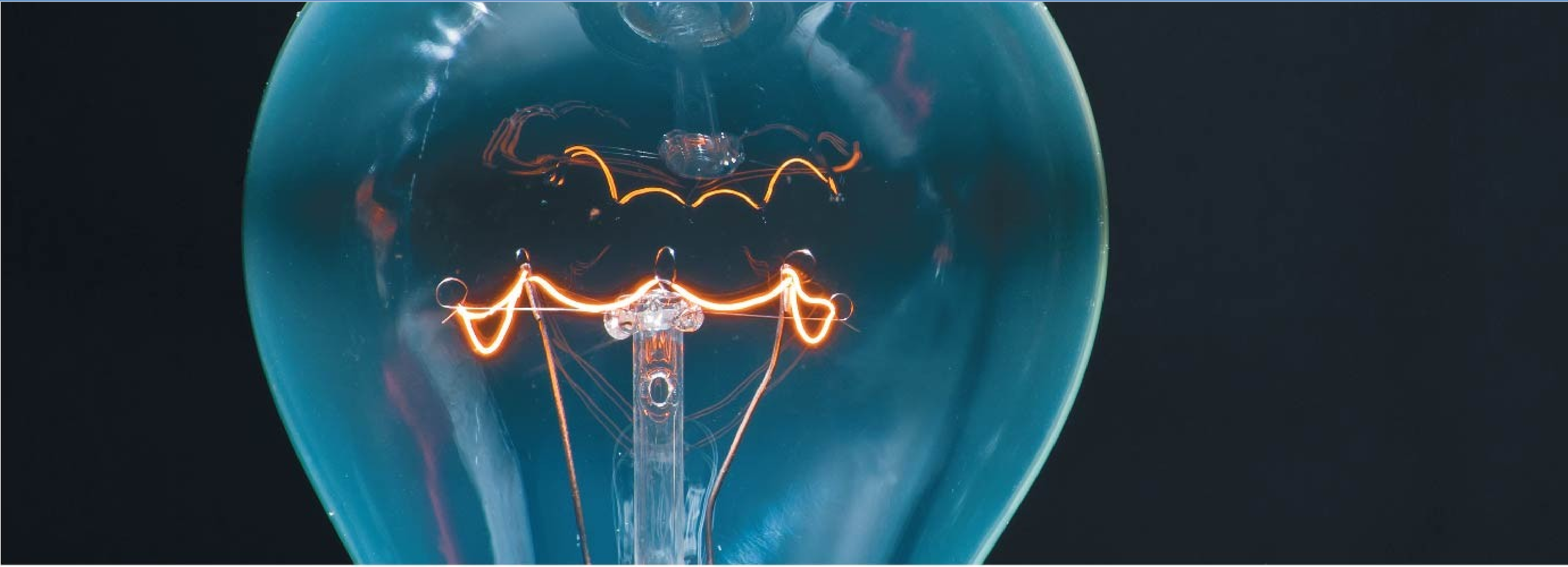
Chapter 9: Current and Resistance

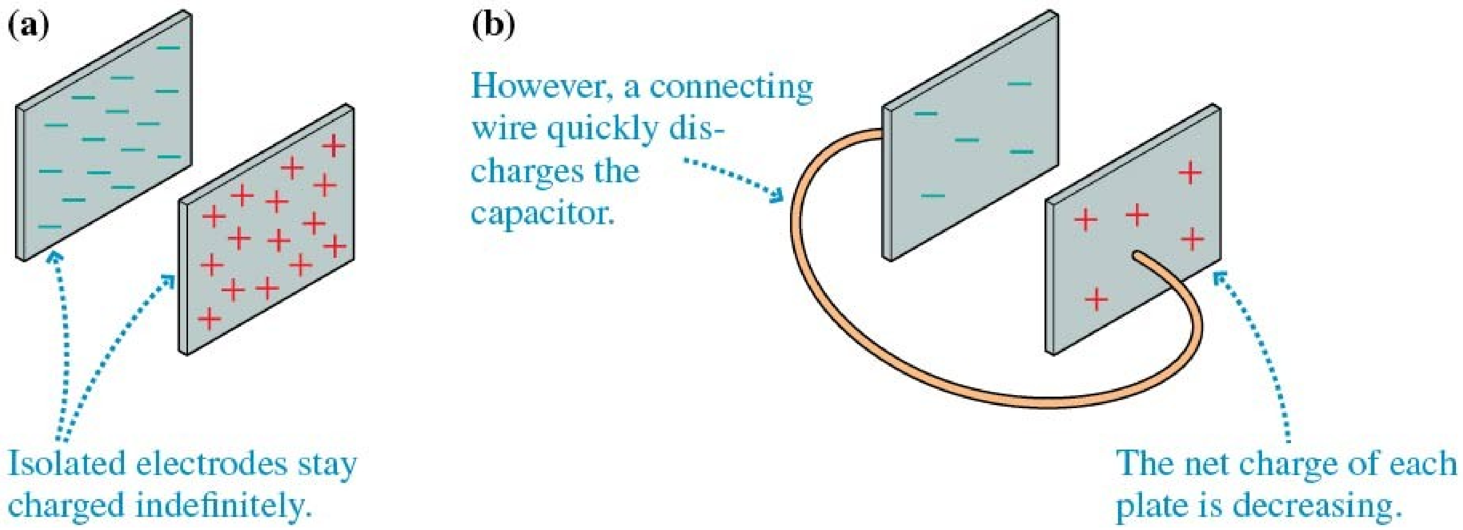


1 Electrical Current
2 Model of Conduction in Metals
3 Resistivity and Resistance
4 Ohm's Law
5 Electrical Energy and Power
6 Superconductors

lecture 6.1.1

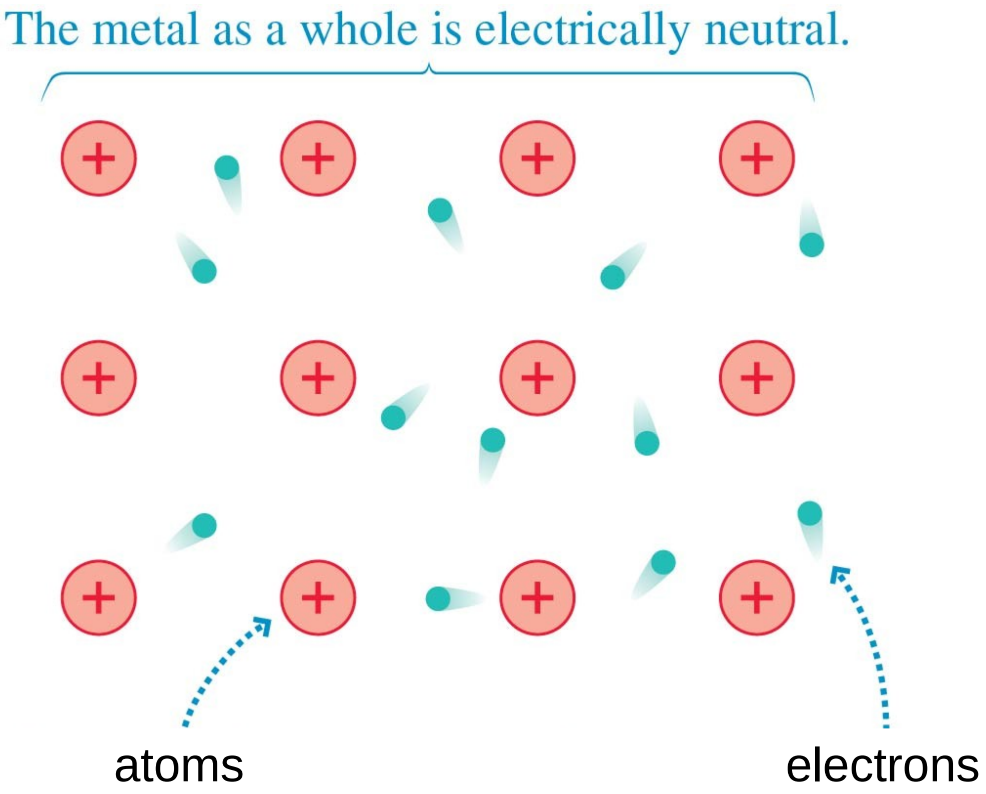
Electric Current

- How does a capacitor get discharged?
- *As* the capacitor is discharging, there is a *current* in the wire.
- When a current is flowing, the conductors are *not* in electrostatic equilibrium.



Electric Current

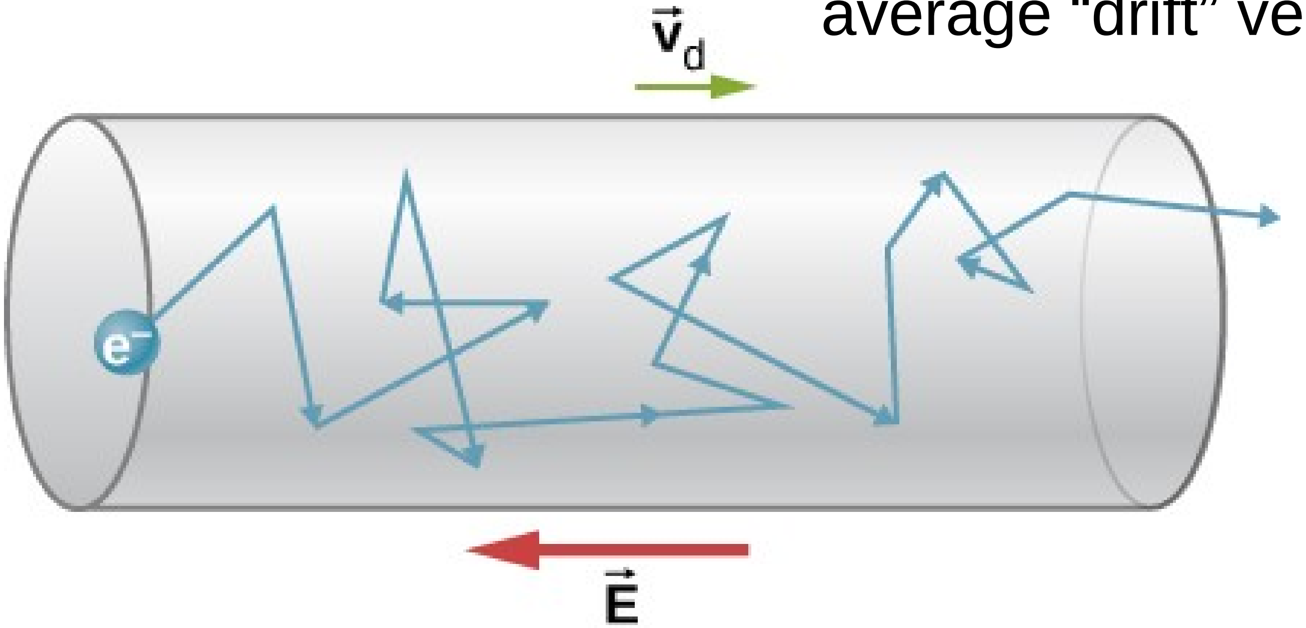
In a conductor, the outer electrons are not attached to their parent nuclei, and form a fluid-like *sea of electrons* that can move through the solid.



Electric Current

Actual path of electrons:

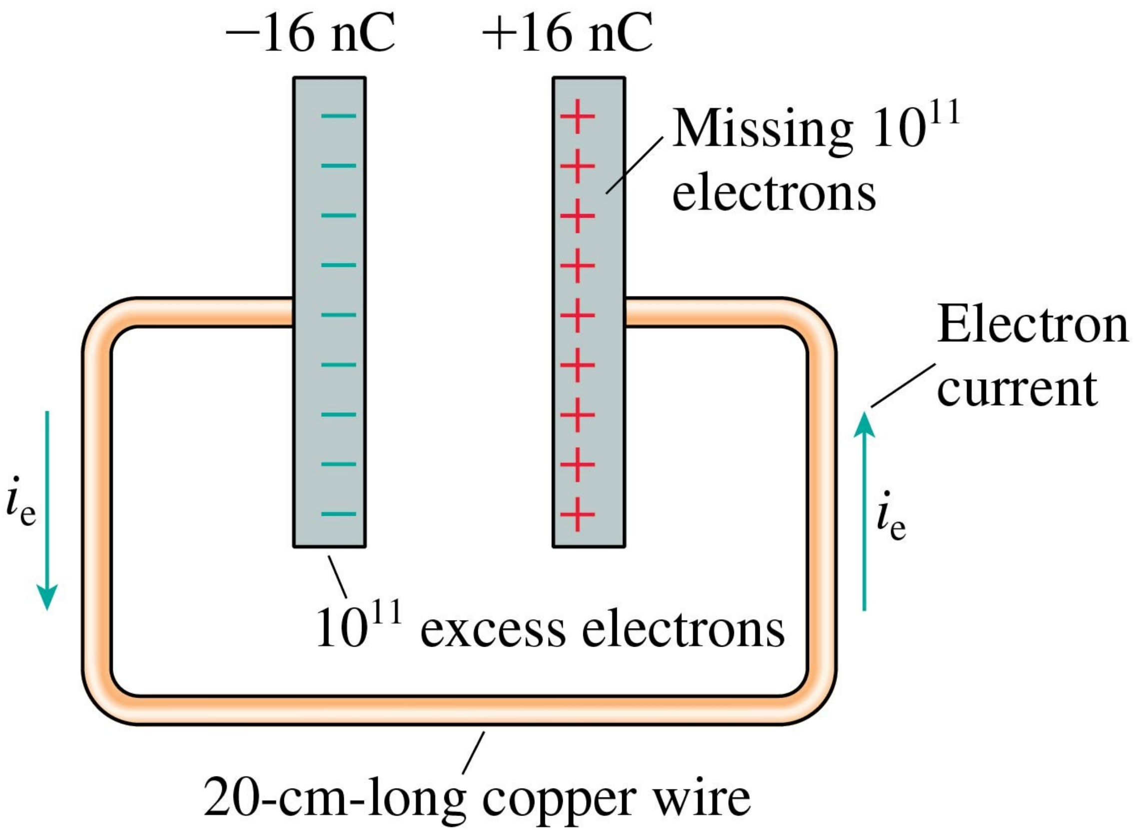
average “drift” velocity



Electric Current

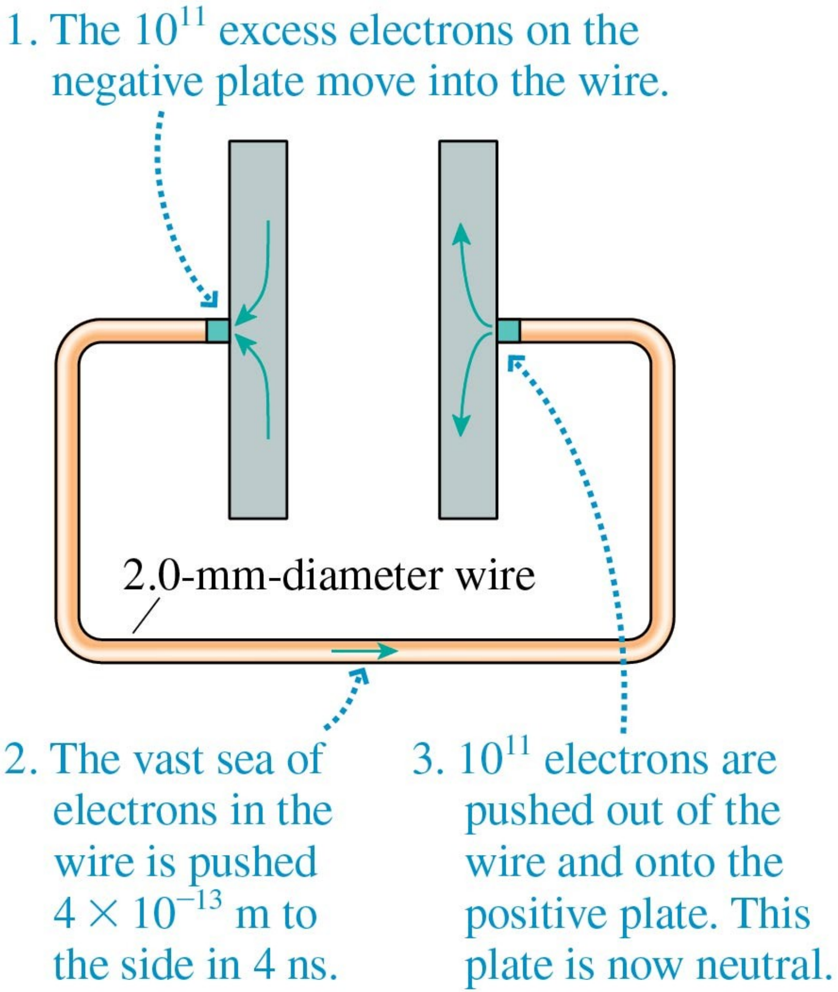
- How long should it take to discharge this capacitor?
- Typical drift speed of electron current through a wire is
  $v_d \approx 10^{-4}$ m/s.
- At this rate, it would take an electron over half an hour to travel 20 cm.

  ??



Electric Current

- The wire is *already full* of electrons.
- We just need to slightly rearrange the charges on the plates *and* in the wire.



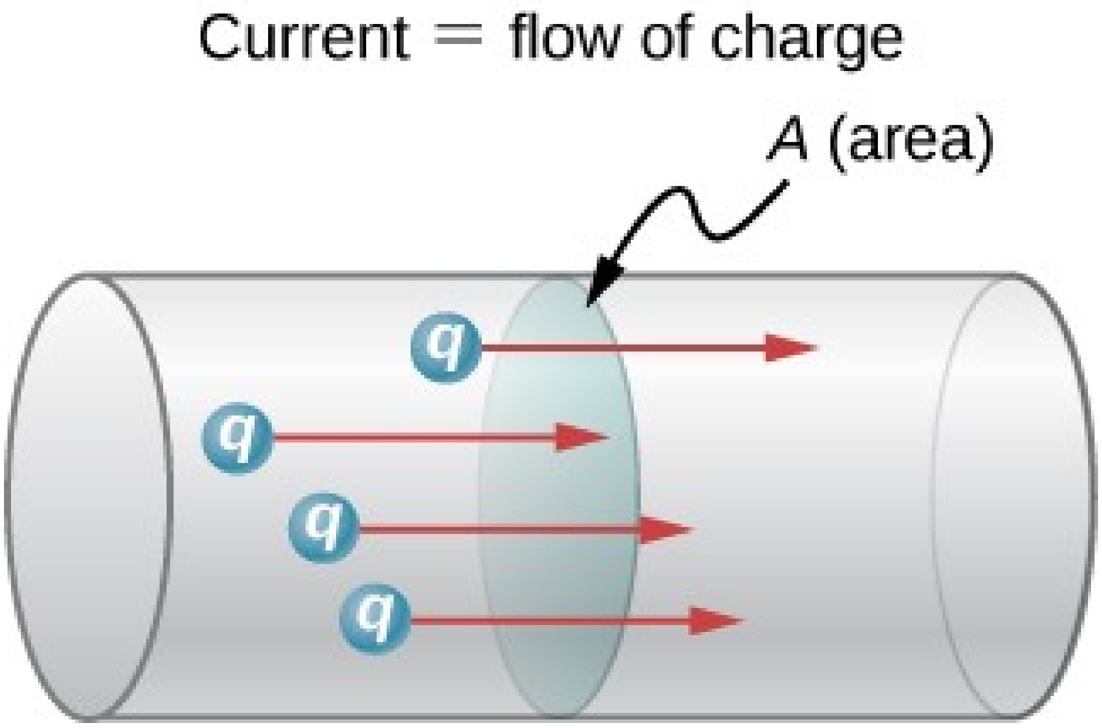
Electric Current

We define the electric current I as the amount of charge passing a given cross-section per second.

$$I = \frac{dQ}{dt}$$

units: Amperes (Amps)

$$1\ \mathrm{A} = 1\ \frac{\mathrm{C}}{\mathrm{s}}$$



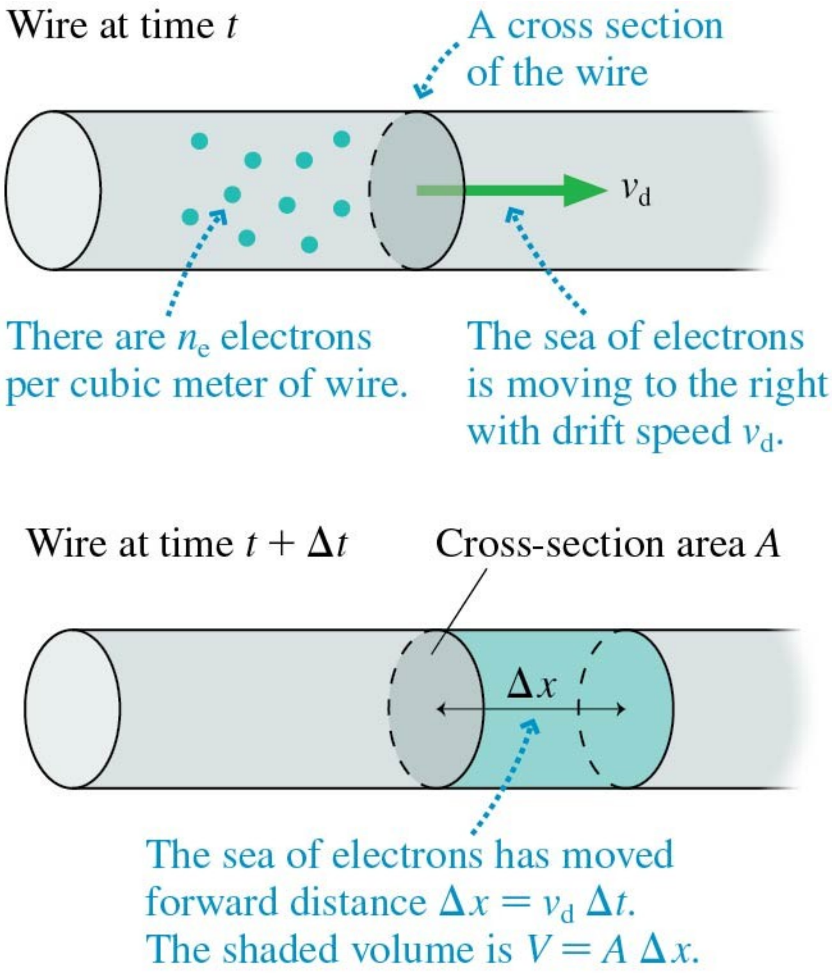
Electric Current

$$I = \frac{dQ}{dt}$$

If there are n charge carriers per unit volume and move at speed v_d then

$$I = \frac{qnV}{\Delta t} = \frac{qnA\Delta x}{\Delta t}$$

$$\boxed{I = qnAv_d}$$



Electric Current

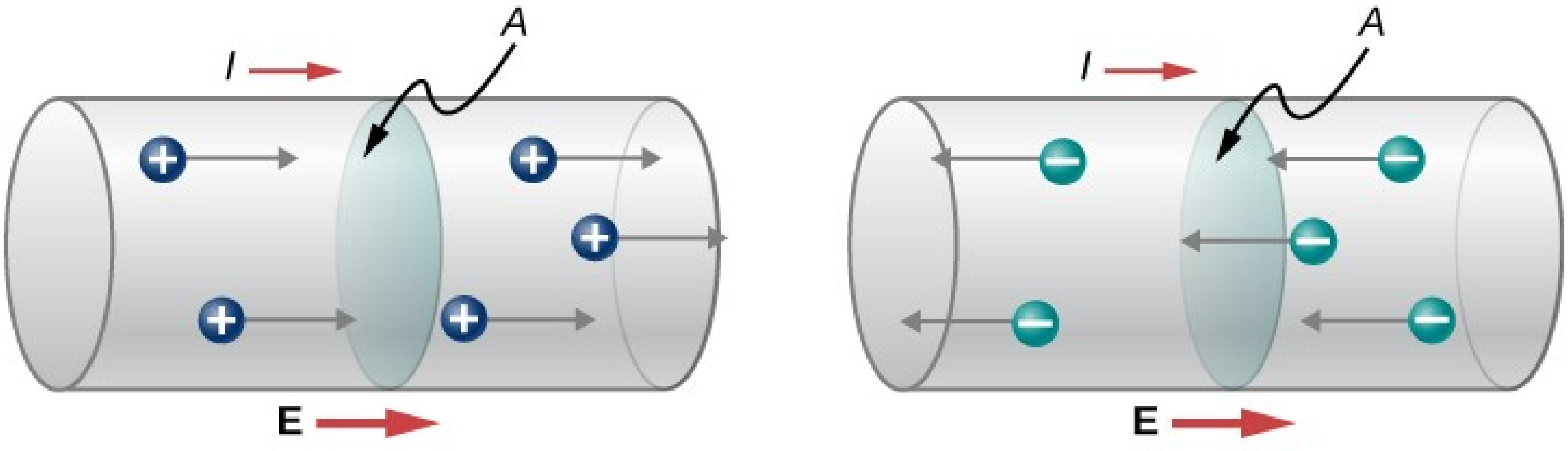
Each atom contributes one valence electron to the sea of electrons.

Metal	Electron density (m^{-3})
Aluminum	6.0×10^{28}
Copper	8.5×10^{28}
Iron	8.5×10^{28}
Gold	5.9×10^{28}
Silver	5.8×10^{28}

Electric Current

If the charge carriers were positive, I would be in the same direction of the drift speed v_d

$$I = qnAv_d$$



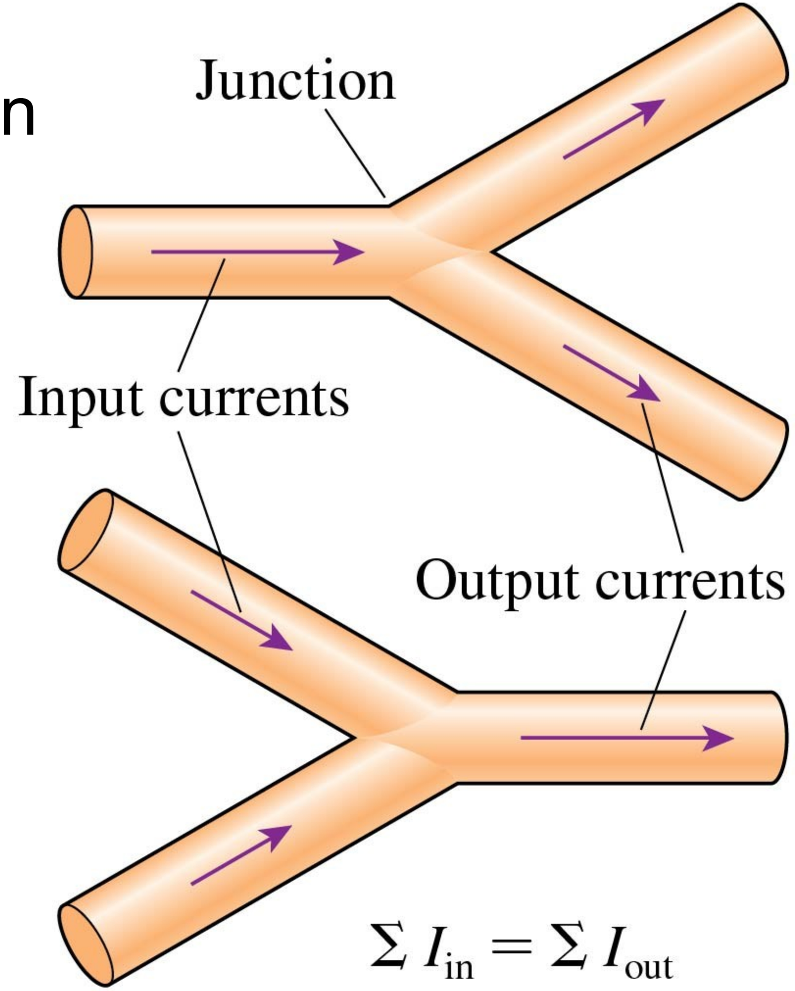
The **conventional current** I is in the opposite direction of the electron flow.

Conservation of Current

- For a *junction*, the law of conservation of current requires that

$$\sum I_{in} = \sum I_{out}$$

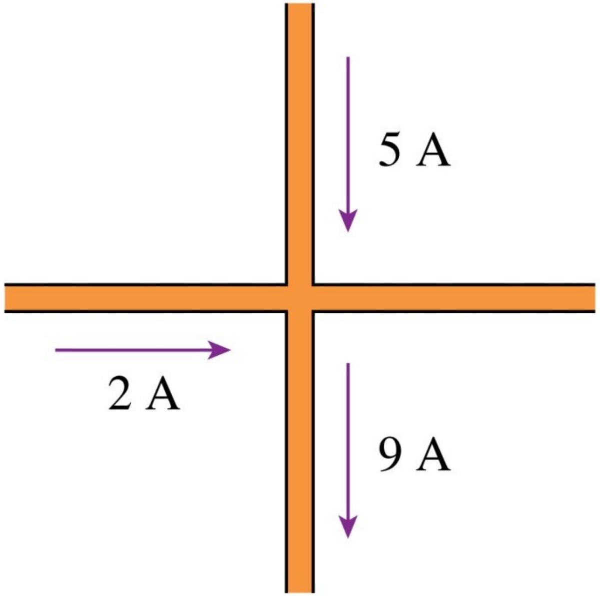
- Called **Kirchhoff's junction law**.



Conservation of Current

The current in the fourth wire is

A. 16 A to the right.
B. 4 A to the left.
C. 2 A to the right.
✔ D. 2 A to the left.
E. Not enough information to tell.



Current Density

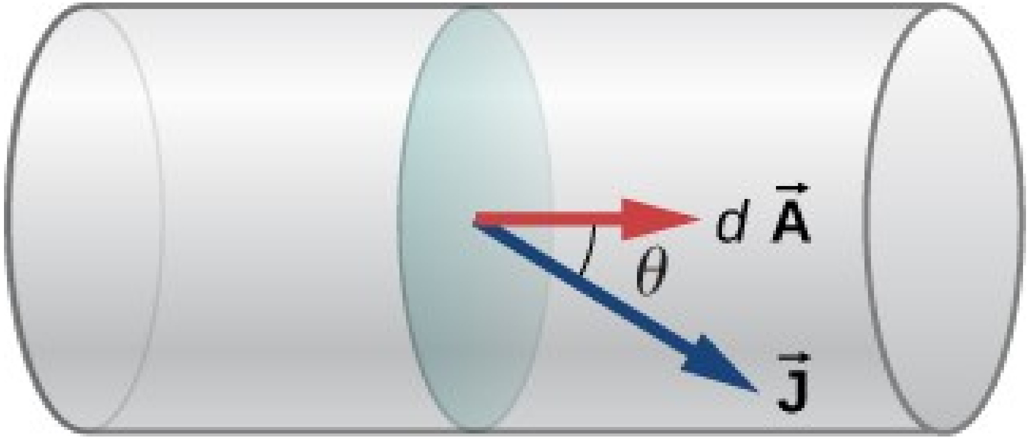
- The **current density** J in a wire is the current per square meter of cross section:

$$J = \frac{I}{A} = \frac{enAv_d}{A} = env_d$$

- The current density has units of $\mathrm{A/m^2}$.

Current Density

$$I = \int \vec{J} \cdot d\vec{A}$$



Current Density

A 1.0 A current passes through a 1.0-mm-diameter aluminum wire. What are the current density and the drift speed of the electrons in the wire?

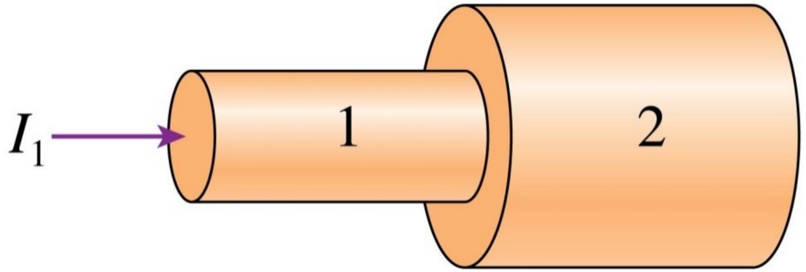
$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{1.0\ \text{A}}{\pi(0.00050\ \text{m})^2} = 1.3 \times 10^6\ \text{A/m}^2$$

$$v_\text{d} = \frac{J}{n_\text{e} e} = 1.3 \times 10^{-4}\ \text{m/s} = 0.13\ \text{mm/s}$$

Metal	Electron density (m^{-3})
Aluminum	6.0×10^{28}
Copper	8.5×10^{28}
Iron	8.5×10^{28}
Gold	5.9×10^{28}
Silver	5.8×10^{28}

Current Density

Both segments of the wire are made of the same metal. Current I_1 flows into segment 1 from the left. How does current I_1 in segment 1 compare to current I_2 in segment 2?



A. $I_1 > I_2$
B. $I_1 = I_2$ ✔
C. $I_1 < I_2$
D. There’s not enough information to compare them.

Current Density

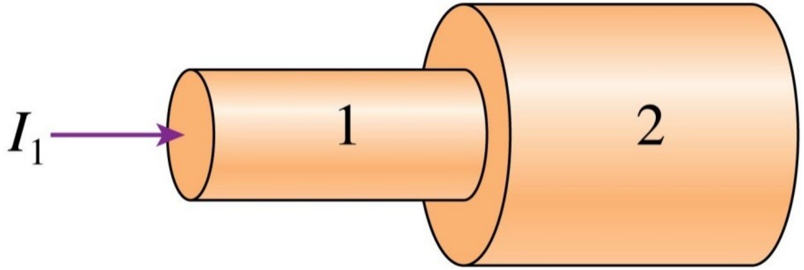
Both segments of the wire are made of the same metal. Current I_1 flows into segment 1 from the left. How does current density J_1 in segment 1 compare to **current density** J_2 in segment 2?

✔ A. $J_1 > J_2$ Smaller cross-section area

B. $J_1 = J_2$

C. $J_1 < J_2$

D. There's not enough information to compare them.



Conductivity and Resistivity

- A given electric field will create greater current in a material with large “conductivity” σ

$$J = \sigma E$$

- The resistivity tells us how reluctantly the electrons move in response to an electric field:

$$\rho = \text{resistivity} = \frac{1}{\sigma}$$

Conductivity and Resistivity

Material	Resistivity ($\Omega\,m$)	Conductivity ($\Omega^{-1}m^{-1}$)
Aluminum	2.8×10^{-8}	3.5×10^{7}
Copper	1.7×10^{-8}	6.0×10^{7}
Gold	2.4×10^{-8}	4.1×10^{7}
Iron	9.7×10^{-8}	1.0×10^{7}
Silver	1.6×10^{-8}	6.2×10^{7}
Tungsten	5.6×10^{-8}	1.8×10^{7}
Nichrome*	1.5×10^{-6}	6.7×10^{5}
Carbon	3.5×10^{-5}	2.9×10^{4}

*Nickel-chromium alloy used for heating wires.

Conductivity and Resistivity

A 2.0-mm-diameter aluminum wire carries a current of 800 mA. What is the electric field strength inside the wire?

$$E = \frac{J}{\sigma} = \frac{I}{\sigma \pi r^2} = \frac{0.80\ \text{A}}{(3.5 \times 10^7\ \Omega^{-1}\ \text{m}^{-1})\pi(0.0010\ \text{m})^2} = 0.0073\ \text{V/m}$$

Material	Resistivity (Ω m)	Conductivity (Ω^{-1} m^{-1})
Aluminum	2.8×10^{-8}	3.5×10^{7}
Copper	1.7×10^{-8}	6.0×10^{7}
Gold	2.4×10^{-8}	4.1×10^{7}
Iron	9.7×10^{-8}	1.0×10^{7}
Silver	1.6×10^{-8}	6.2×10^{7}
Tungsten	5.6×10^{-8}	1.8×10^{7}

Resistance and Ohm's Law

- A electric field E is creating current I by pushing the charge carriers.
- The field strength is

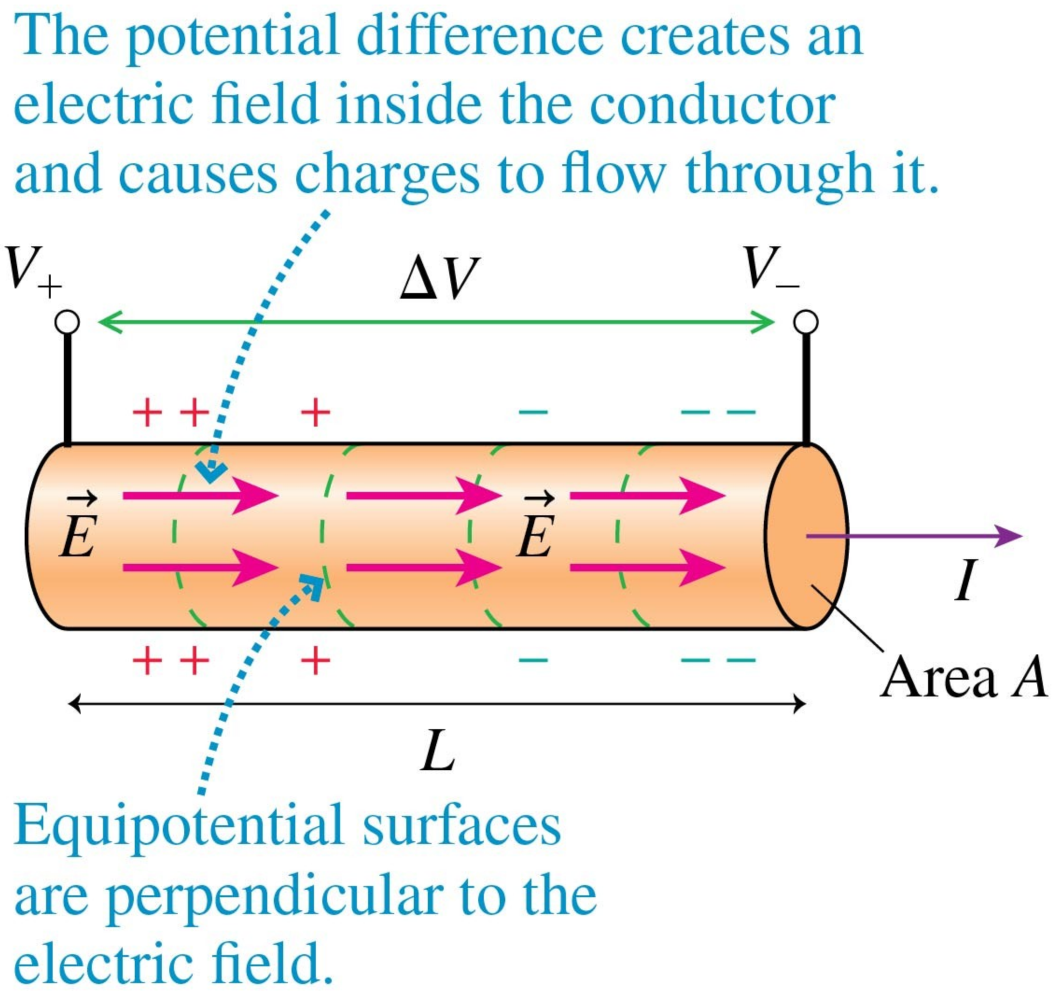
$$E = \frac{\Delta V}{\Delta s} = \frac{\Delta V}{L}$$

- The current density is

$$J = I/A = E/\rho$$

- So the current is related to ΔV by

$$I = \frac{A}{\rho L}\Delta V$$



Resistance and Ohm's Law

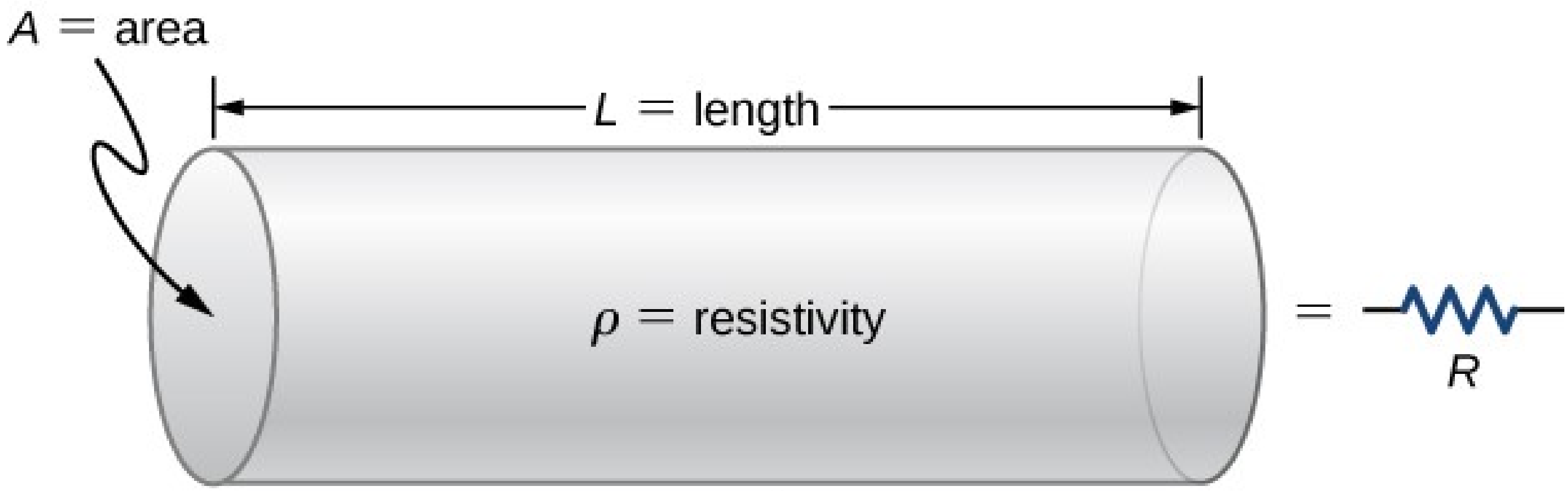
- We define the **resistance** R of a long, thin conductor of length L and cross-sectional area A to be

$$R = \frac{\rho L}{A}$$

- Unit: *ohm*
- 1 ohm = 1 Ω = 1 V/A
- The current through a conductor is determined by the potential difference ΔV along its length:

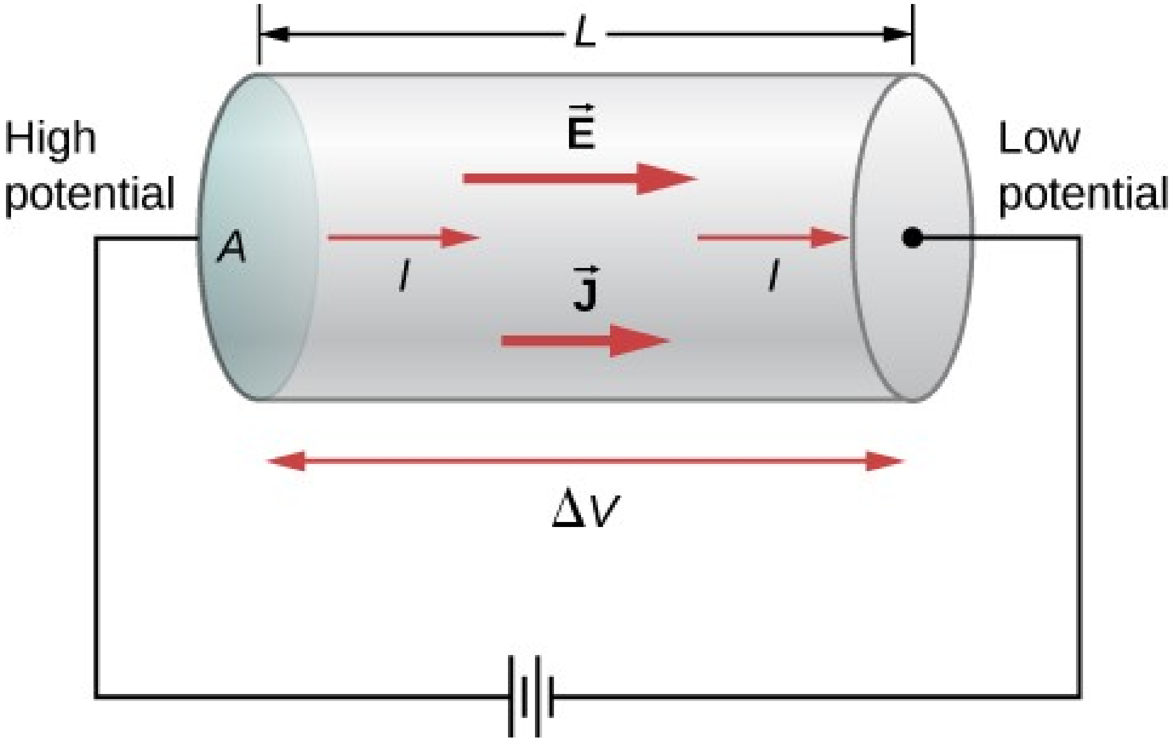
$$I = \frac{\Delta V}{R} \qquad \text{(Ohm's law)}$$

Resistance and Ohm's Law



$$R = \frac{\rho L}{A}$$

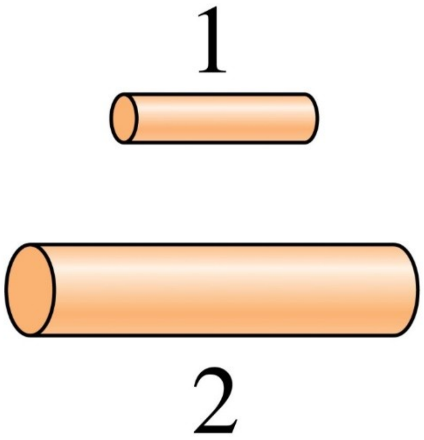
Resistance and Ohm's Law



Resistance and Ohm's Law

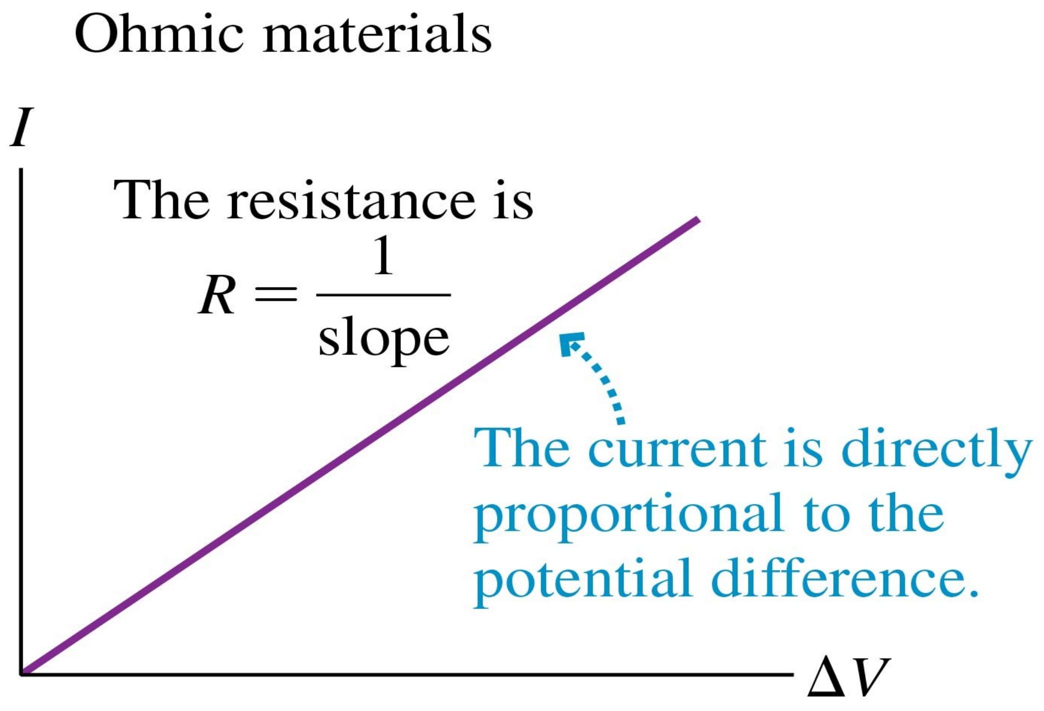
Wire 2 is twice the length and twice the diameter of wire 1. What is the ratio R_2/R_1 of their resistances?

A. 1/4
✔ B. 1/2
C. 1
D. 2
E. 4



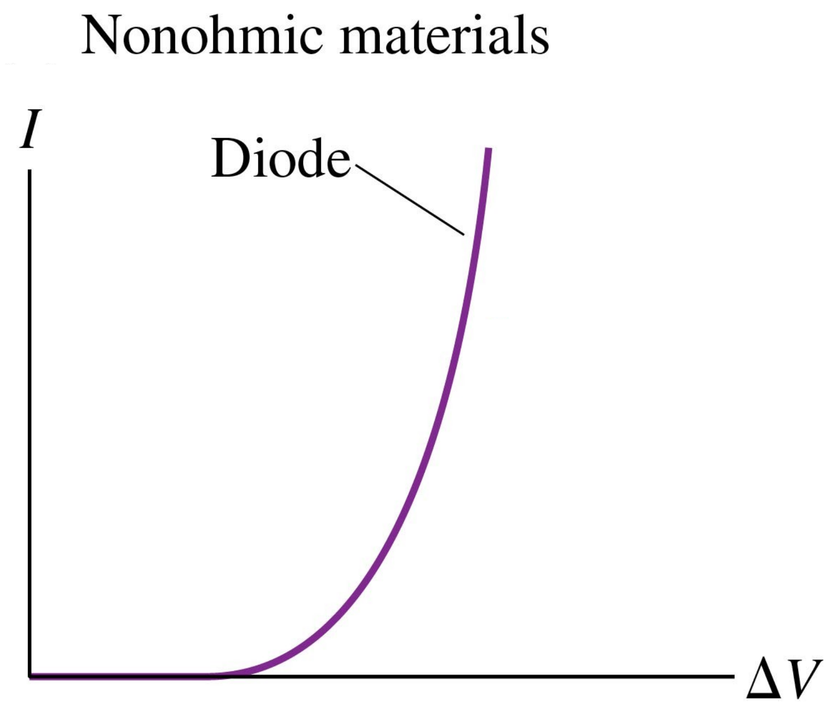
$$R = \frac{\rho L}{A}$$

Resistance and Ohm’s Law



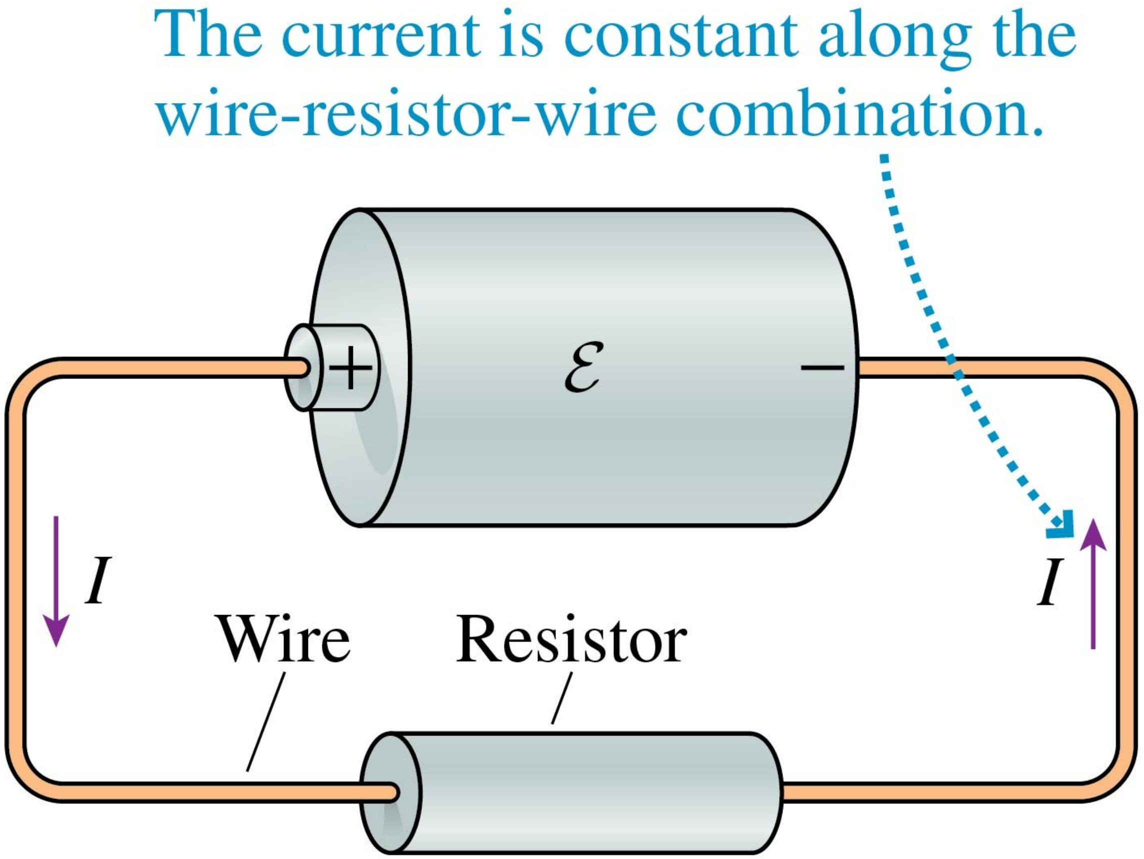
Resistance and Ohm's Law

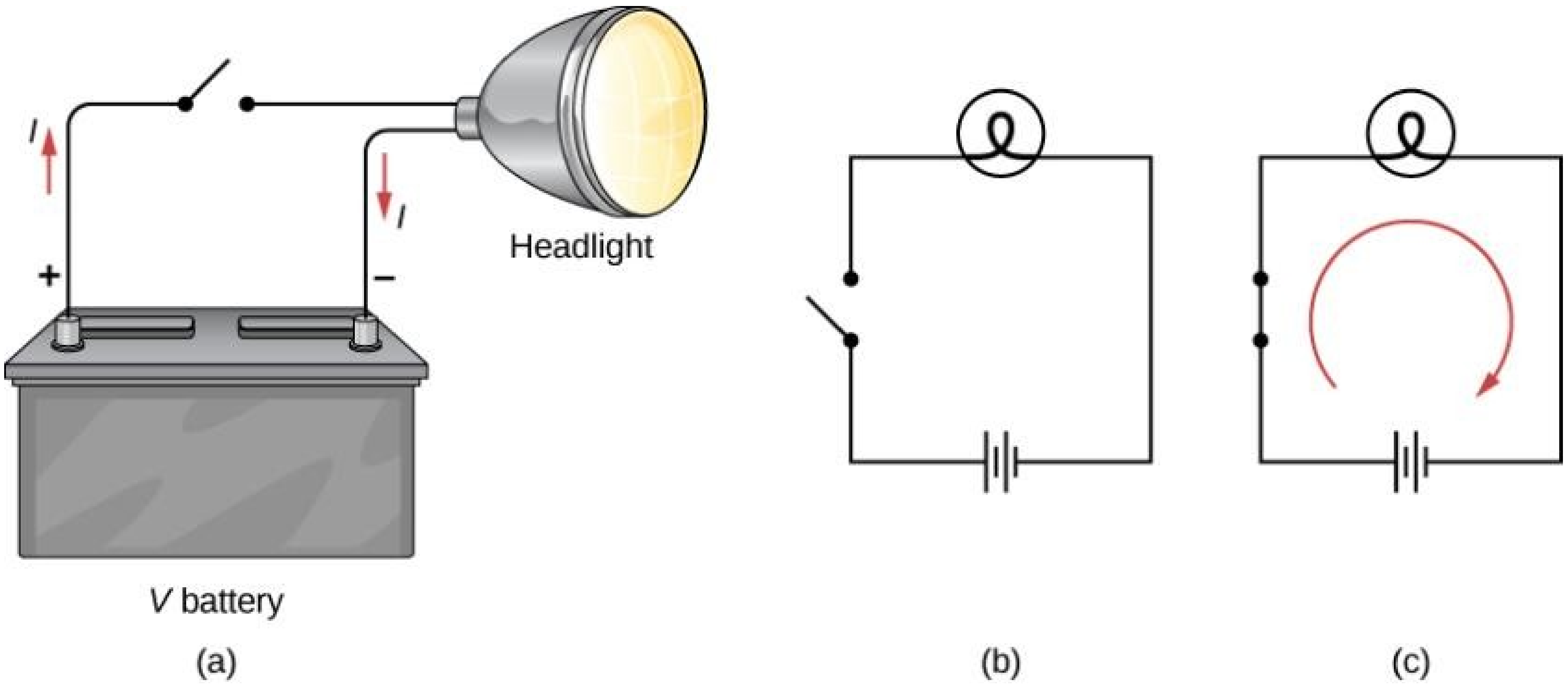
- *Nonohmic* materials have current that is *not* directly proportional to the potential difference.
- Diodes, batteries, and capacitors are all nonohmic devices.



Resistance and Ohm's Law

- A resistor connected to a battery with current-carrying wires.
- Current must be conserved; hence the current I through the resistor is the same as the current in each wire.




I
I
Headlight
+
−
V battery
(a)
(b)
(c)

Energy and Power

- The power supplied by a battery is

$$P_{\text{bat}} = I\mathcal{E} \qquad \text{(power delivered by an emf)}$$

- The units of power are J/s or W.
- The power dissipated by a resistor is

$$P_R = \frac{dE_{\text{th}}}{dt} = \frac{dq}{dt}\Delta V_R = I\,\Delta V_R$$

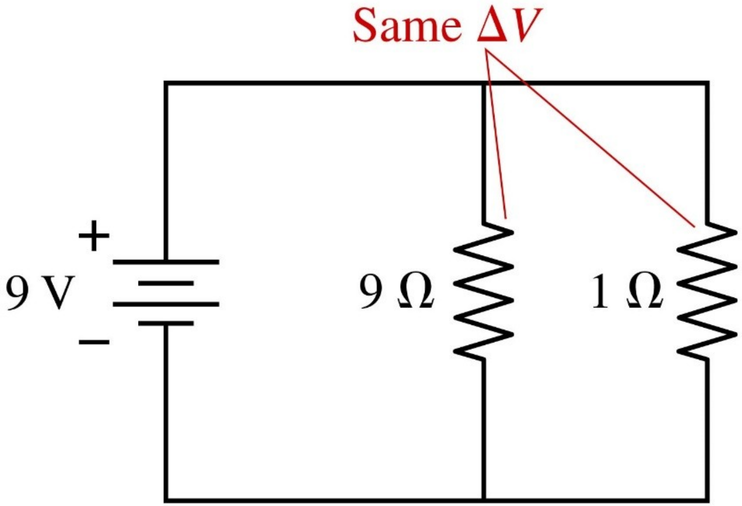
- Or, in terms of the potential drop across the resistor,

$$P_R = I\Delta V_R = I^2 R = \frac{(\Delta V_R)^2}{R} \qquad \text{(power dissipated by a resistor)}$$

Energy and Power

Which resistor dissipates more power?

A. The $9\ \Omega$ resistor
✔ B. The $1\ \Omega$ resistor
C. They dissipate the same power.



$$P = \frac{(\Delta V)^2}{R}$$

Energy and Power

Which has a larger resistance, a 60 W light bulb or a 100 W light bulb?

✔ A. The 60 W bulb

$$P = \frac{(\Delta V)^2}{R} \text{ with both used at } \Delta V = 120 \text{ V}$$

B. The 100 W bulb

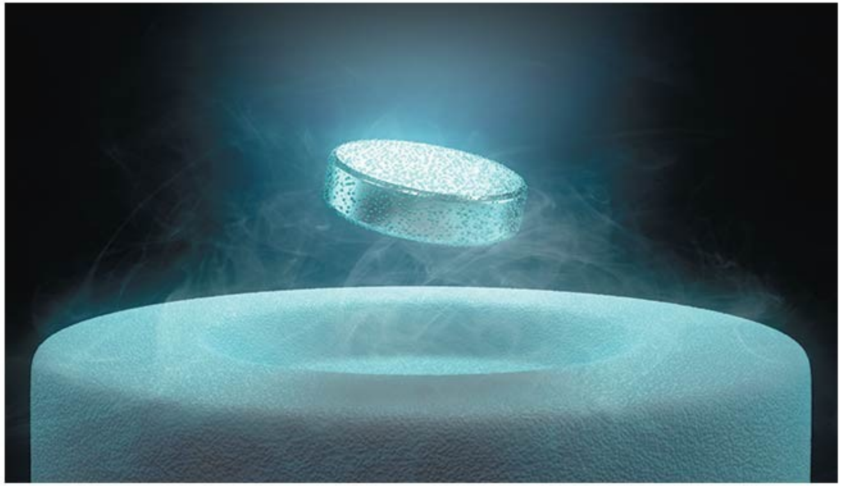
C. Their resistances are the same.

Energy and Power

- Watts times seconds is joules of energy
- Electric companies prefers to use the **kilowatt hour**, to measure the energy you use each month.
- Example:
  - A $4000\ \mathrm{W}$ electric water heater uses $40\ \mathrm{kWh}$ of energy in 10 hours.

Superconductivity

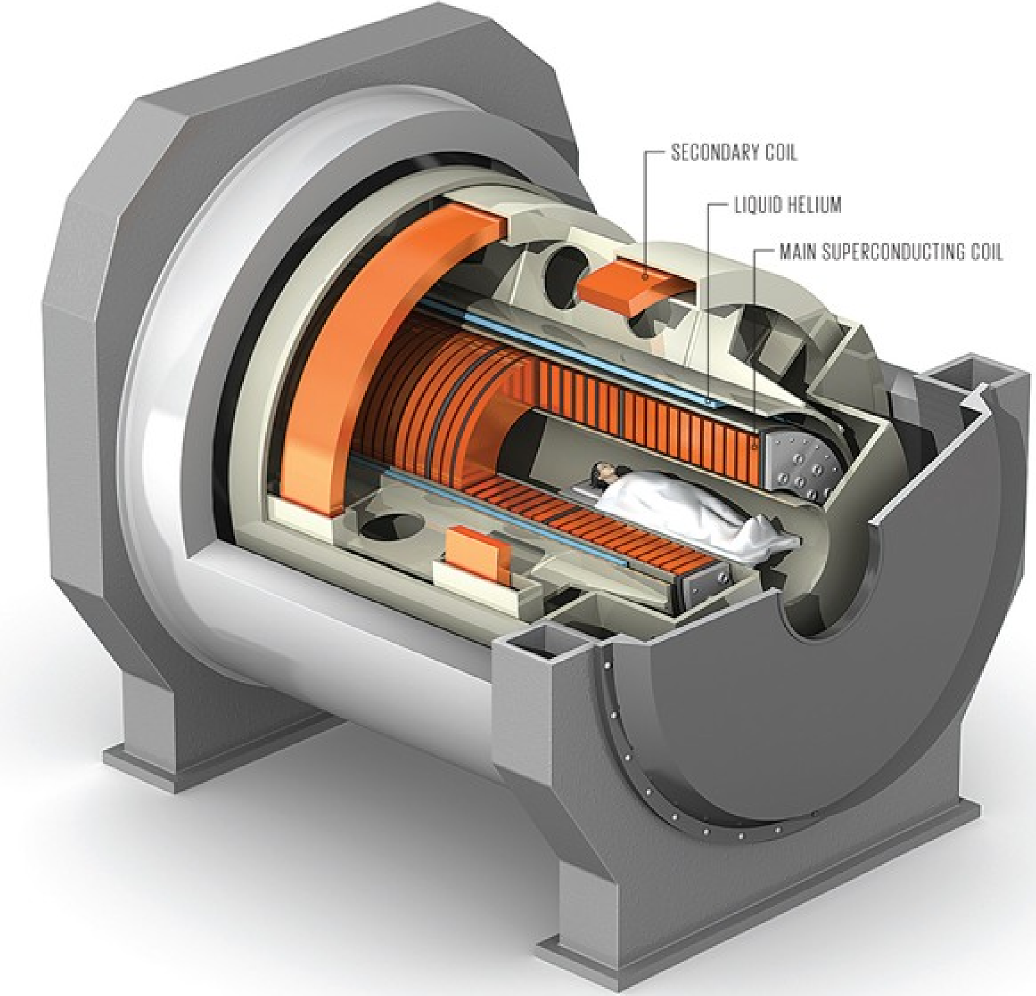
- Certain materials suddenly lose *all* resistance to current when cooled below a certain temperature.
- Called **superconductivity**.



Superconductors have unusual magnetic properties. Here a small permanent magnet levitates above a disk of the high temperature superconductor $YBa_2Cu_3O_7$ that has been cooled to liquid-nitrogen temperature.

Superconductivity

An MRI uses superconducting coils to create a strong magnetic field.



Superconductivity

Meissner Effect: Superconductors will “lock in” their magnetic field flux.

Superconductivity

Meissner Effect: Superconductors will "lock in" their magnetic field flux.