = \mu_0 n I$$

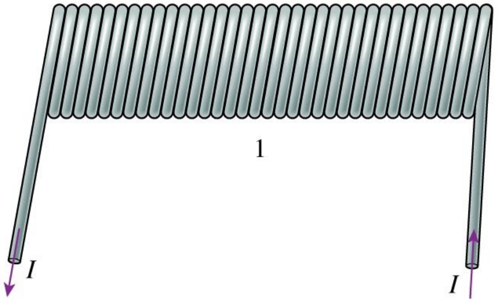
where n is the number of loops per meter

$$n = \frac{N}{\ell}$$



Magnetic Field of a Solenoid

Solenoid 2 has twice the diameter, twice the length, and twice as many turns as solenoid 1. How does the field B_2 at the center of solenoid 2 compare to B_1 at the center of solenoid 1?



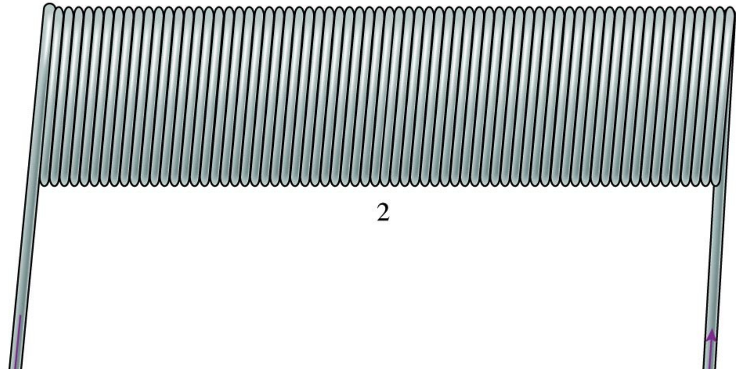
A. $B_2 = B_1/4$

B. $B_2 = B_1/2$

C. $B_2 = B_1$

D. $B_2 = 2B_1$

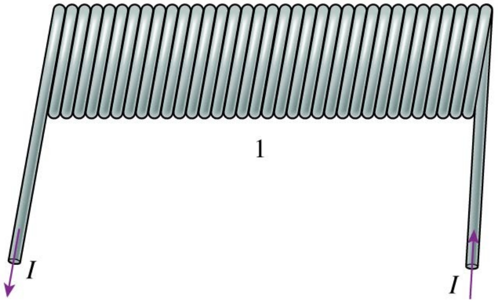
E. $B_2 = 4B_1$



$$B_{\text{sol}} = \mu_0 n I$$

Magnetic Field of a Solenoid

Solenoid 2 has twice the diameter, twice the length, and twice as many turns as solenoid 1. How does the field B_2 at the center of solenoid 2 compare to B_1 at the center of solenoid 1?



A. $B_2 = B_1/4$

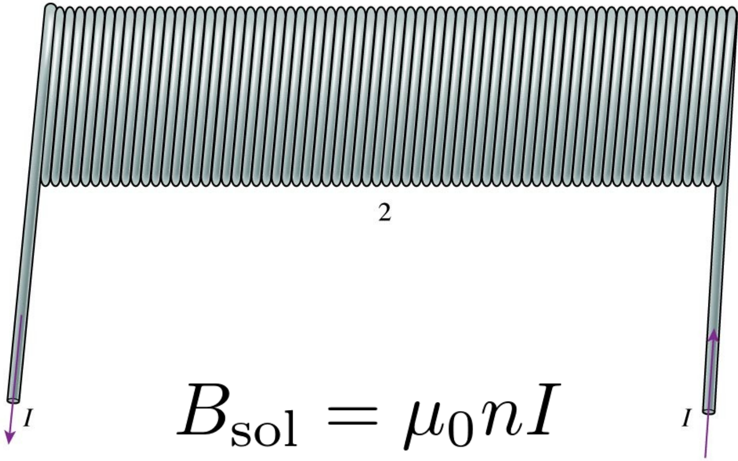
B. $B_2 = B_1/2$

✔ **C. $\boldsymbol{B_2 = B_1}$**

D. $B_2 = 2B_1$

E. $B_2 = 4B_1$

Same turns-per-length; diameter is irrelevant

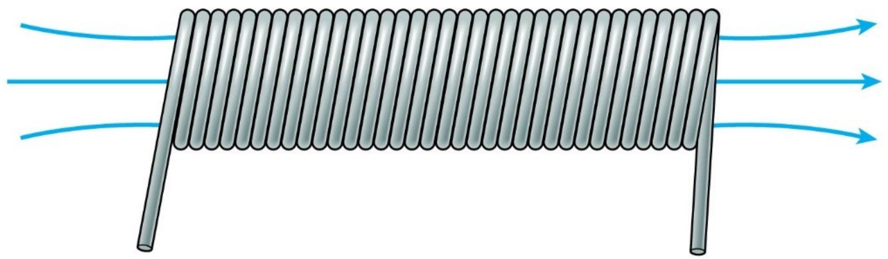


$$B_{\text{sol}} = \mu_0 n I$$

Magnetic Field of a Solenoid

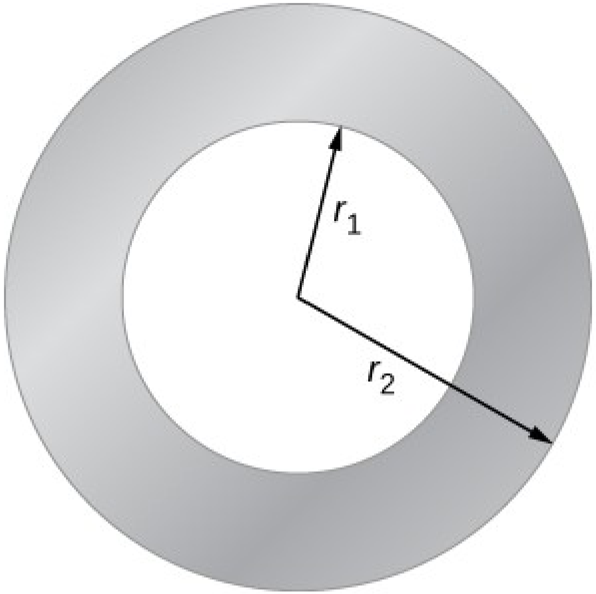
The current in this solenoid

A. Enters on the left, leaves on the right.
B. Enters on the right, leaves on the left.
C. Either A or B would produce this field.



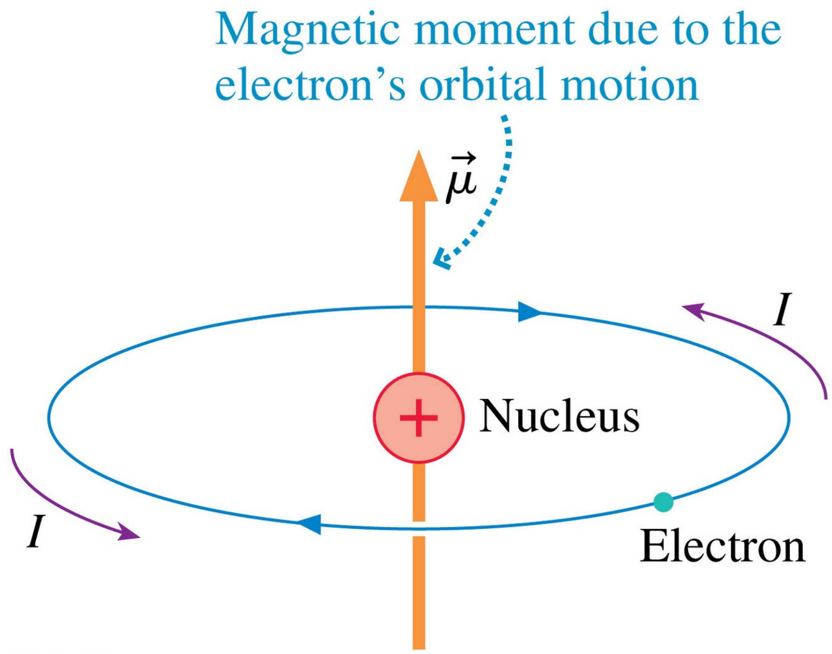
Ampère’s Law

The accompanying figure shows a cross-section of a long, hollow, cylindrical conductor of inner radius $r_1 = 3$ cm and outer radius $r_2 = 5.5$ cm. A 39-A current distributed uniformly over the cross-section flows into the page. Calculate the magnetic field at $r = 0.5$ cm, $r = 3.5$ cm, and $r = 10$ cm.



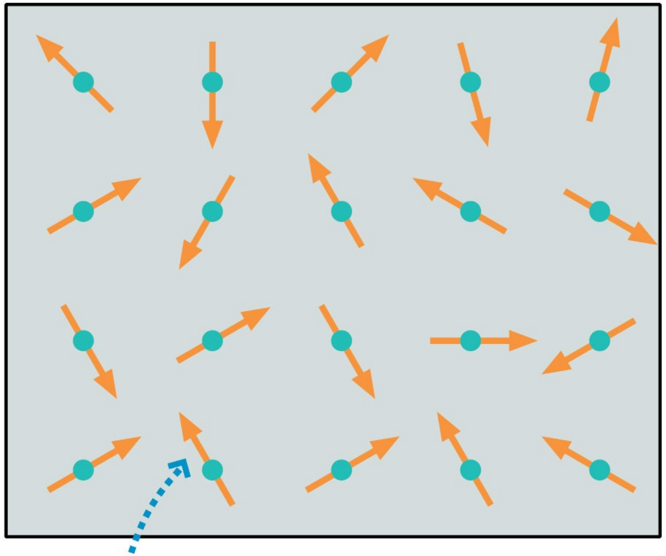
Magnetism in Matter

- A plausible explanation for the magnetic properties of materials is the orbital motion of the atomic electrons.
- In this picture of the atom, the electron's motion is that of a current loop.
- An orbiting electron acts as a tiny magnetic dipole, with a north pole and a south pole.



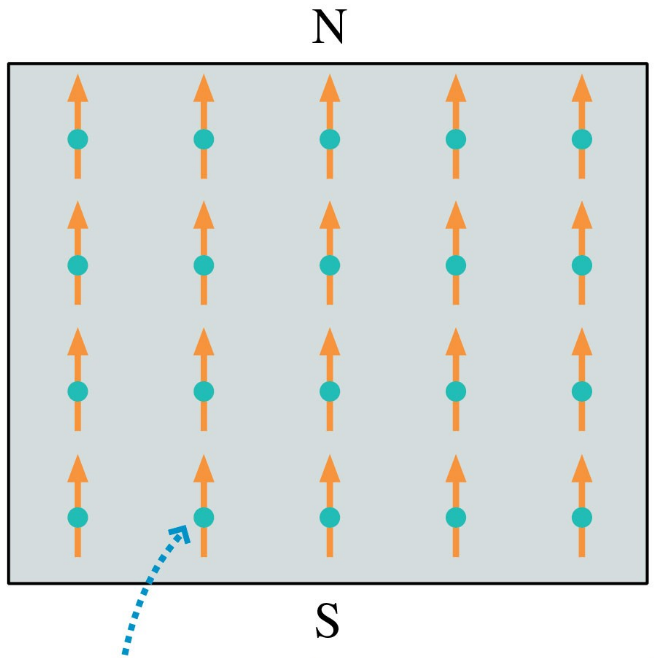
Magnetism in Matter

For most elements, the random arrangement magnetic moments produces a non-magnetic solid.



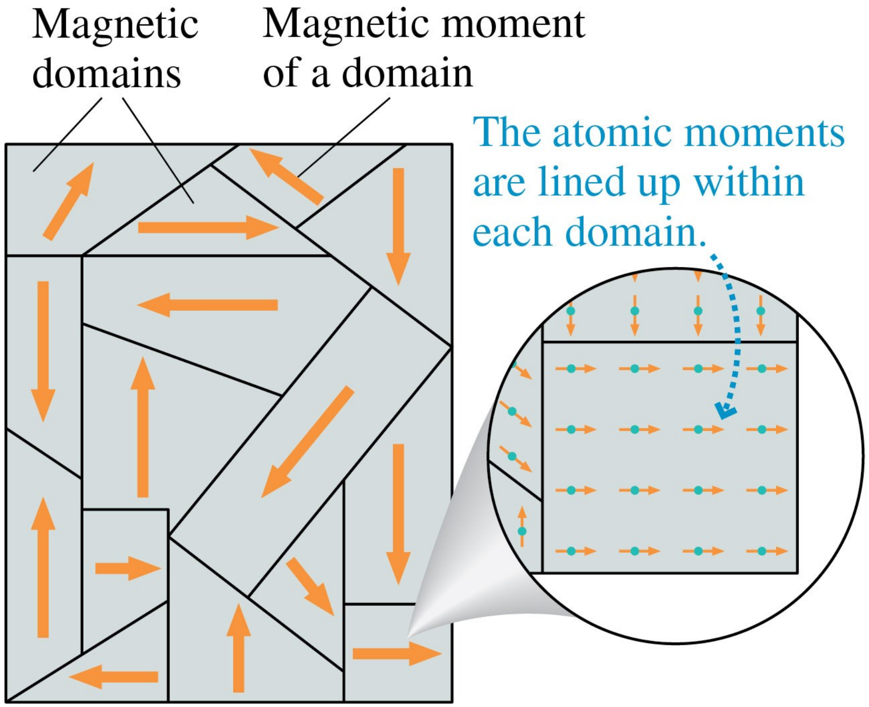
Magnetism in Matter

- In iron, and a few other substances, the atomic magnetic moments tend to all line up in the *same* direction, as shown in the figure.
- Materials that behave in this fashion are called **ferromagnetic**.



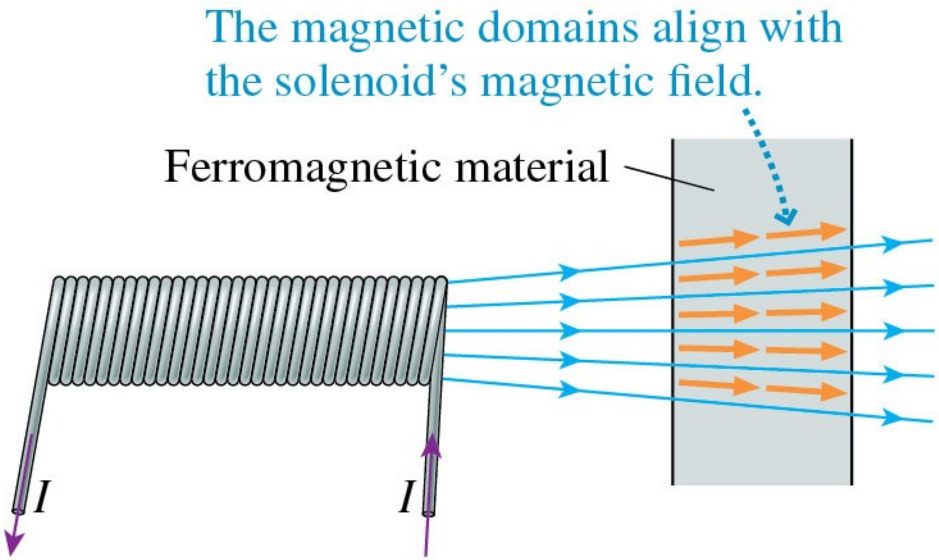
Magnetism in Matter

- A typical piece of iron is divided into small regions, typically less than $100\mu m$ in size, called **magnetic domains**.
- The magnetic moments of all the iron atoms within each domain are perfectly aligned, so each individual domain is a strong magnet.
- However, the various magnetic domains that form a larger solid are randomly arranged.



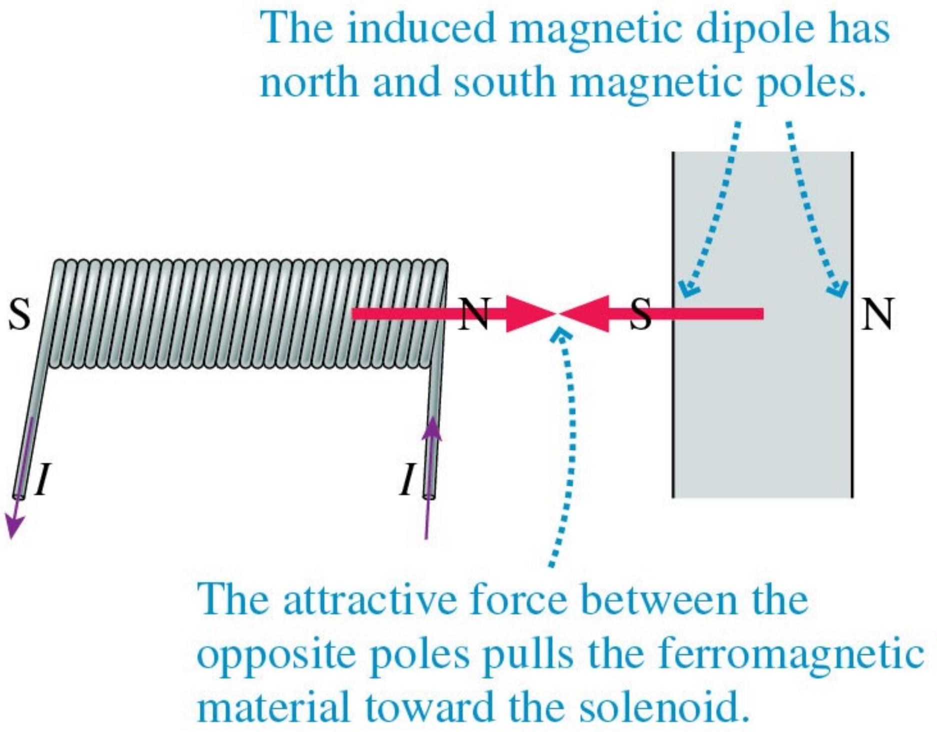
Magnetism in Matter

- An *external* magnetic field exerts a torque on the magnetic dipole of each domain.
- The torque causes many of the domains to rotate and become aligned with the external field.



Magnetism in Matter

- The induced magnetic dipole always has an *opposite* pole facing the solenoid.
- Consequently the magnetic force between the poles *pulls* the ferromagnetic object to the electromagnet.



Magnetism in Matter

This is how a magnet attracts and picks up ferromagnetic objects.