

Chapter 11: Magnetic Forces and Fields

1 Magnetism and Its Historical Discoveries

2 Magnetic Fields and Lines

3 Motion of a Charged Particle in a Magnetic Field

4 Magnetic Force on a Current-Carrying Conductor

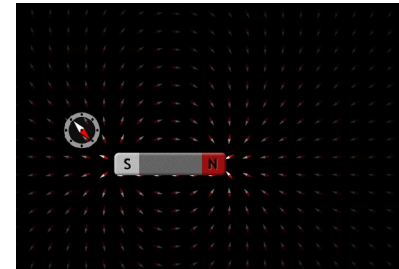
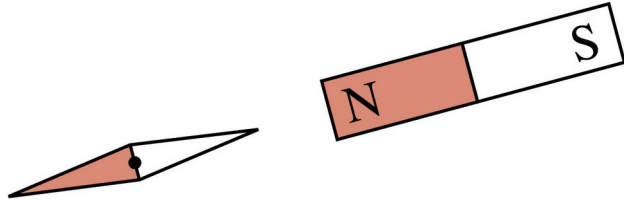
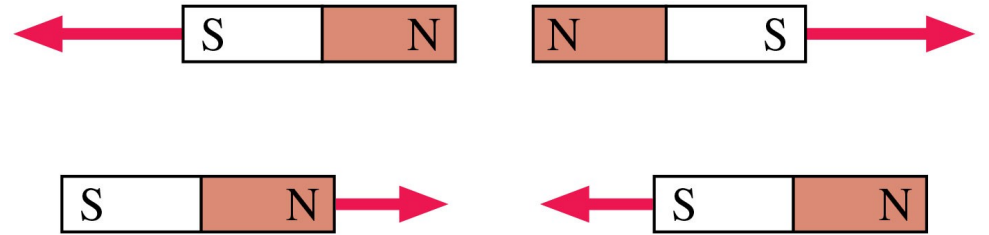
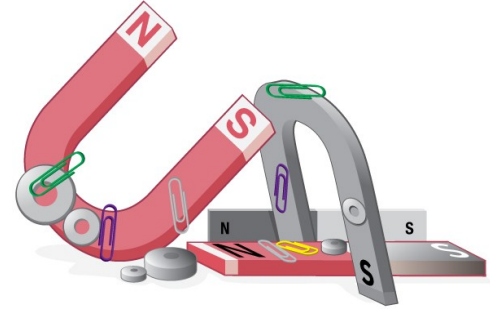
5 Force and Torque on a Current Loop

6 The Hall Effect

7 Applications of Magnetic Forces and Fields

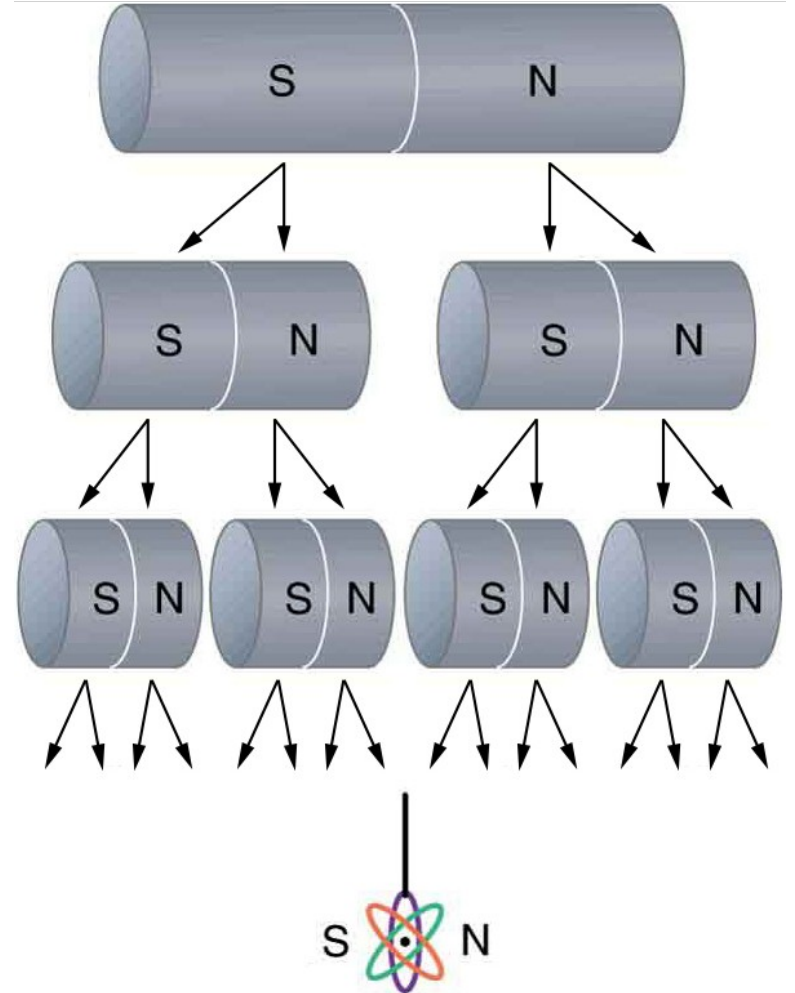
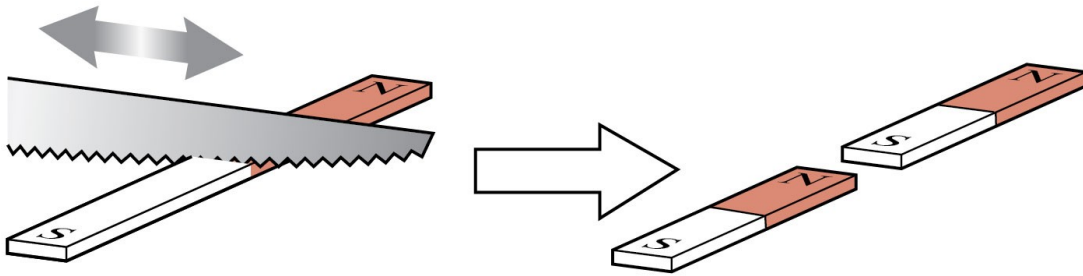
Magnetism

- magnets have North and South poles
- like poles repel; unlike poles attract
- a magnetic compass needle is a small magnet



Magnetism

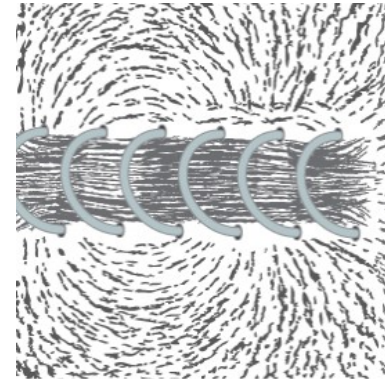
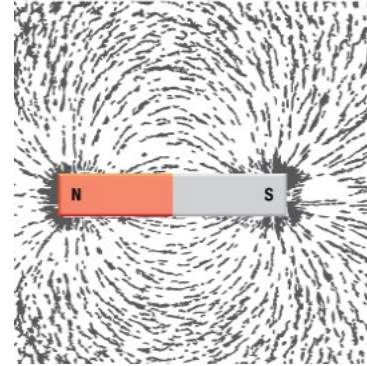
- magnets always have N-S poles
- there are no magnetic monopoles



Magnetism

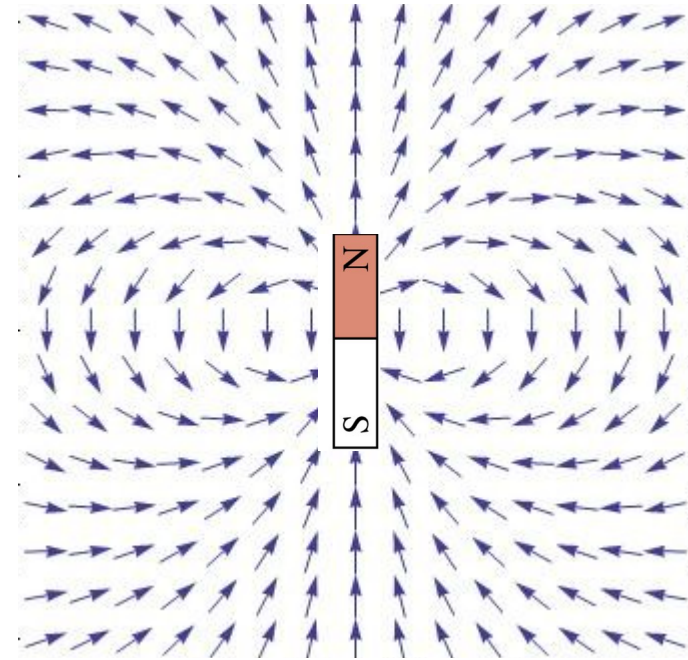
There are 2 sources of magnetism:

- magnetic materials
 - each atom is a tiny magnet
- moving charges
 - there is a magnetic field around every moving charge
 - electric current creates magnetic field



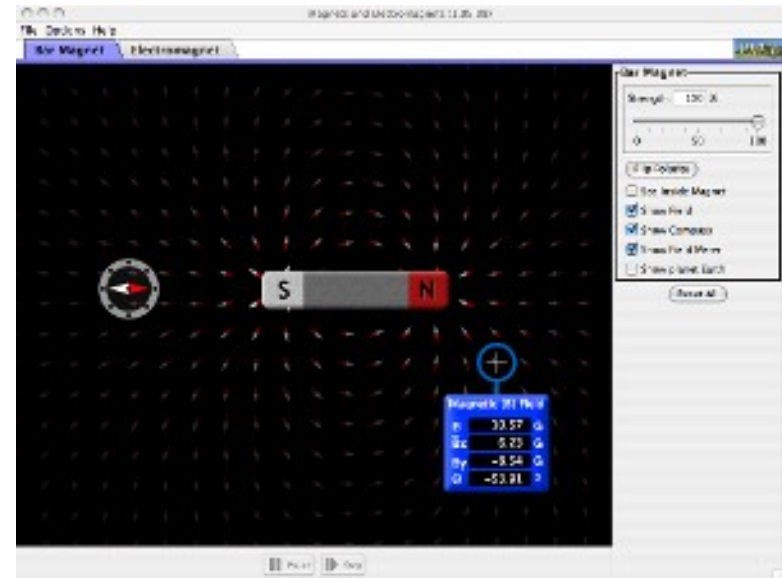
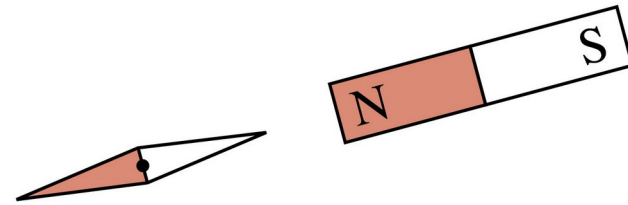
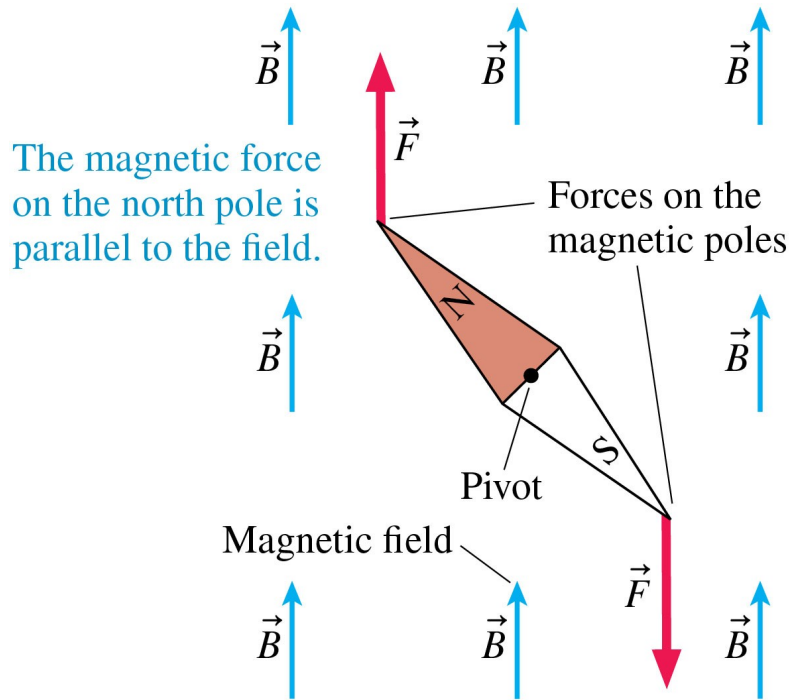
The Magnetic Field

- magnetic field is a vector: \vec{B}
- direction of field:
 - away from N
 - toward S
- Unit: Tesla (T)
- Another common unit is a Gauss (G):
 $1 \text{ T} = 10^4 \text{ G}$
- Some typical B Values:
 - Conventional laboratory magnets: 25000 G or 2.5 T
 - Superconducting magnets: 300000 G or 30 T
 - Earth's magnetic field: 0.5 G or $5 \times 10^{-5} \text{ T}$



The Magnetic Field

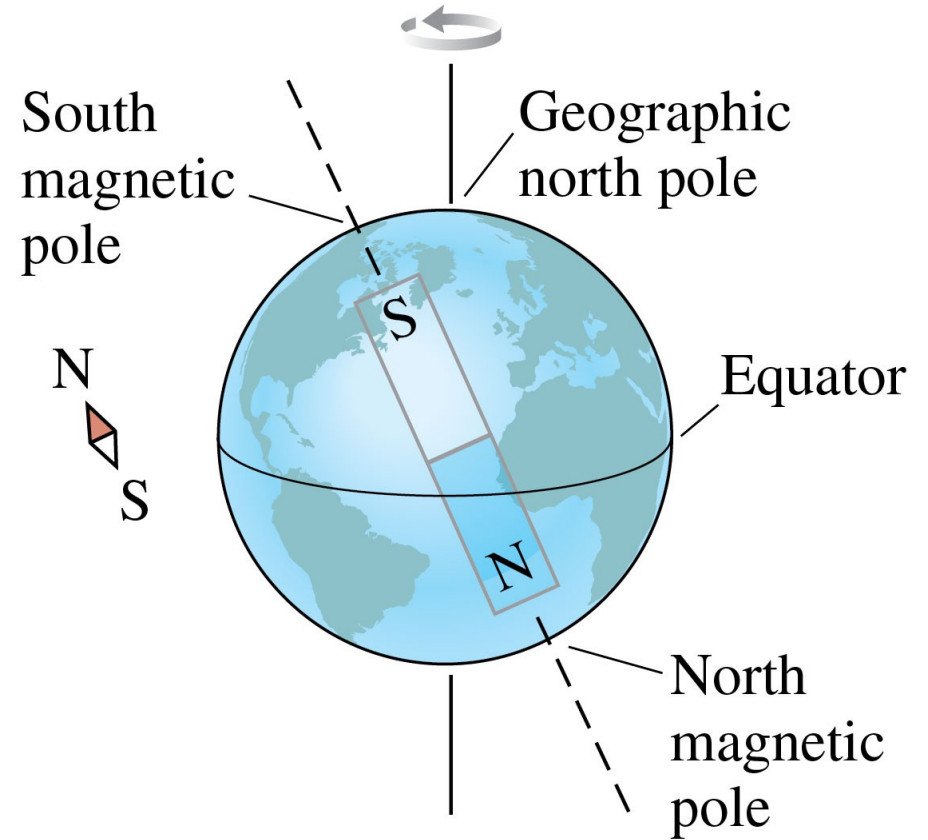
a compass needle points in the direction of the magnetic field



The Magnetic Field

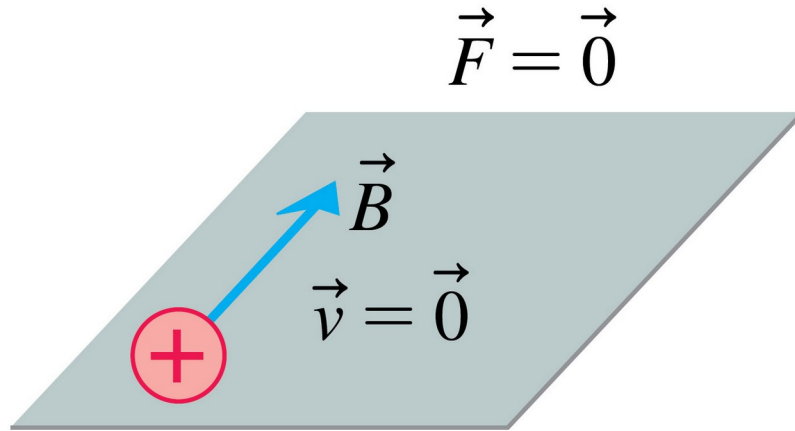
The Earth's geographic north pole corresponds to a magnetic south pole.

(and vice versa)



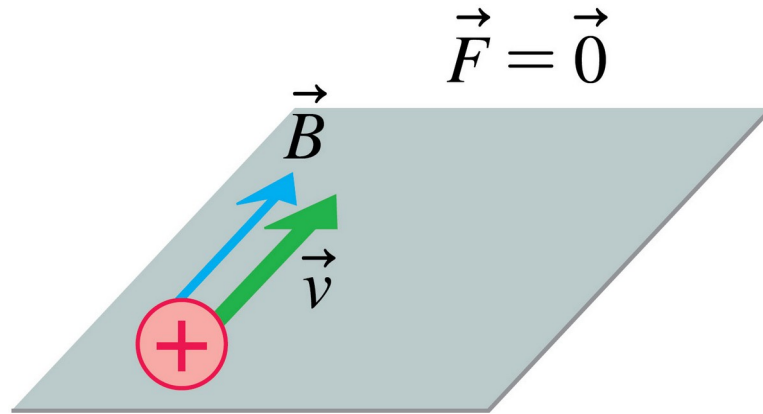
Magnetic Force

There is no magnetic force on a charged particle at rest.

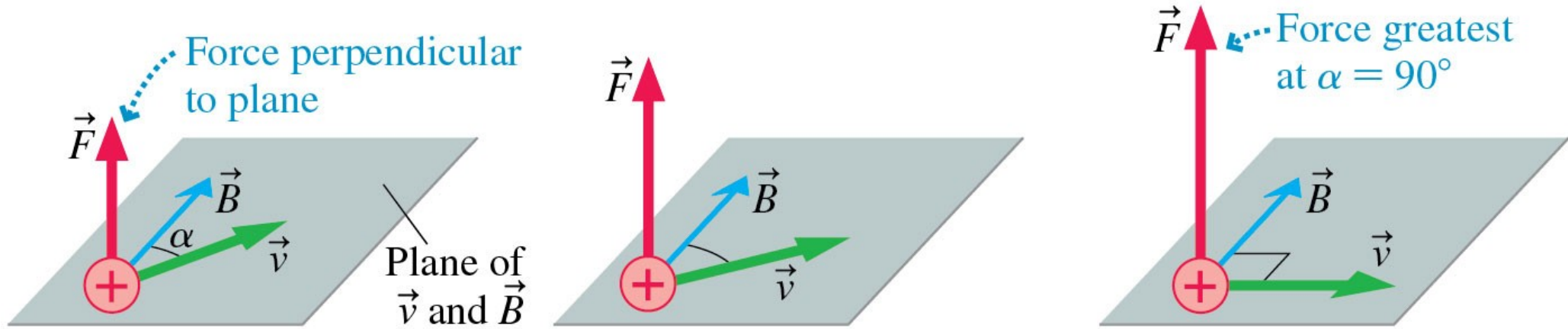


Magnetic Force

There is no magnetic force on a charged particle moving *parallel* to a magnetic field.



Magnetic Force



- As the angle α between the velocity and the magnetic field increases, the magnetic force also increases.
- The force is greatest when the angle is 90° .
- The force is perpendicular to the plane containing v and B .

Magnetic Force

The magnetic force on a charge is

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{F} = qvB \sin \theta \hat{u}_{\perp}$$

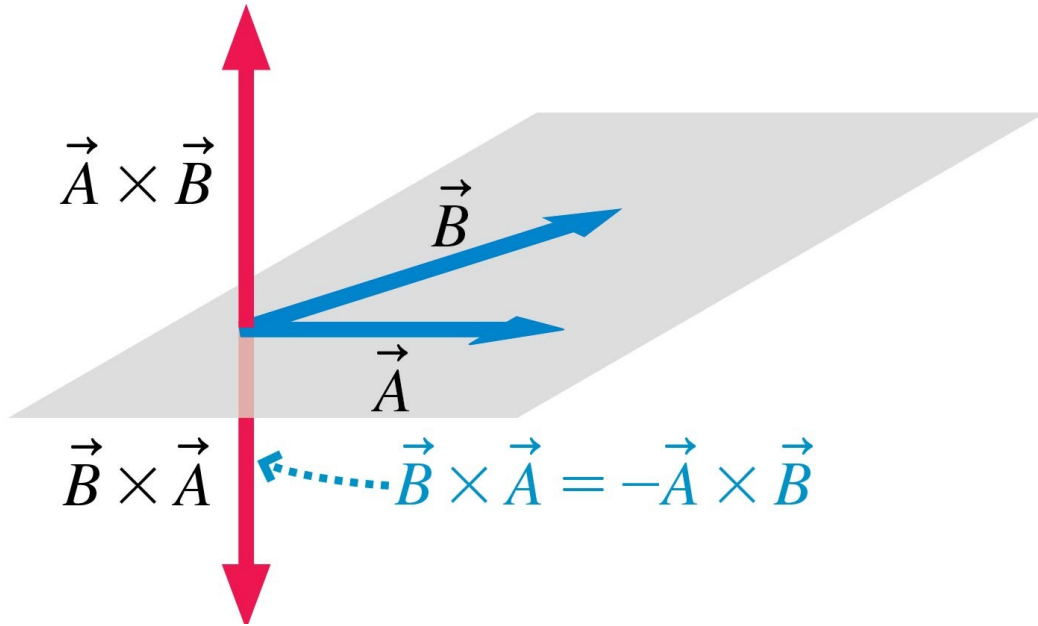
The total force on a charge is

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

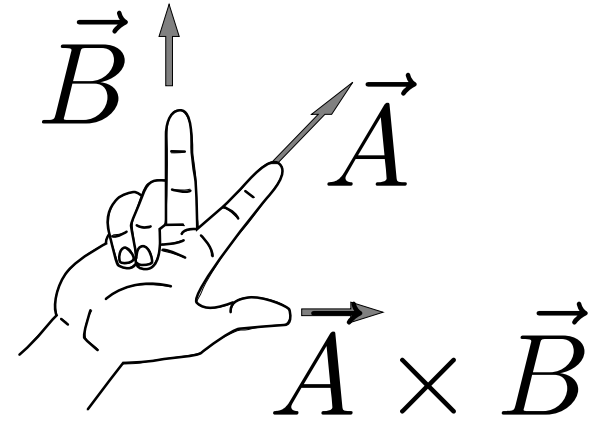
called the “Lorentz Force”

Magnetic Force

Vector cross product



right hand rule

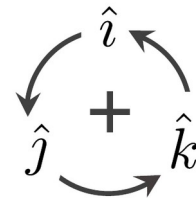
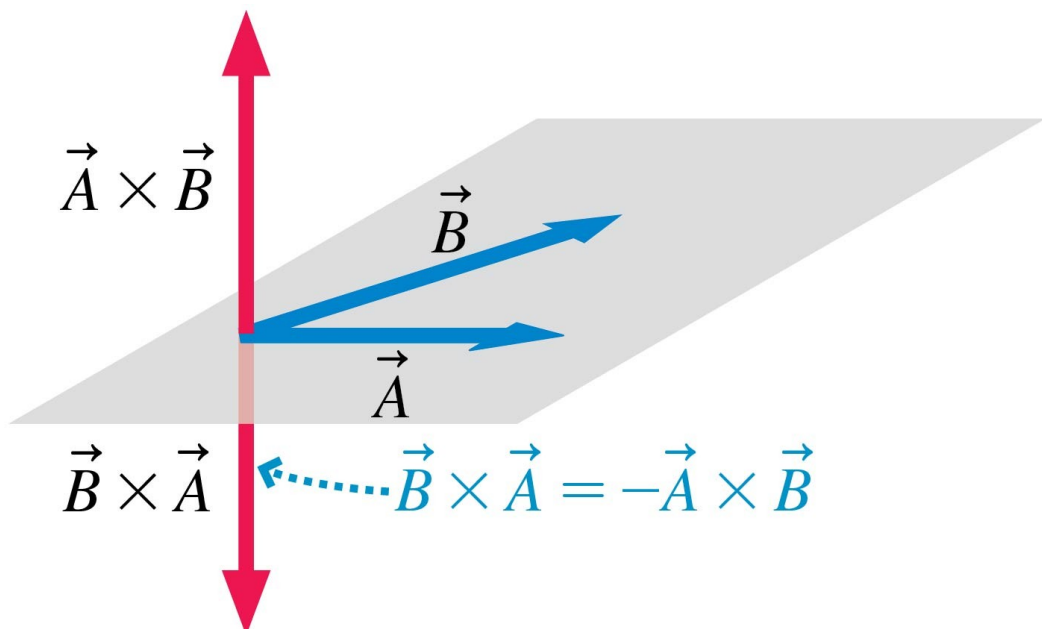


Magnetic Force

Vector cross product

$$\vec{A} \times \vec{B} = (AB \sin \theta) \hat{u}_{\perp}$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

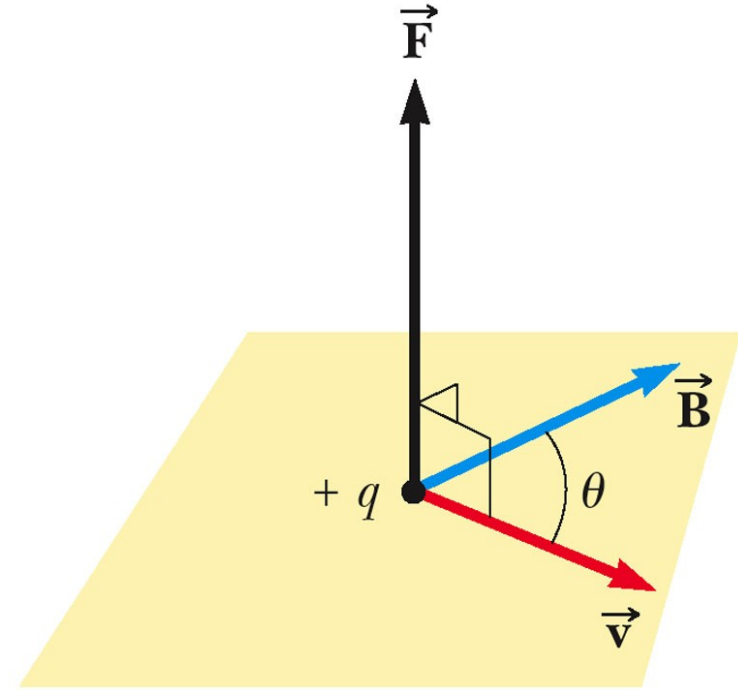


Magnetic Force

Magnetic force is proportional to charge q .

$$\vec{F} = q\vec{v} \times \vec{B}$$

The force on a negative charge points in the opposite direction of $\vec{v} \times \vec{B}$



Magnetic Force

How to show vectors perpendicular to the page:



Vectors into page



Vectors out of page



Current into page

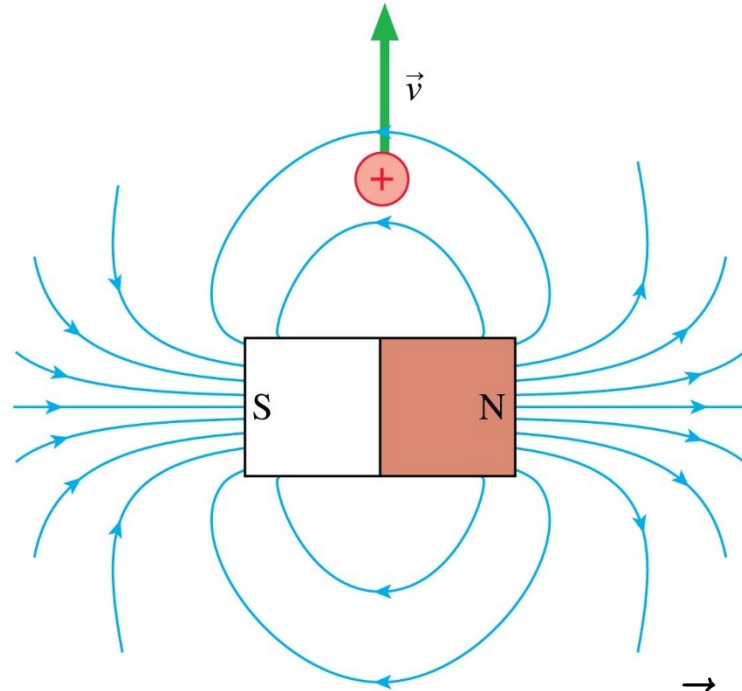


Current out of page

Magnetic Force

The direction of the magnetic force on the proton is

- A. To the right.
- B. To the left.
- C. Into the screen.
- ✓ D. Out of the screen.
- E. The magnetic force is zero.

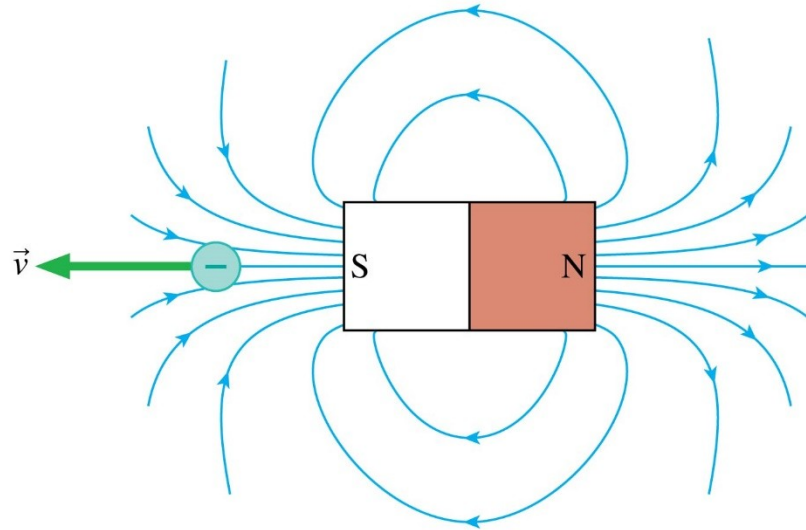


$$\vec{F} = q\vec{v} \times \vec{B}$$

Magnetic Force

The direction of the magnetic force on the electron is

- A. Upward.
- B. Downward.
- C. Into the screen.
- D. Out of the screen.
- ✓ E. The magnetic force is zero.



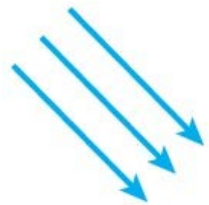
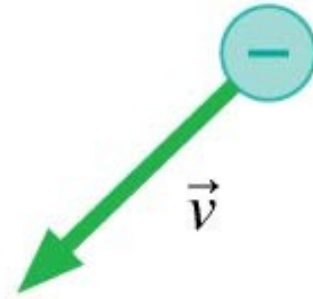
$$\vec{F} = q\vec{v} \times \vec{B}$$

Magnetic Force

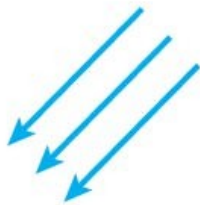
Which magnetic field causes the observed force?

$$\vec{F} = q\vec{v} \times \vec{B}$$

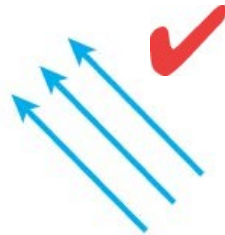
\vec{F} out of screen



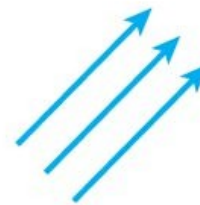
A.



B.



C.



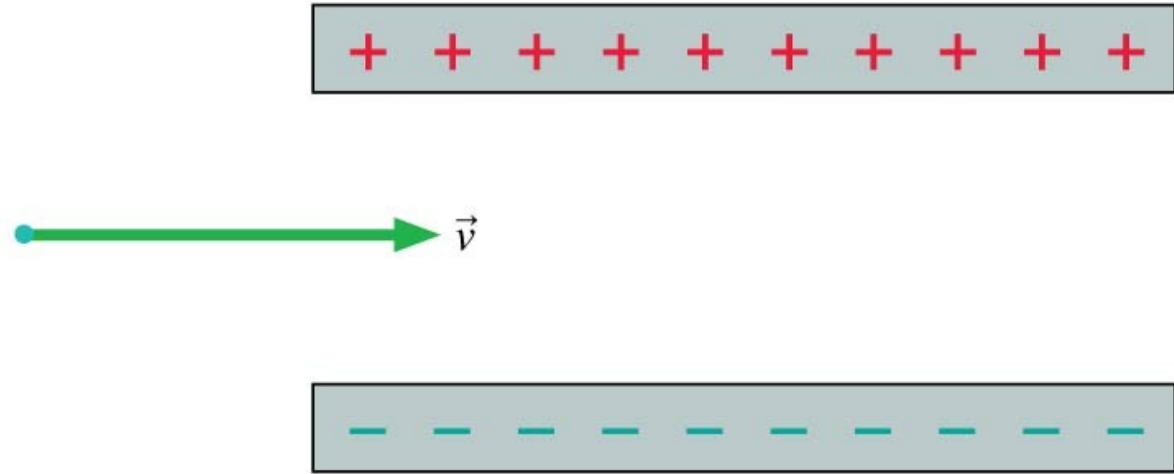
D.



E.

Magnetic Force

Which magnetic field (if it's the correct strength) allows the electron to pass through the charged electrodes without being deflected?



A.



B.



C.



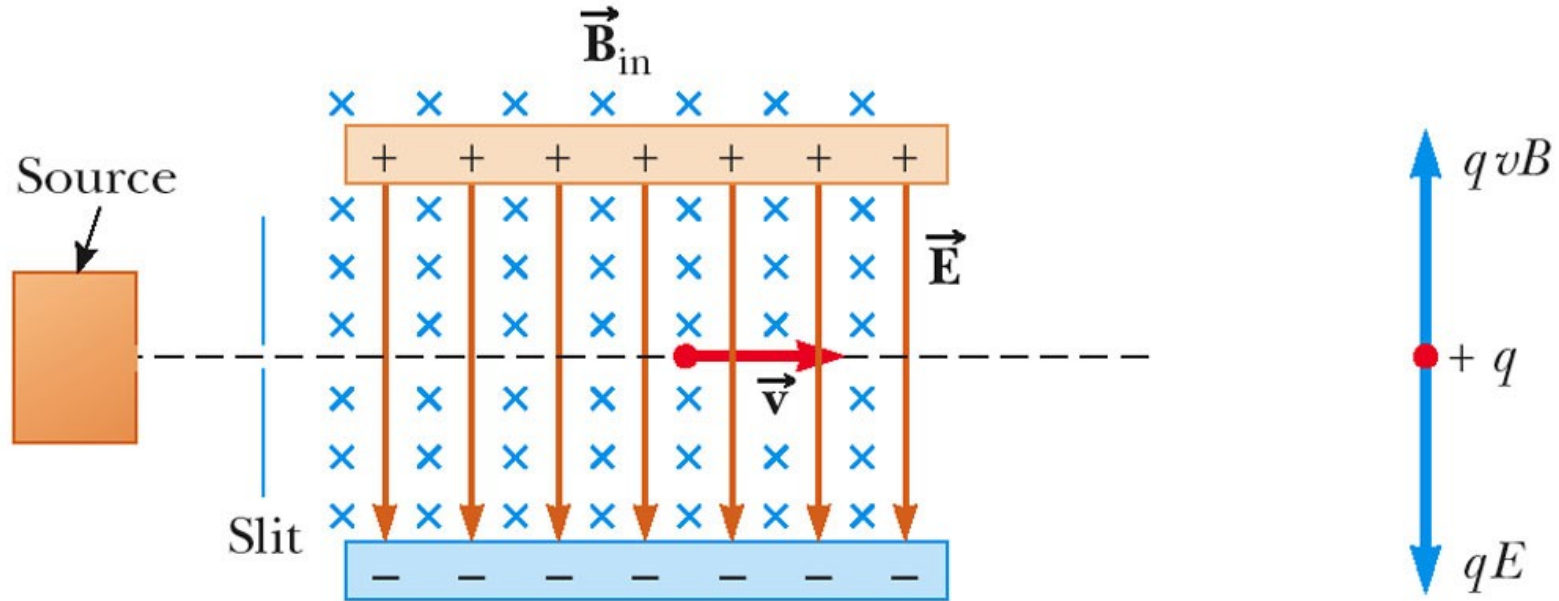
D.



E.

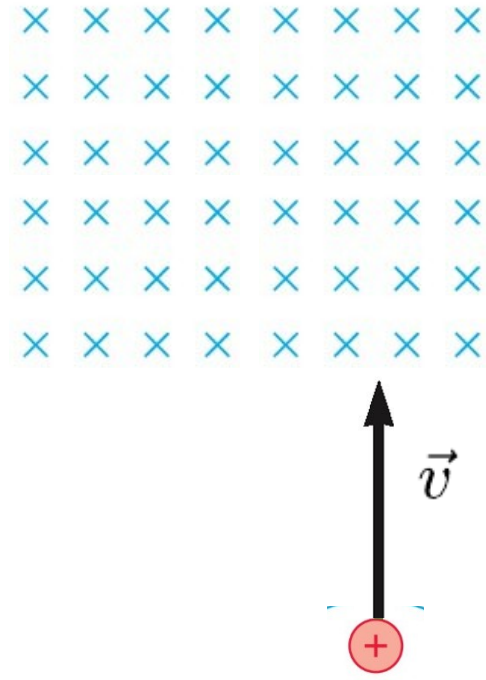
Magnetic Force

A “velocity selector” will only pass through particles of the desired speed.



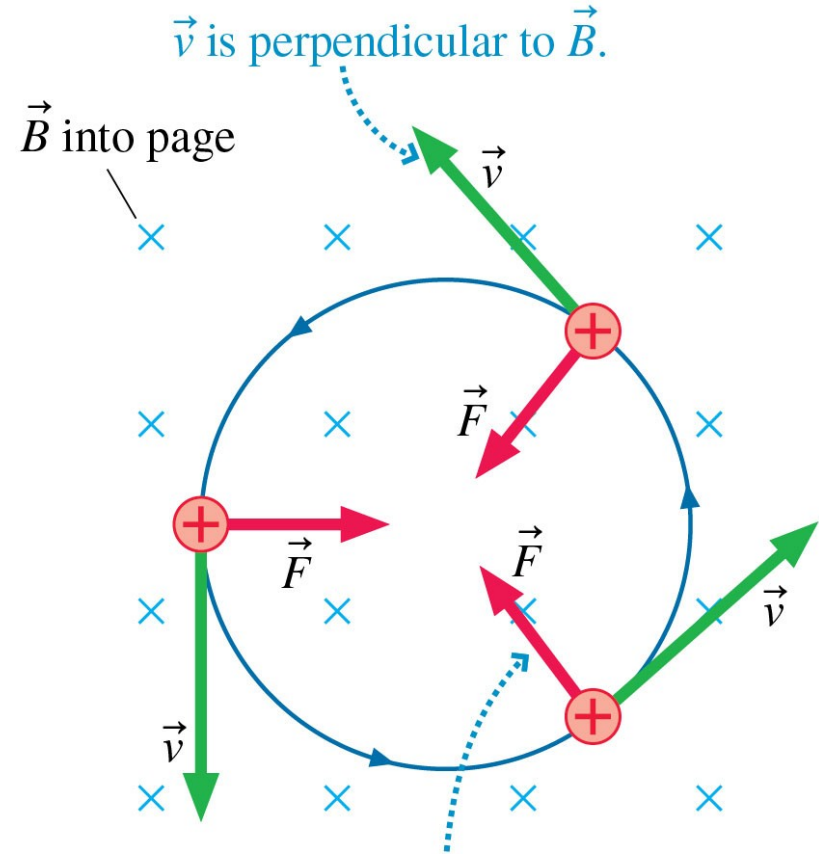
Magnetic Force

What will be the motion of this particle after it enters the magnetic field?



Cyclotron Motion

- a positive charge moving in a plane perpendicular to a *uniform* magnetic field.
- the charge undergoes **uniform circular motion**.
- called the **cyclotron motion**



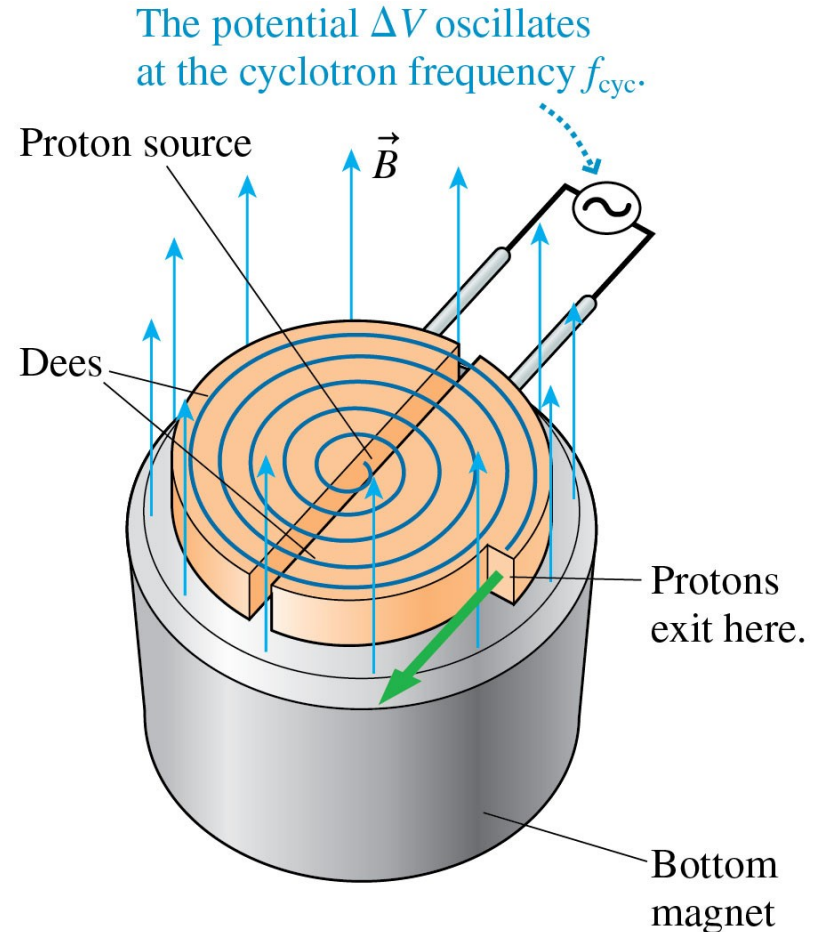
The magnetic force is always perpendicular to \vec{v} , causing the particle to move in a circle.

Cyclotron Motion

- a particle with mass m and charge q moving with a speed v in a plane that is perpendicular to a uniform magnetic field of strength B .
- for circular motion: $F = ma_c$ $qvB = m \frac{v^2}{r}$
- radius of the cyclotron orbit is $r = \frac{mv}{qB}$
- cyclotron frequency $v = \frac{2\pi r}{T}$ $f = \frac{1}{T} = \frac{qB}{2\pi m}$

Cyclotron Motion

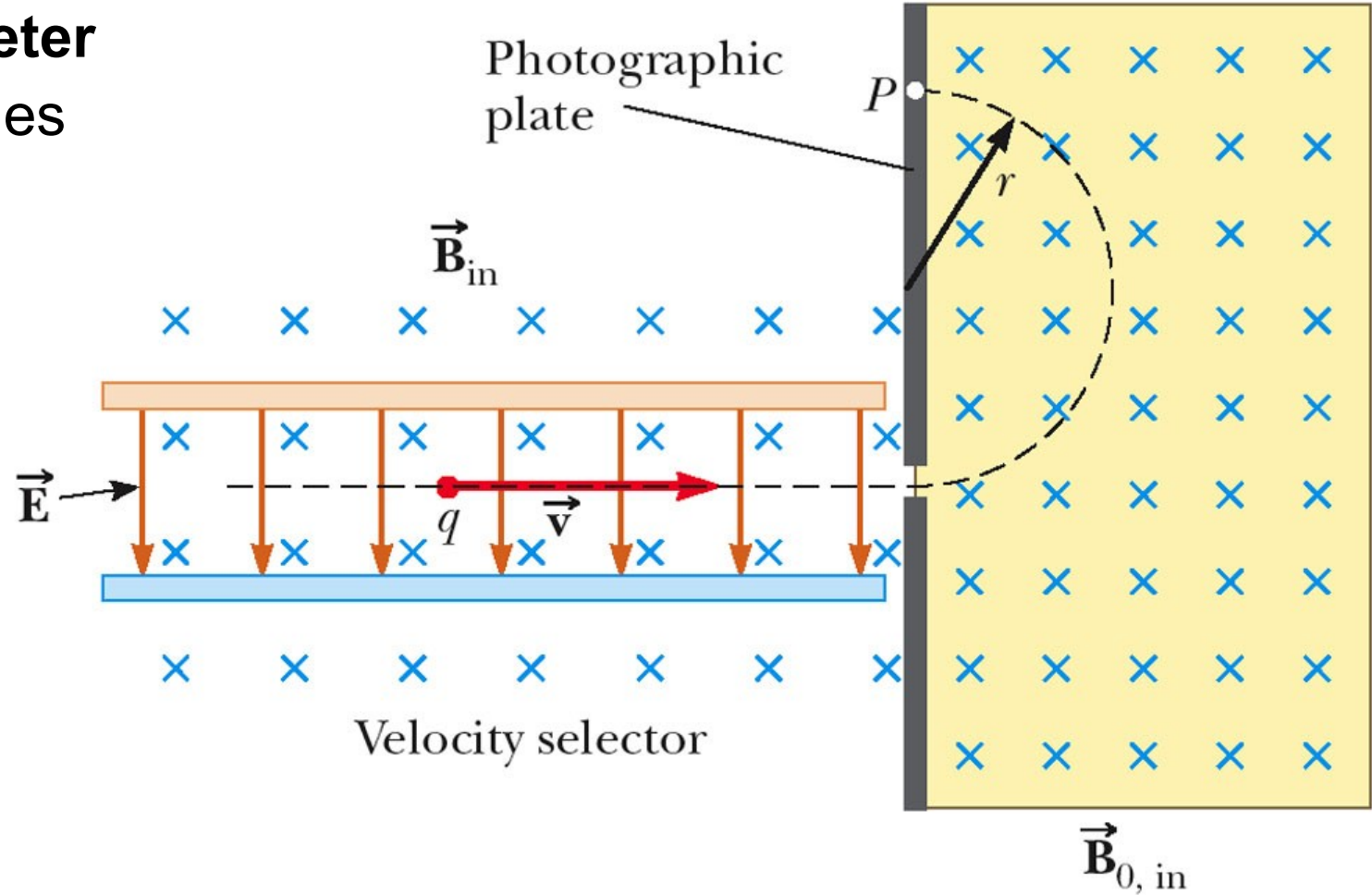
The first practical particle accelerator, invented in the 1930s, was the **cyclotron**.



Cyclotron Motion

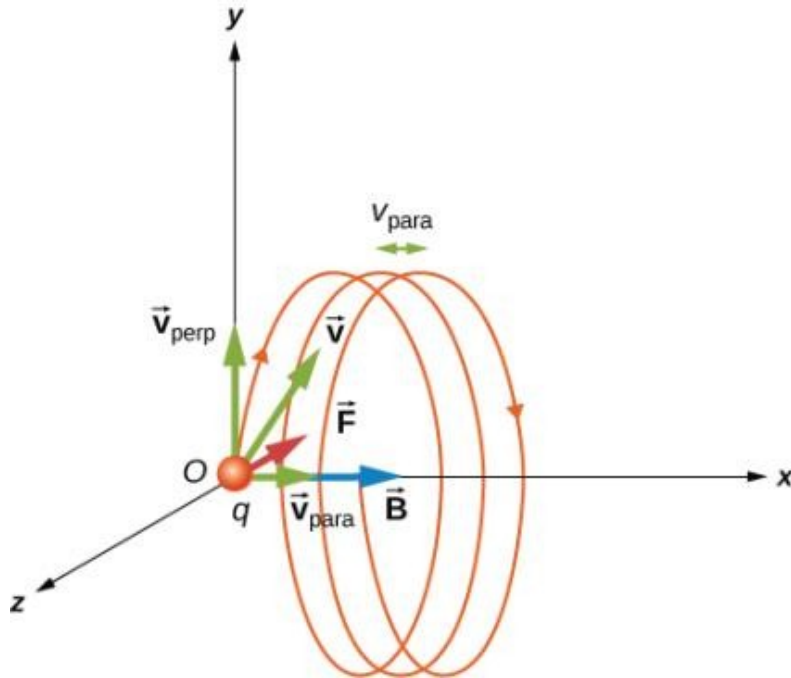
A mass spectrometer will separate particles by mass.

$$r = \frac{mv}{qB}$$

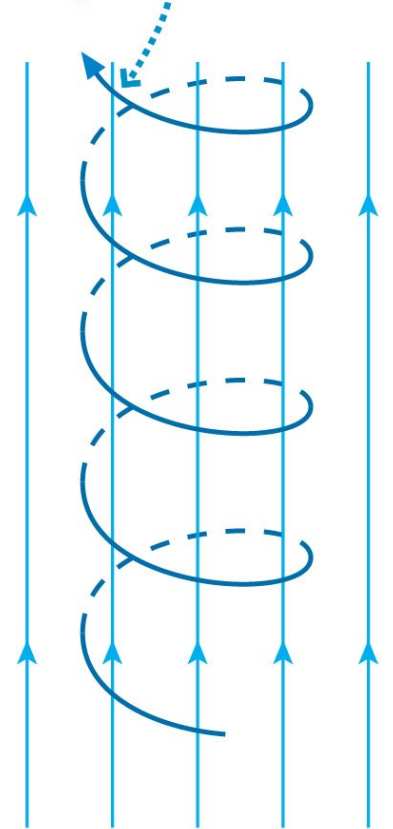


Cyclotron Motion

- if the velocity has a component parallel to the field, that component is constant
- the result is a helix

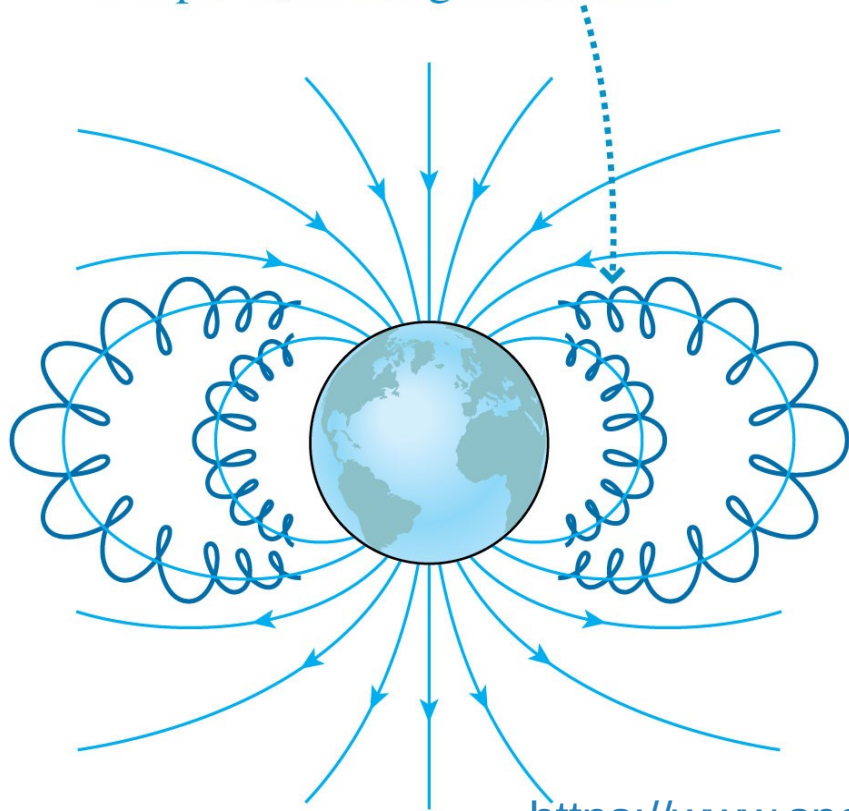


Charged particles spiral around the magnetic field lines.



Cyclotron Motion

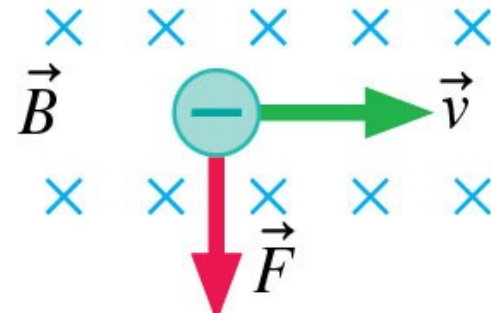
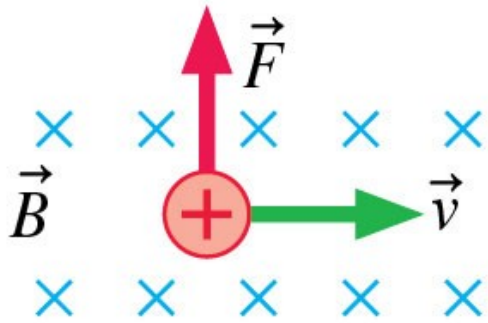
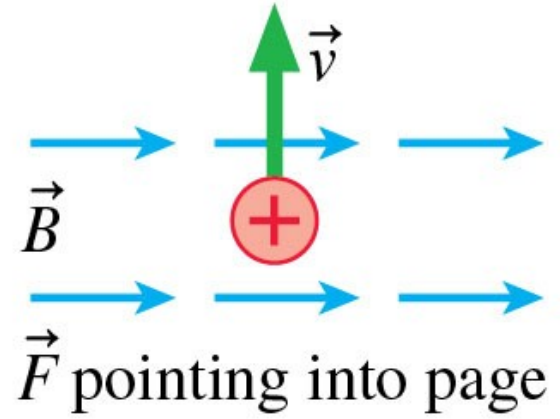
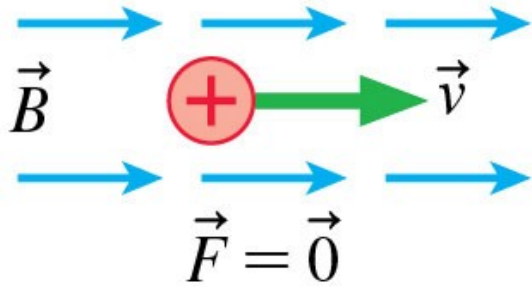
The earth's magnetic field leads particles into the atmosphere near the poles, causing the aurora.



<https://www.spaceweatherlive.com/en/auroral-activity/aurora-forecast>

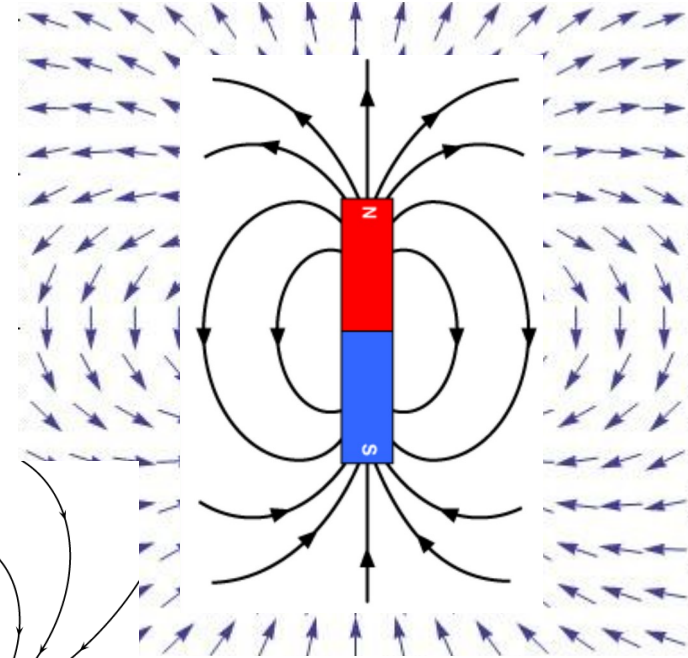
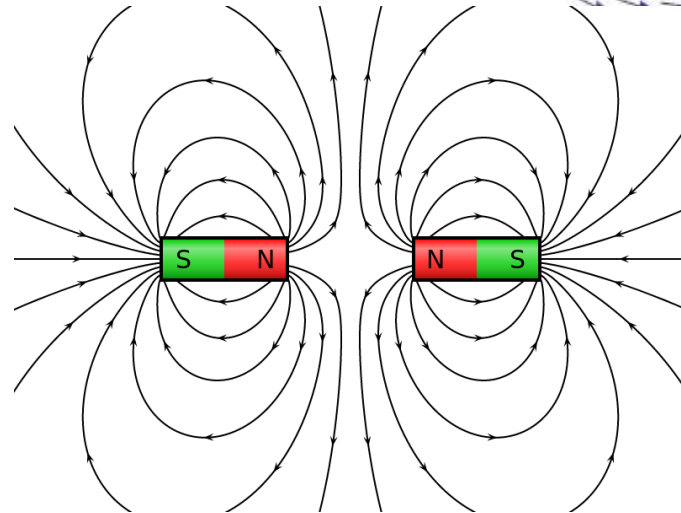
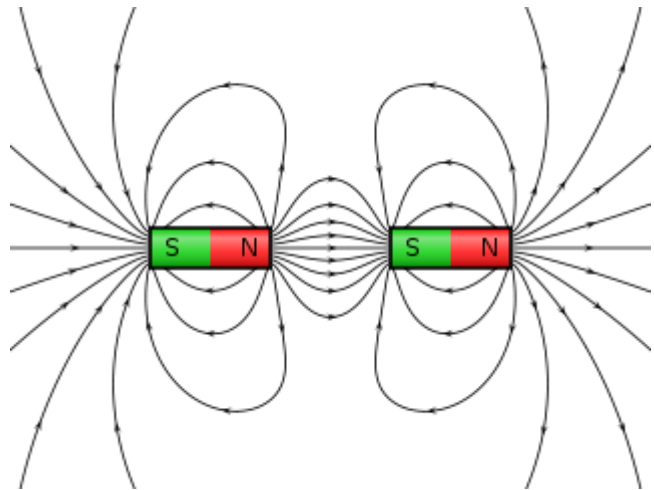
Magnetic Force

$$\vec{F} = q\vec{v} \times \vec{B}$$

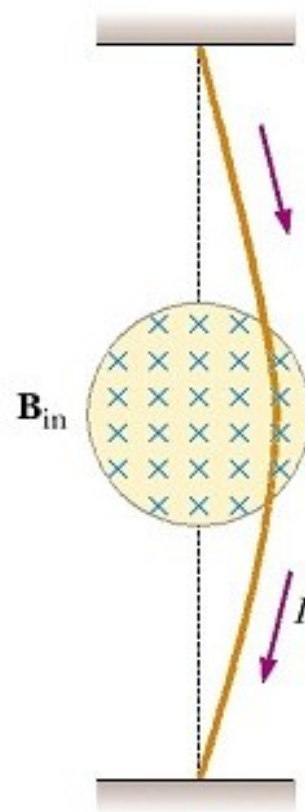
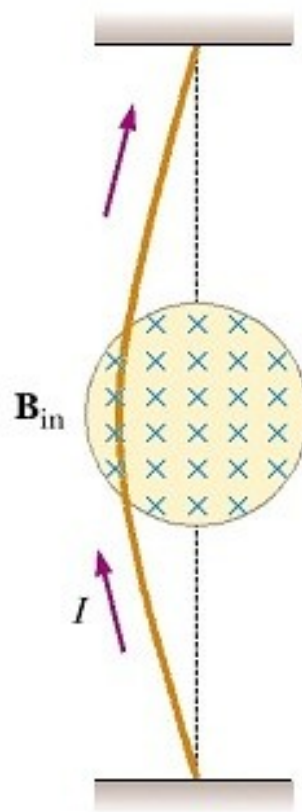
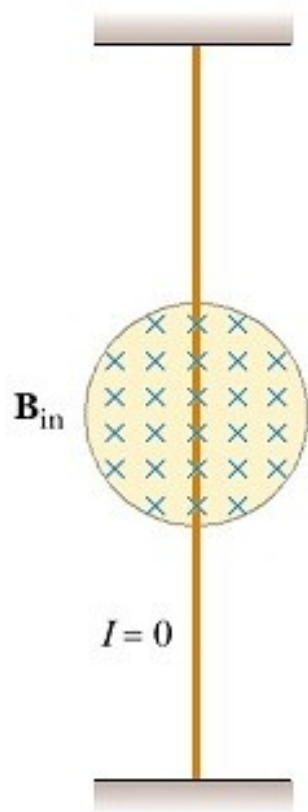
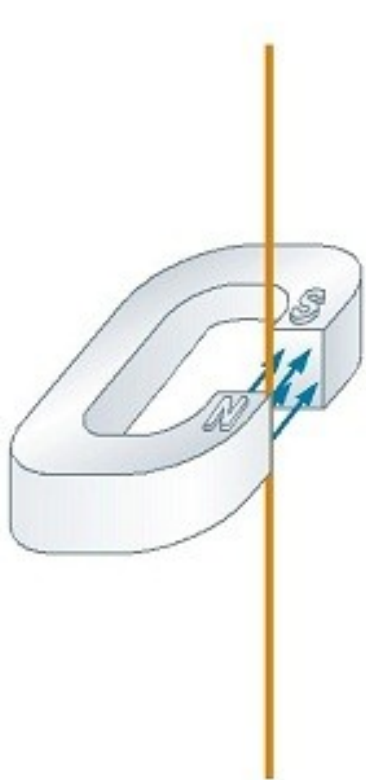


The Magnetic Field

- The vector field \vec{B} is often represented with magnetic field lines
- field lines point away from N, toward S
- line density shows field strength

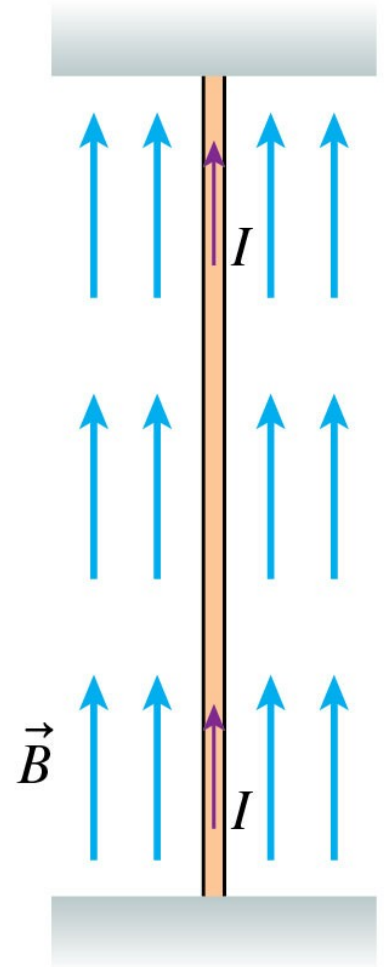


Magnetic Forces on Current-Carrying Wires



Magnetic Forces on Current-Carrying Wires

There's *no* force on a current moving parallel to a magnetic field.



Magnetic Forces on Current-Carrying Wires

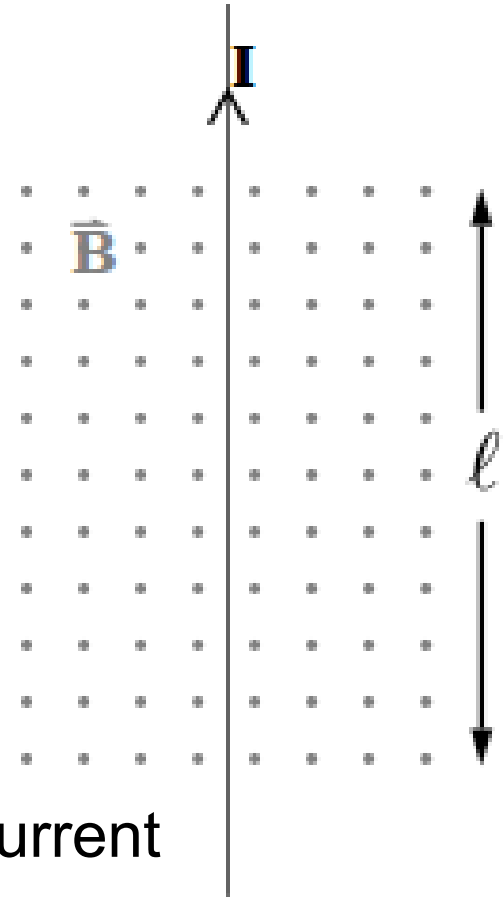
For current I : $I = nqAv_d$

$$\vec{F} = q \left(\frac{I}{nqA} \hat{u} \right) \times \vec{B} = \text{the force on one moving charge}$$

Number of charges in length ℓ :

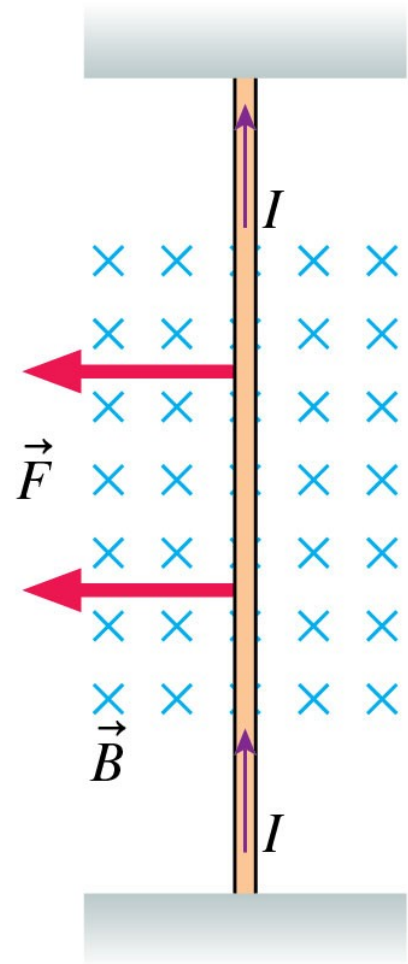
$$\vec{F} = \left(\frac{InA\vec{\ell}}{nA} \right) \times \vec{B} = I\vec{\ell} \times \vec{B}$$

where $\vec{\ell}$ points in direction of current



Magnetic Forces on Current-Carrying Wires

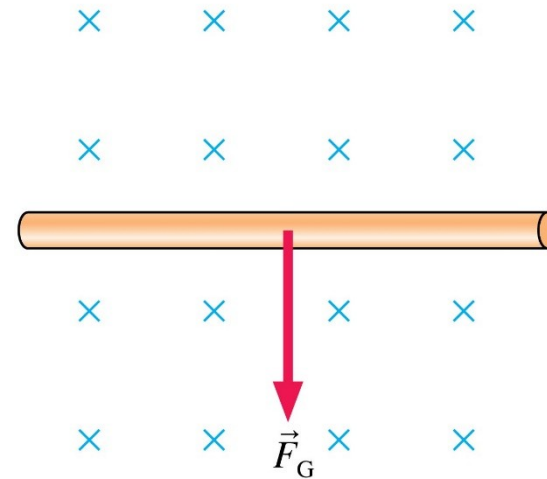
$$\vec{F} = I\vec{\ell} \times \vec{B}$$



Magnetic Forces on Current-Carrying Wires

The horizontal wire can be levitated—held up against the force of gravity—if the current in the wire is

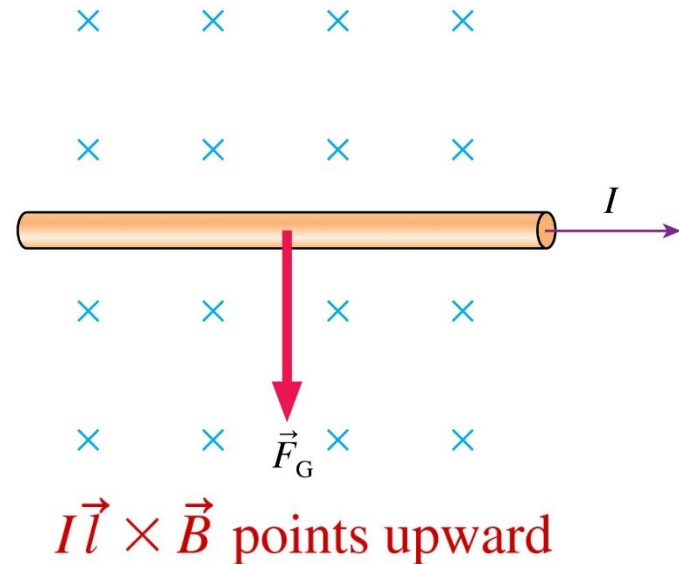
- A. Right to left.
- B. Left to right.
- C. It can't be done with this magnetic field.



Magnetic Forces on Current-Carrying Wires

The horizontal wire can be levitated—held up against the force of gravity—if the current in the wire is

- A. Right to left.
- ✓ B. **Left to right.**
- C. It can't be done with this magnetic field.



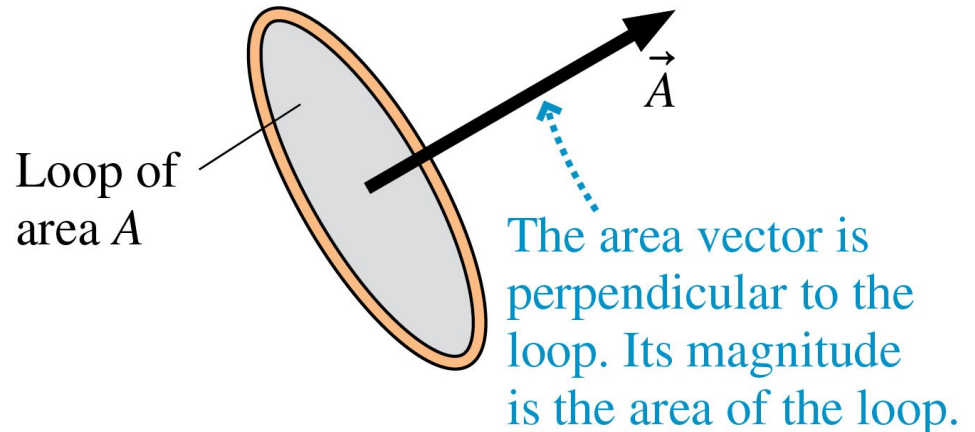
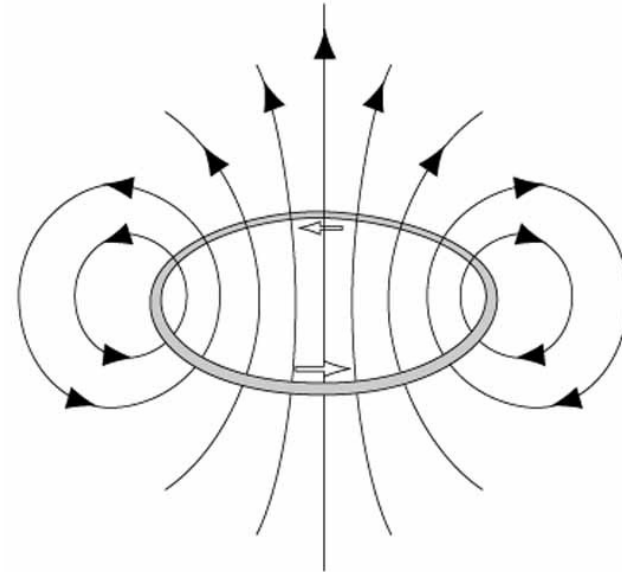
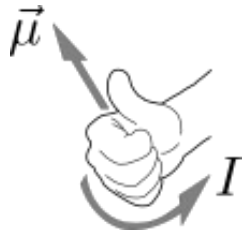
Magnetic Moment

A current loop creates a dipole field.

Define the “magnetic moment”:

$$\vec{\mu} = I\vec{A}$$

Where I is the current and A is the area, with direction given by the direction of circulation of the current:



Magnetic Moment

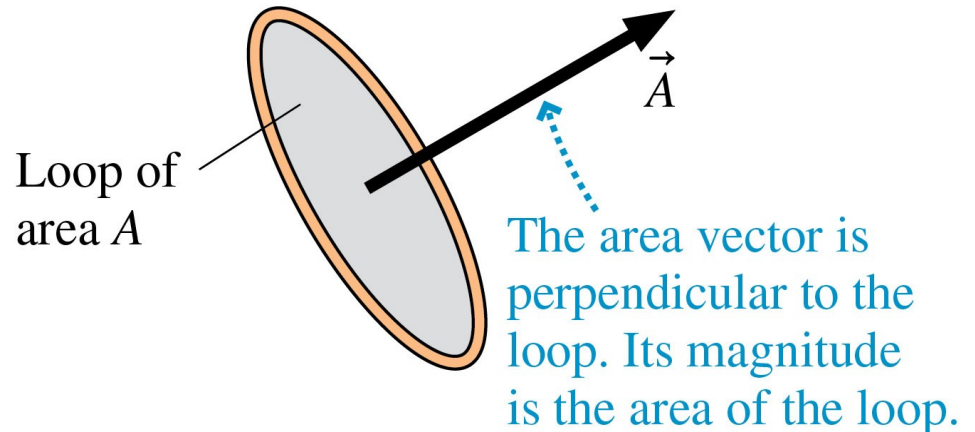
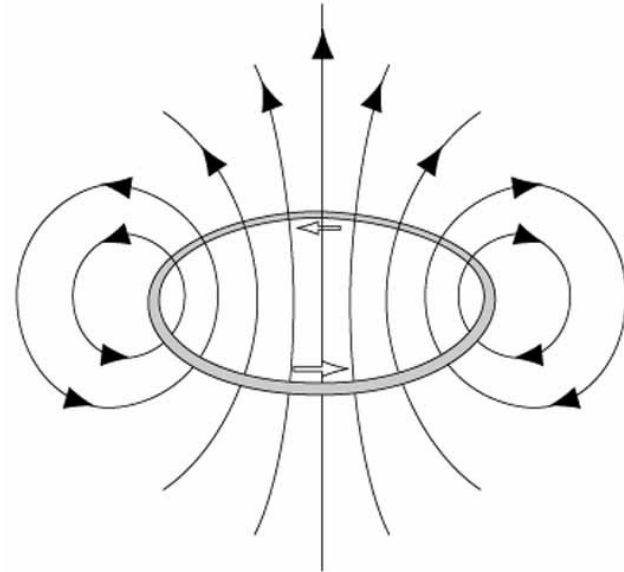
A current loop creates a dipole field.

Define the “magnetic moment”:

$$\vec{\mu} = I\vec{A}$$

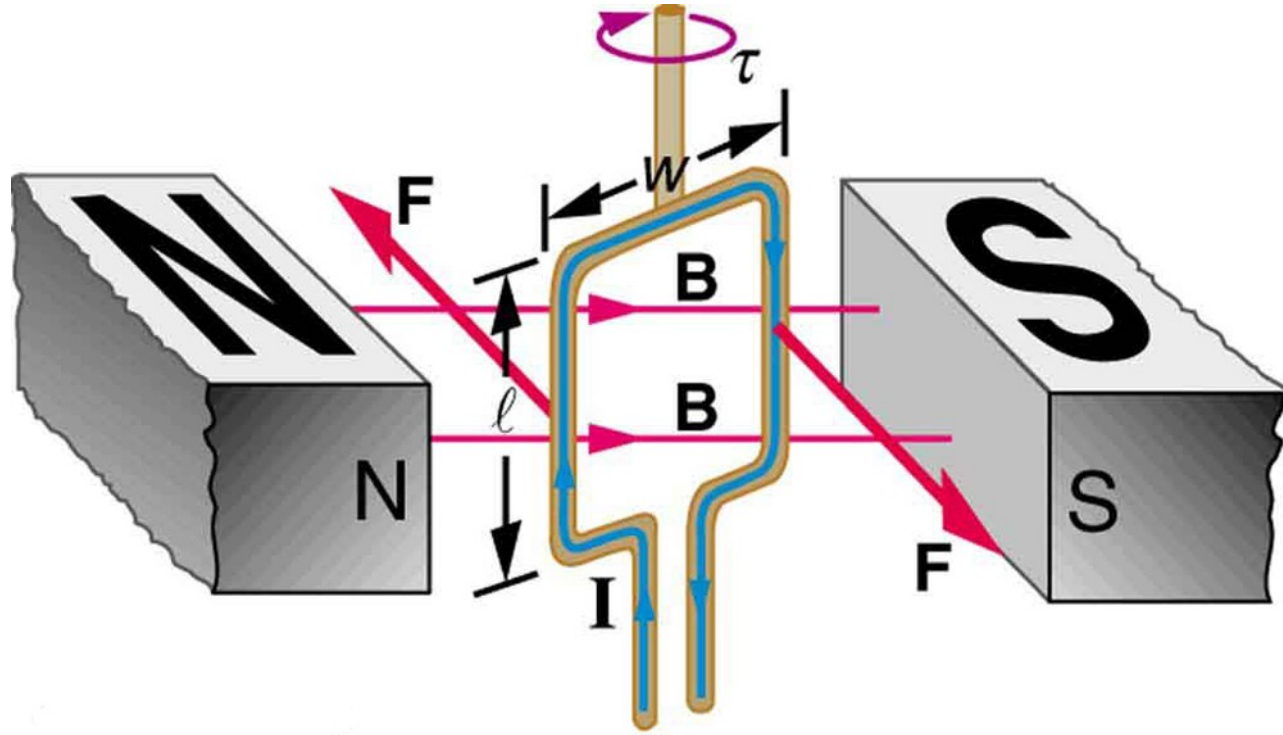
For N loops of wire, the magnetic moment becomes

$$\vec{\mu} = NI\vec{A}$$

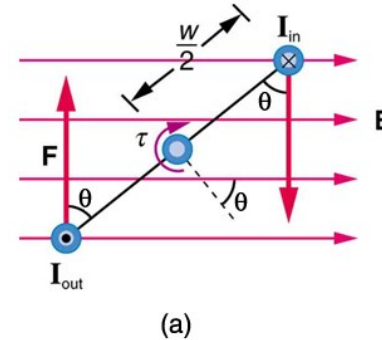
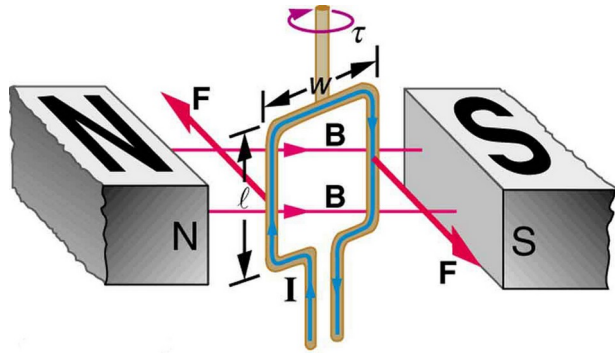


Torque on a Current Loop

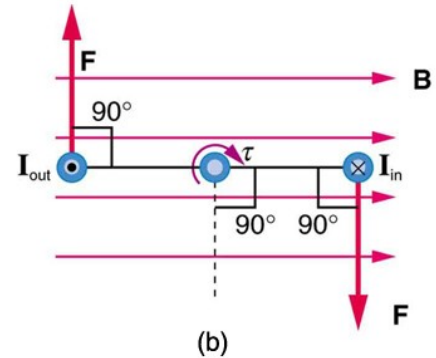
A current loop in a magnetic field experiences a torque.



Torque on a Current Loop

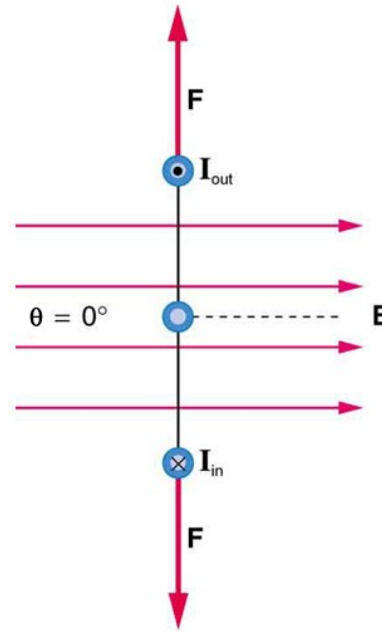


$$\tau = \frac{w}{2} I l B \sin \theta$$

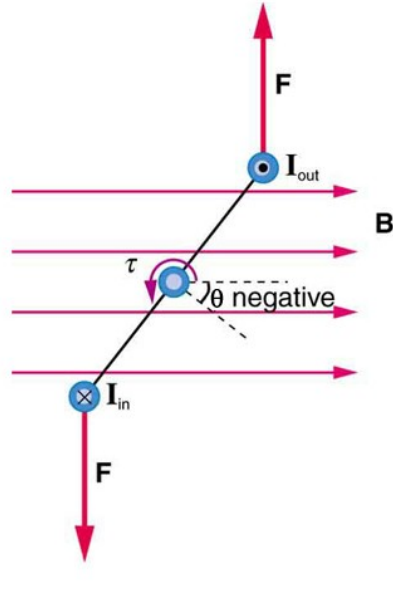


$$\tau = \frac{w}{2} I l B = \tau_{\max}$$

$$\begin{aligned}
 |\tau| &= 2 \left| \vec{r} \times \vec{F} \right| \\
 &= 2 \left(\frac{1}{2} w \cdot B I l \cdot \sin \theta \right) \\
 &= I l w \cdot B \cdot \sin \theta \\
 &= \left| \vec{\mu} \times \vec{B} \right|
 \end{aligned}$$



$\tau = 0$ since $\theta = 0^\circ$



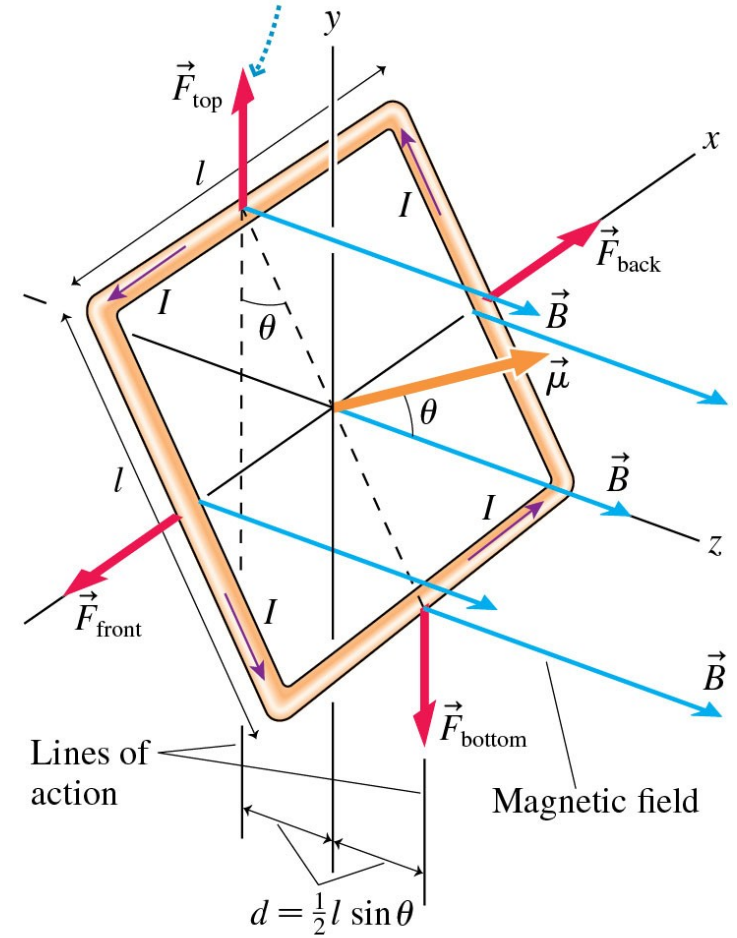
τ is negative

Torque on a Current Loop

A magnetic moment
(such as a current loop)
in a B field feels a torque

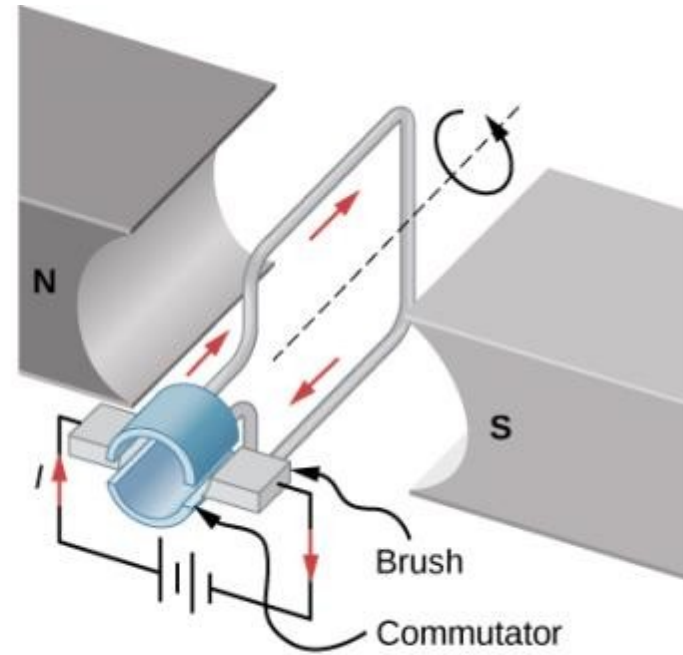
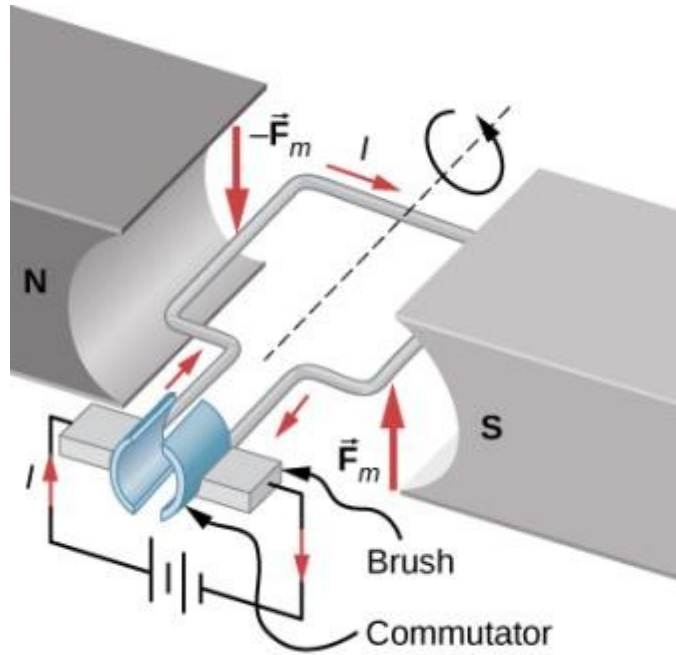
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

\vec{F}_{top} and \vec{F}_{bottom} exert a torque that rotates the loop about the x-axis.



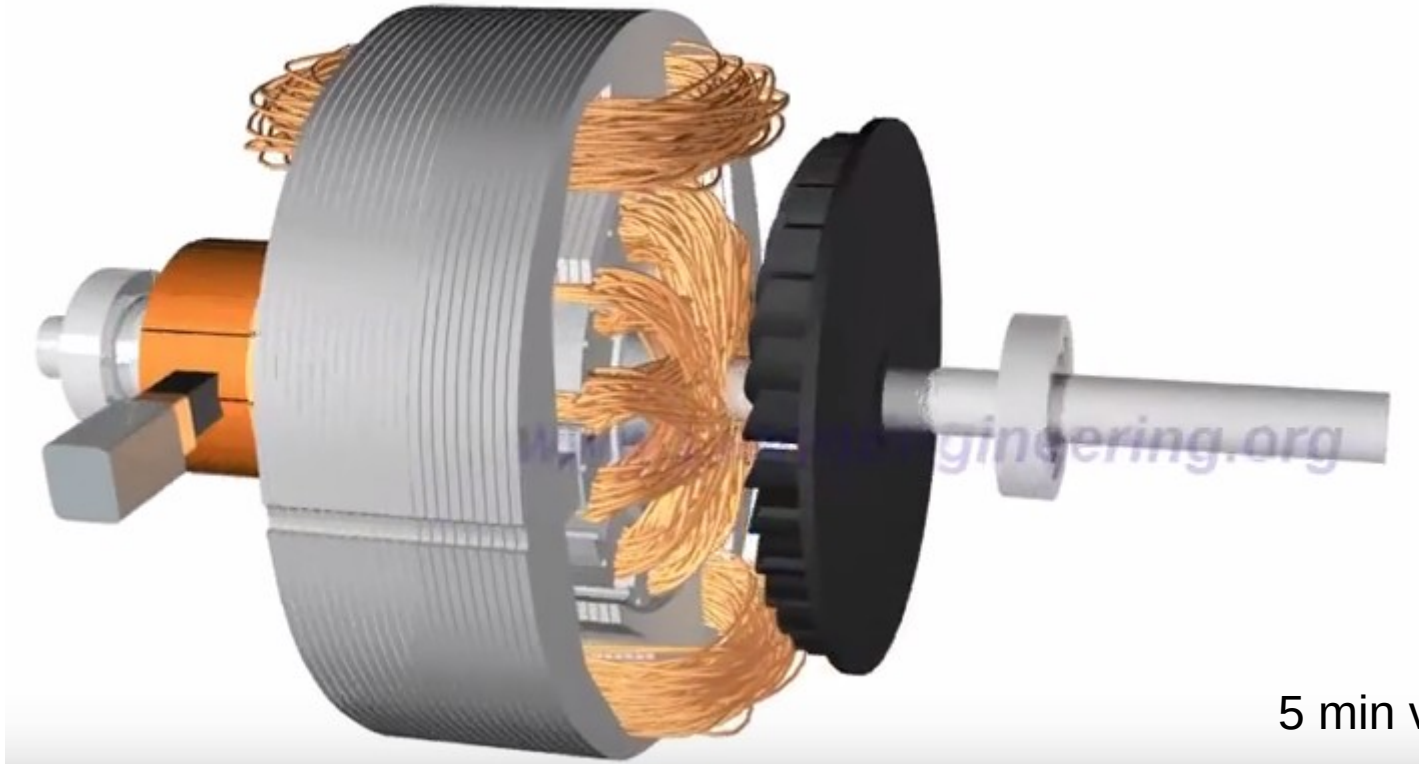
Torque on a Current Loop

Electric motor:



Torque on a Current Loop

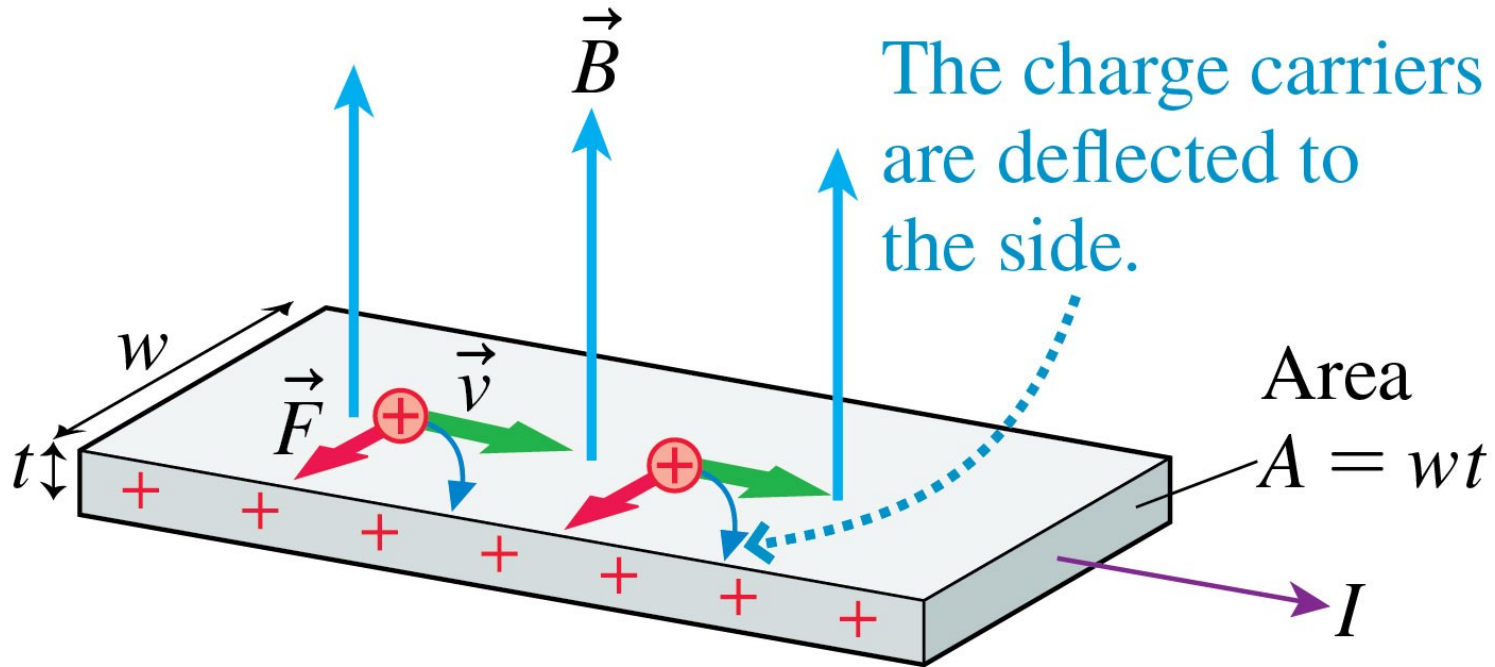
Electric motor:



5 min video

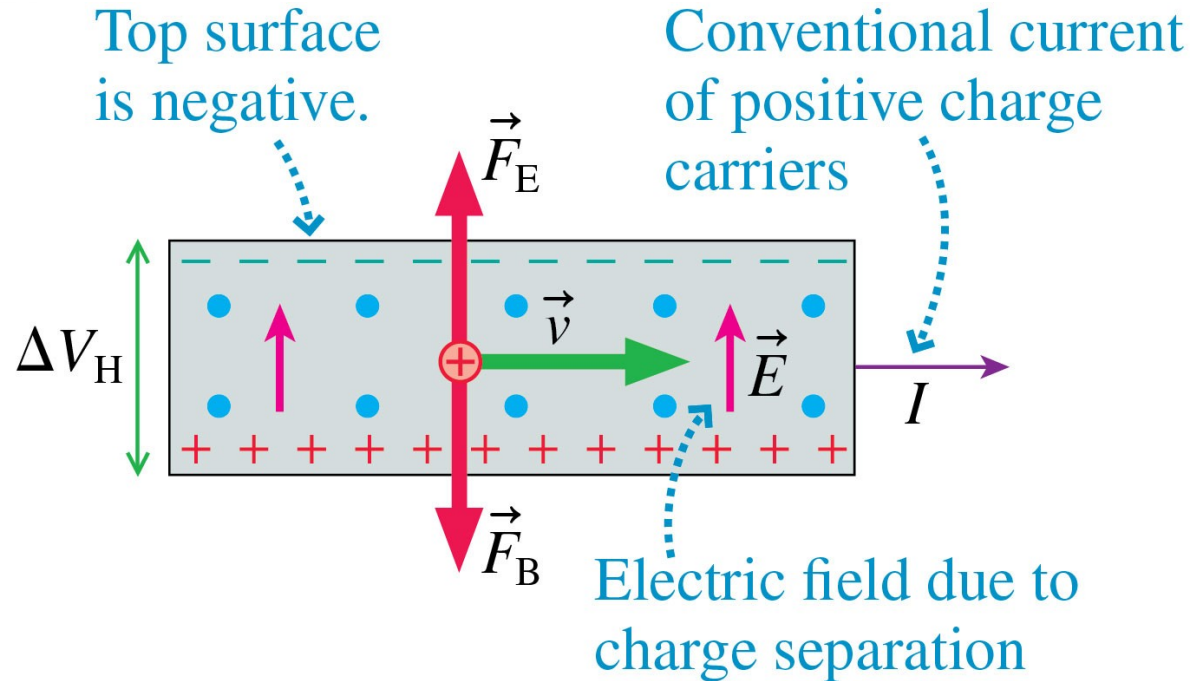
The Hall Effect

As the charge carriers move at the drift speed v_d , they will experience a magnetic force $F_B = ev_d B$ perpendicular to the field and the current.



The Hall Effect

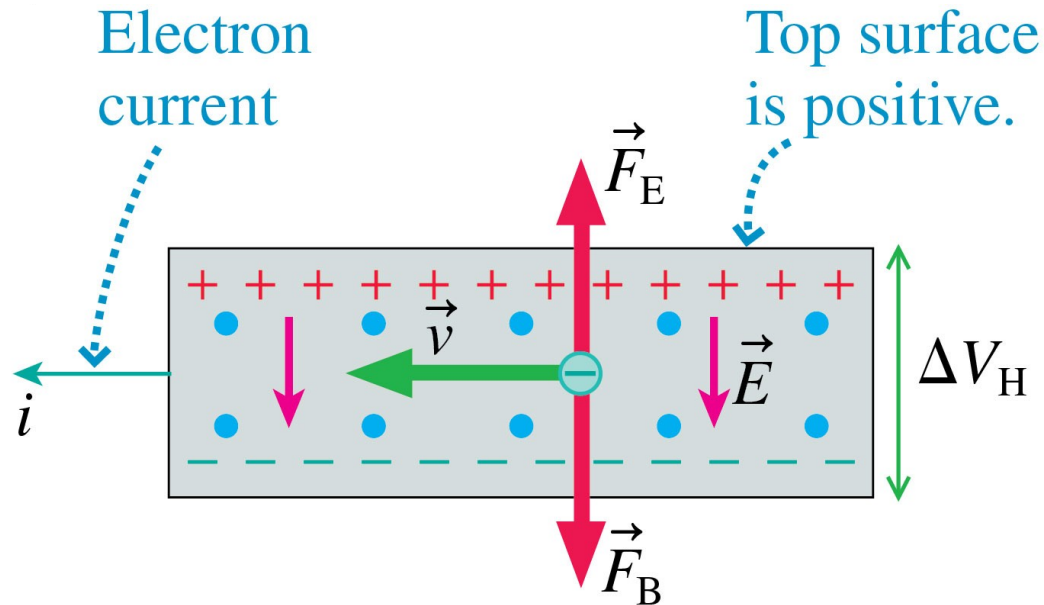
- If the moving charges are *positive*:



- creates a Hall voltage ΔV_H which is higher on the *bottom edge*.

The Hall Effect

- If the moving charges are *negative*:



- creates a Hall voltage ΔV_H which is higher on the top edge.