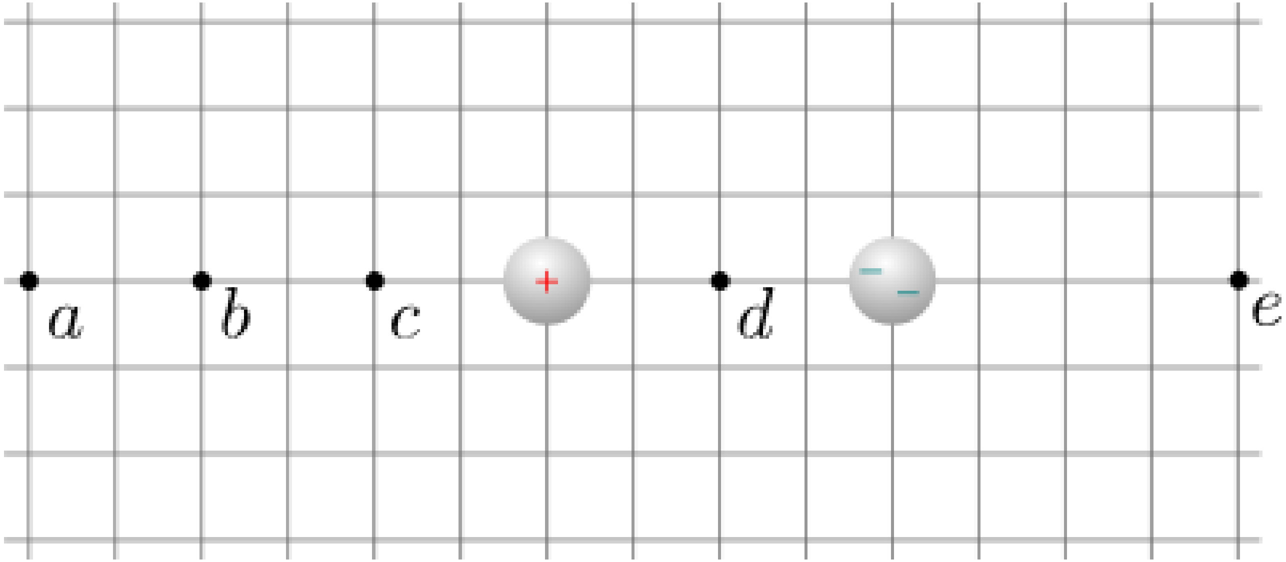
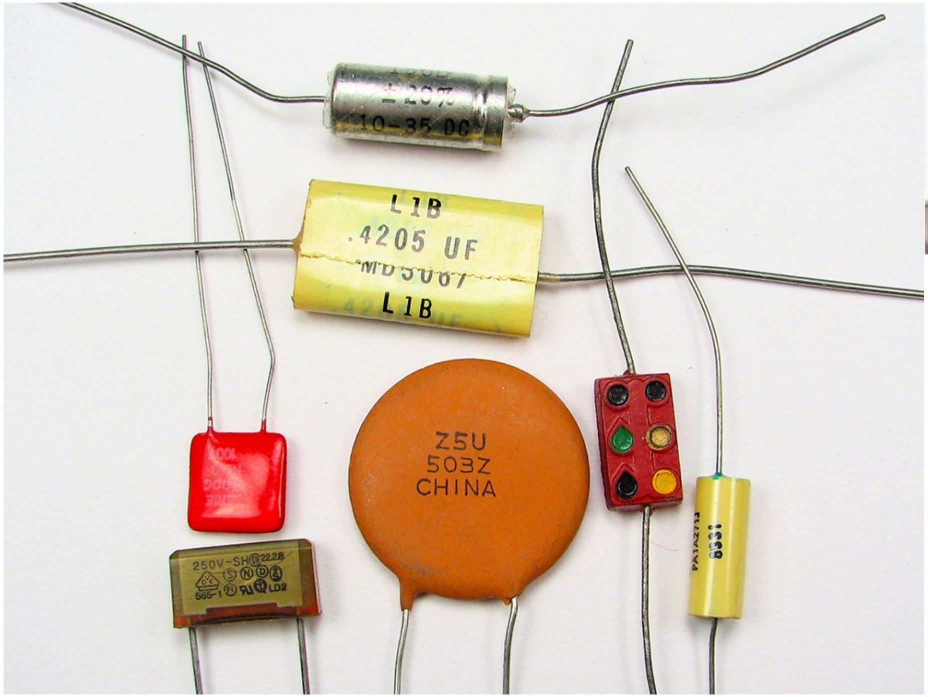
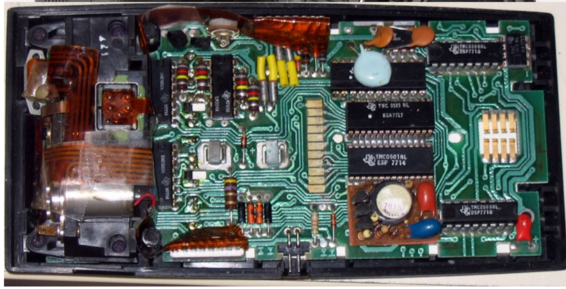
Which point has zero electric potential?



Chapter 8: Capacitance

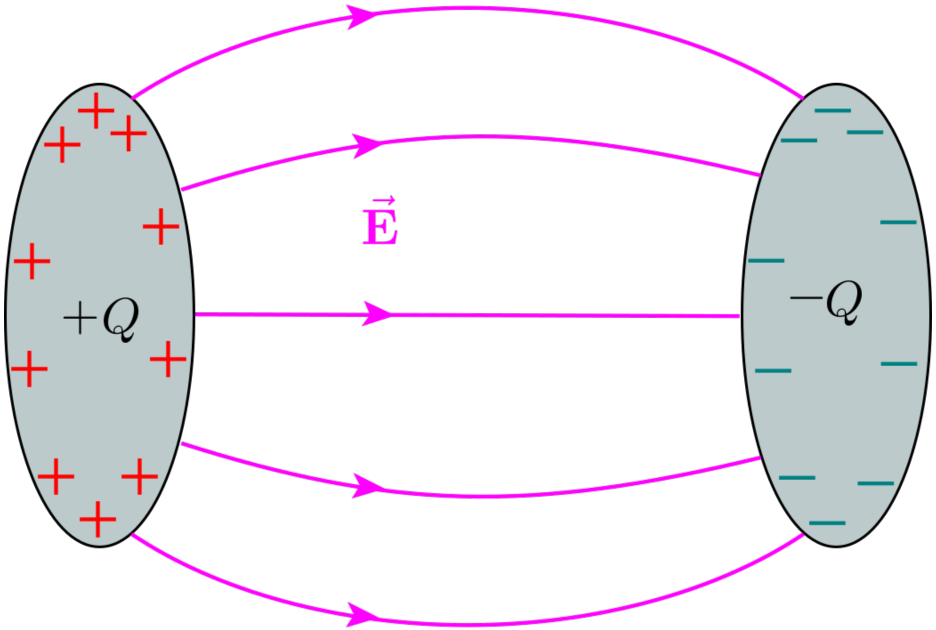


1 Capacitors and Capacitance
2 Capacitors in Series and in Parallel
3 Energy Stored in a Capacitor
4 Capacitor with a Dielectric
5 Molecular Model of a Dielectric

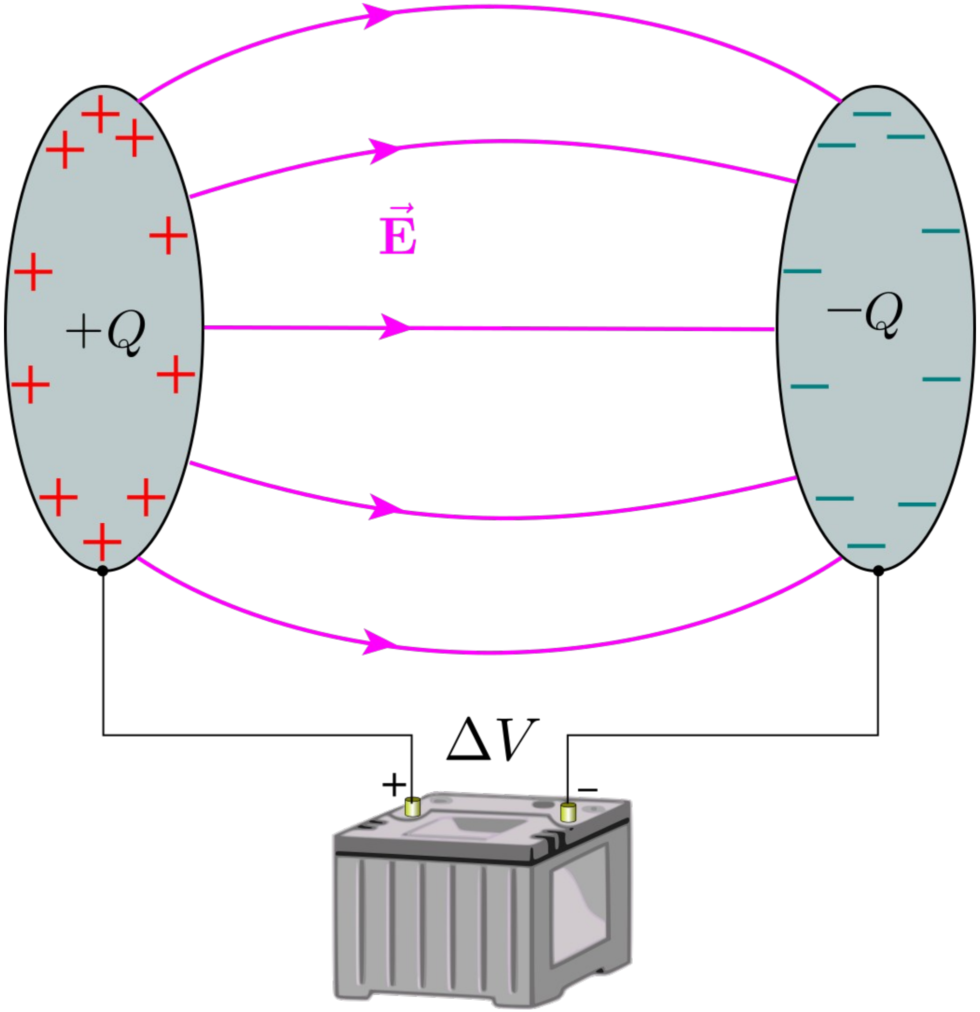
Capacitance and Capacitors

- Separating charge Q onto two “electrodes” creates a potential difference ΔV between them



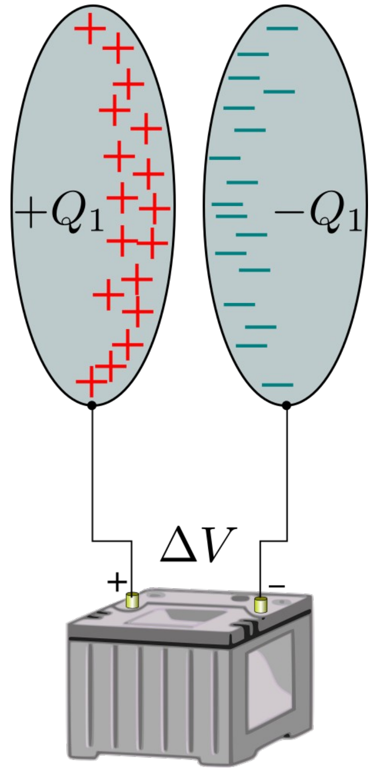
Capacitance and Capacitors

- A voltage source (like a battery) does work on charges to move them
- The voltage source maintains a constant ΔV between the electrodes

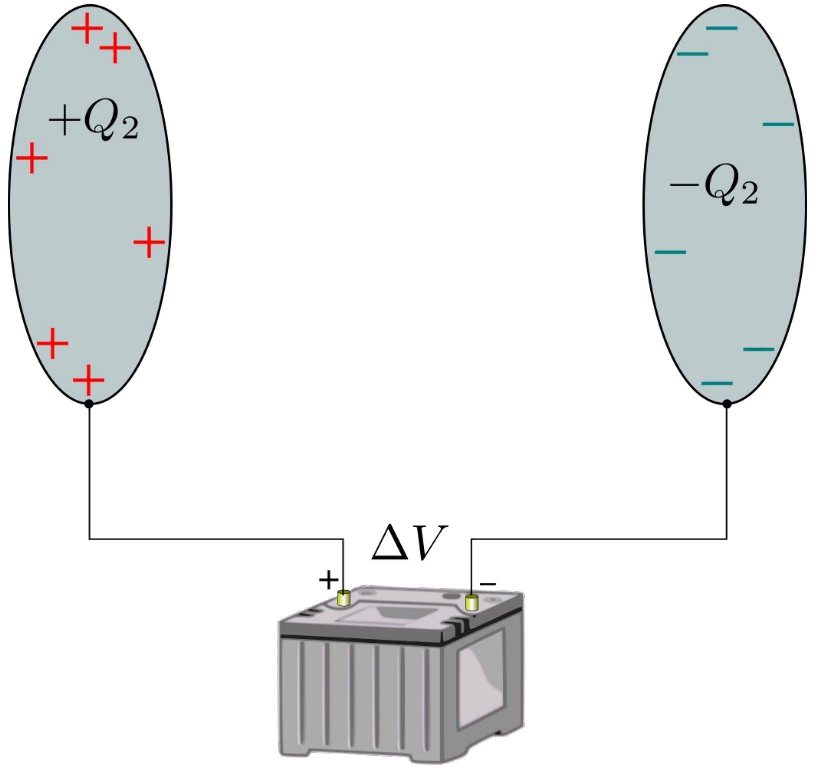


Capacitance and Capacitors

The amount of charge Q that voltage ΔV can separate depends on the geometry of the electrodes

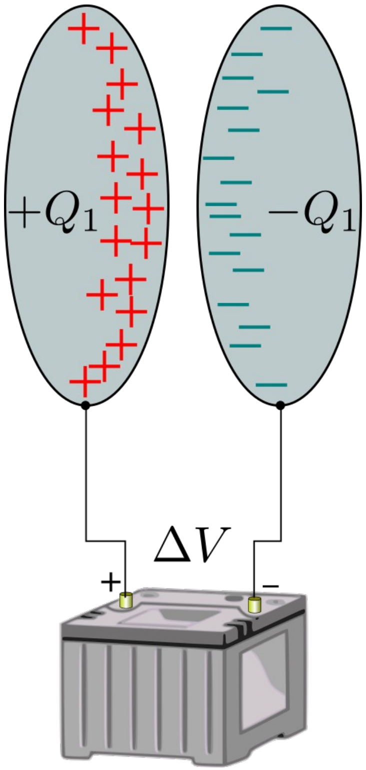


$$Q_1 > Q_2$$

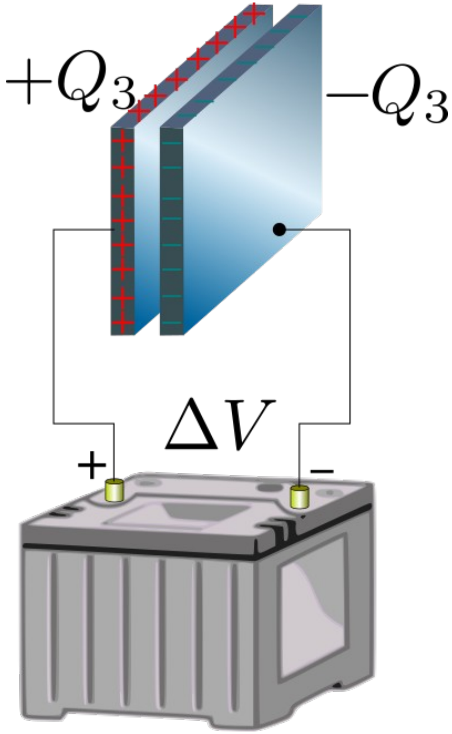


Capacitance and Capacitors

The parallel-plate capacitor can store a lot of charge per volt applied.



$$Q_3 > Q_1$$



Capacitance and Capacitors

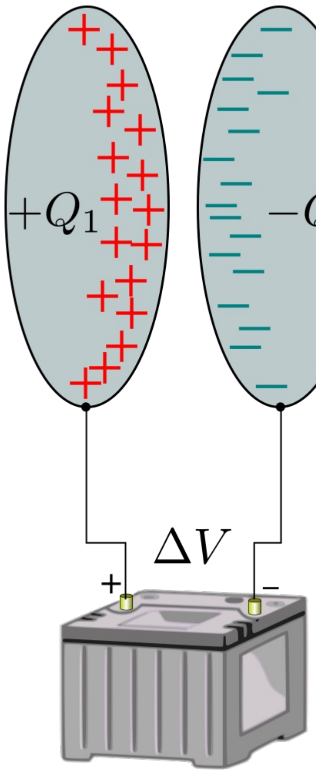
- The amount of charge per volt that a pair of electrodes can store is called its "capacitance".

$$C = \frac{Q}{\Delta V}$$

- unit: Farads (F)

$$1\text{F} = 1\frac{\text{C}}{\text{V}}$$

- Capacitance depends on physical configuration of the conductors, not on Q or ΔV



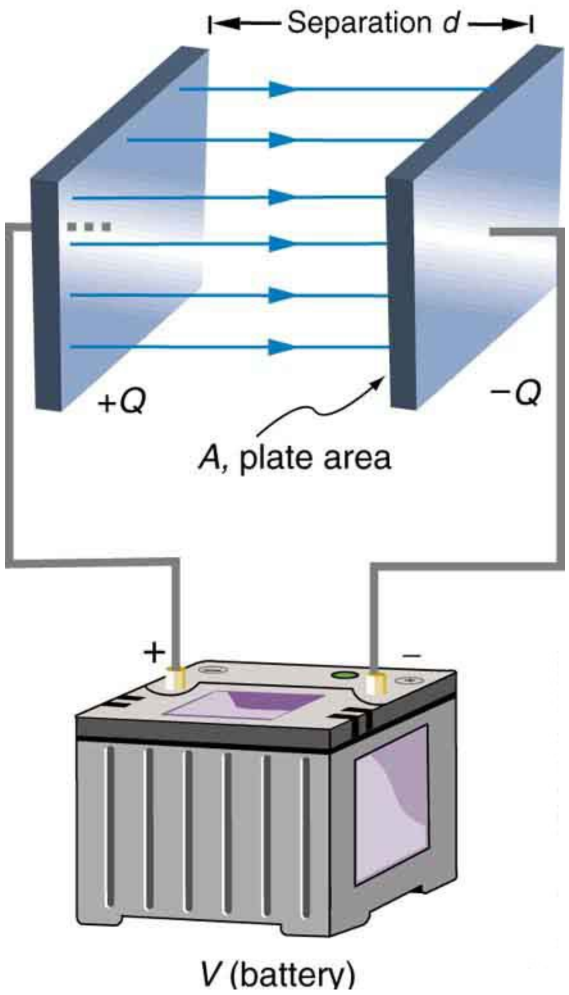
Capacitance and Capacitors

Derive the capacitance of a parallel-plate capacitor

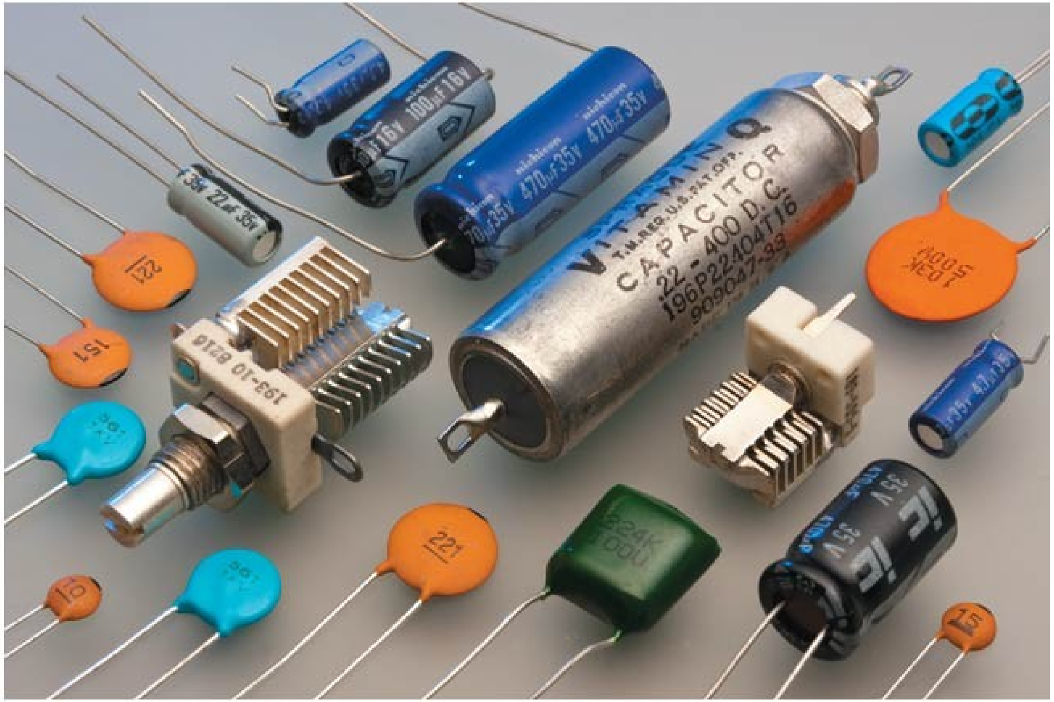
$$\Delta V = -\int_i^f E\, ds$$

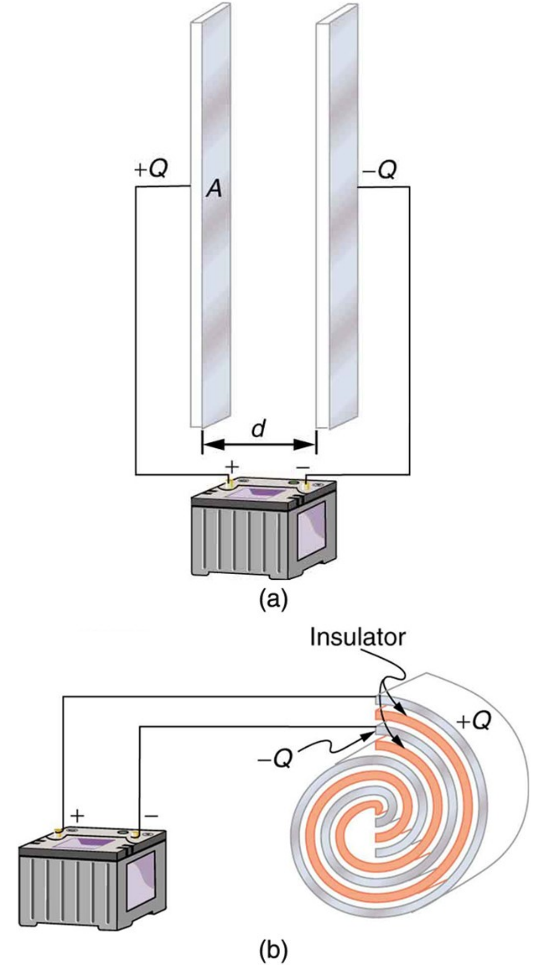
$$= -\int_i^f \frac{\sigma}{\epsilon_0}\, ds = -\frac{\sigma}{\epsilon_0} d = -\frac{Q}{A\epsilon_0} d$$

$$C = \frac{\epsilon_0 A}{d}$$



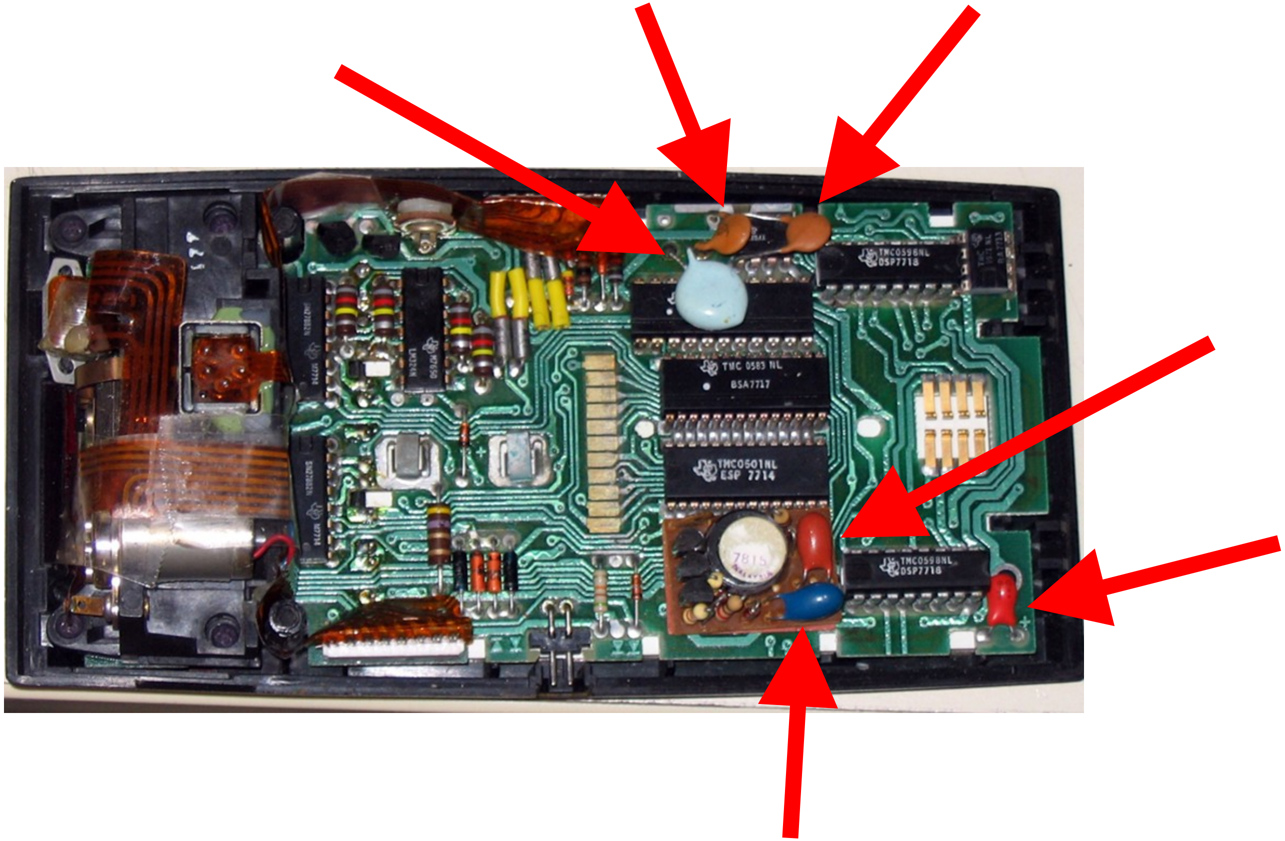
Capacitance and Capacitors





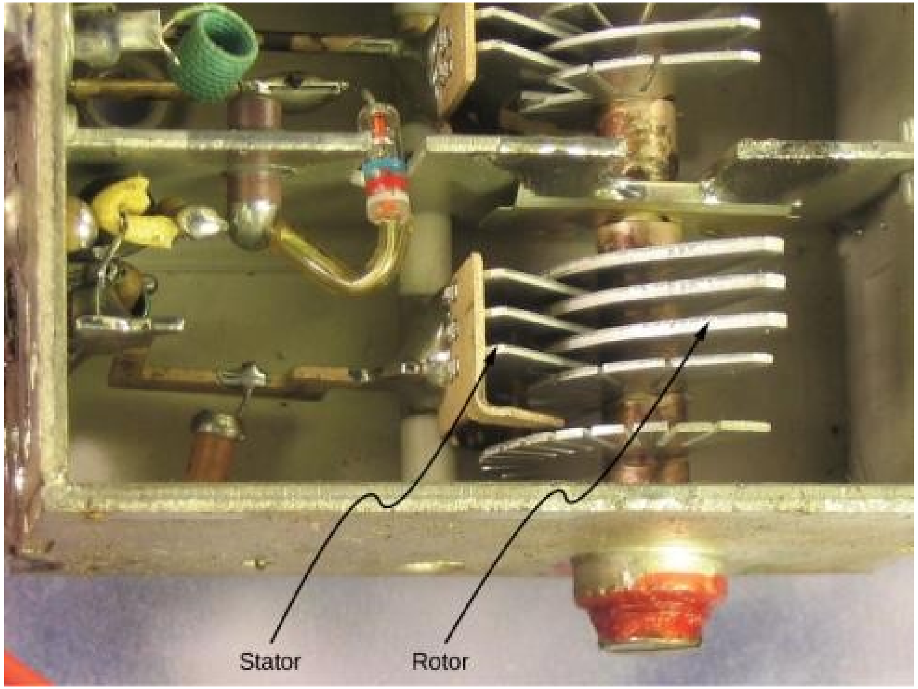
Capacitors come in a variety of sizes and shapes.

Capacitance and Capacitors

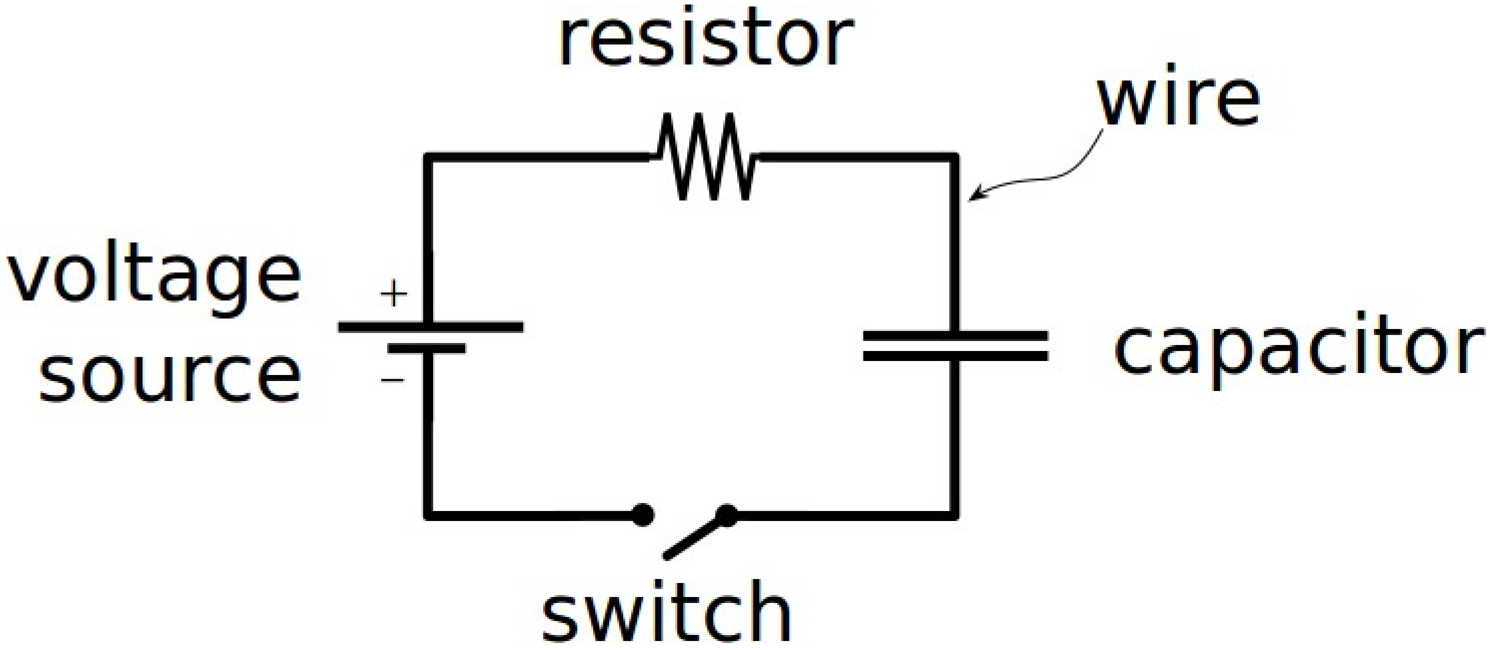


Capacitance and Capacitors

This is a variable capacitor

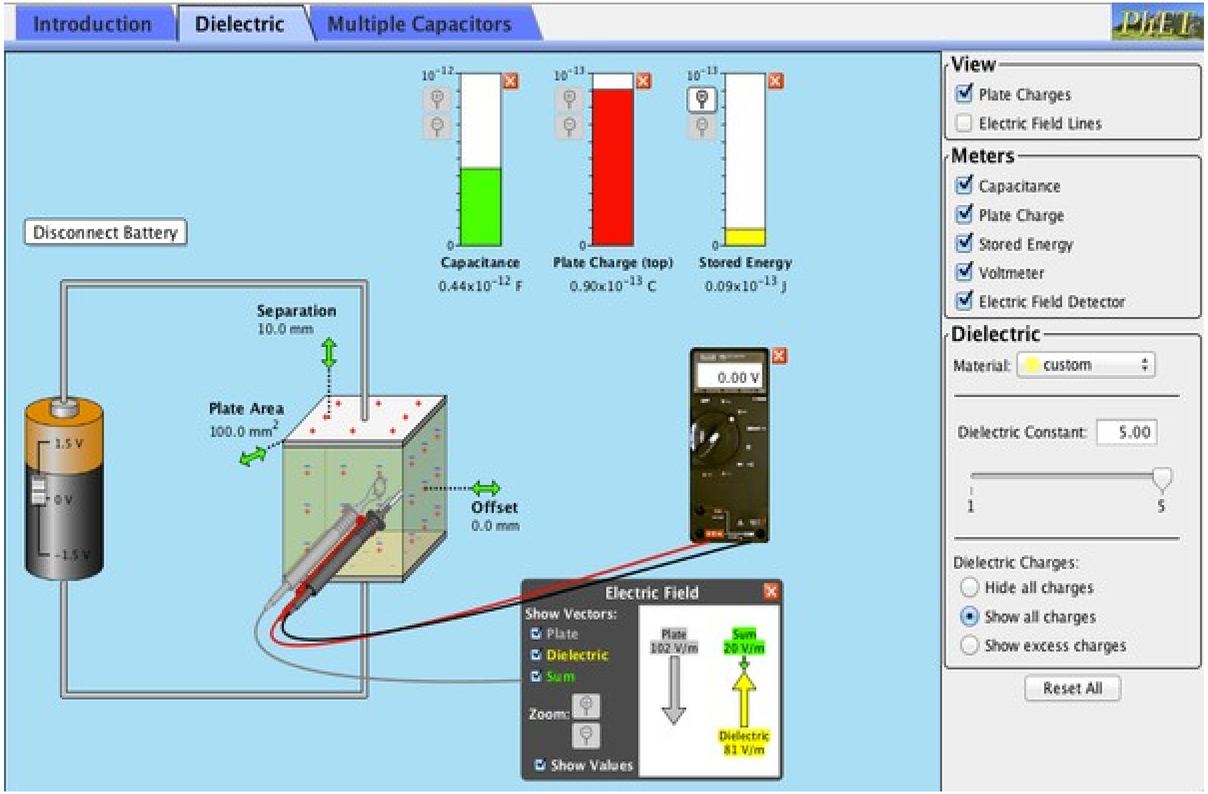


Circuit Diagrams



Circuit Diagrams

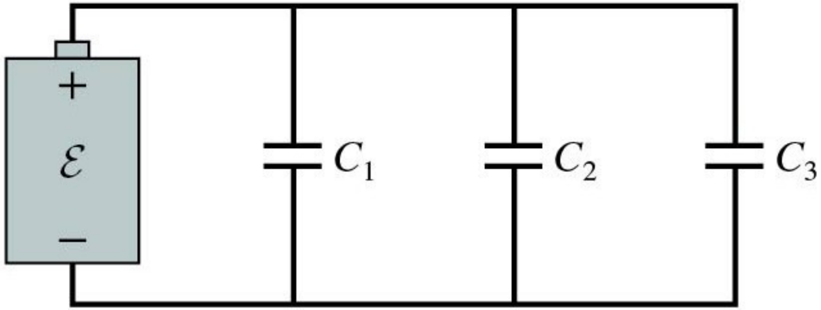
Capacitor Sim



https://phet.colorado.edu/en/simulation/legacy/capacitor-lab

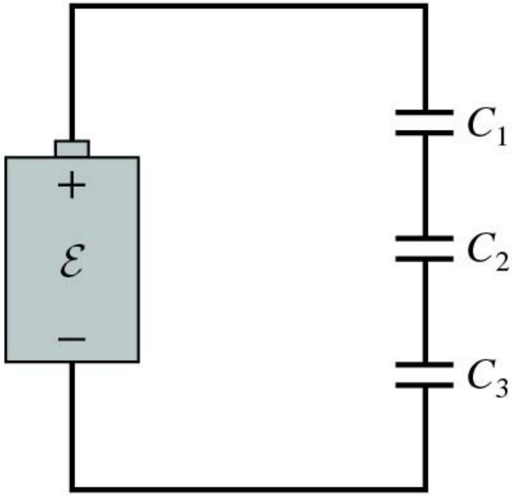
Capacitors in Combination

parallel capacitors



$$V_{\text{total}} = V_1 = V_2 = V_3$$

series capacitors



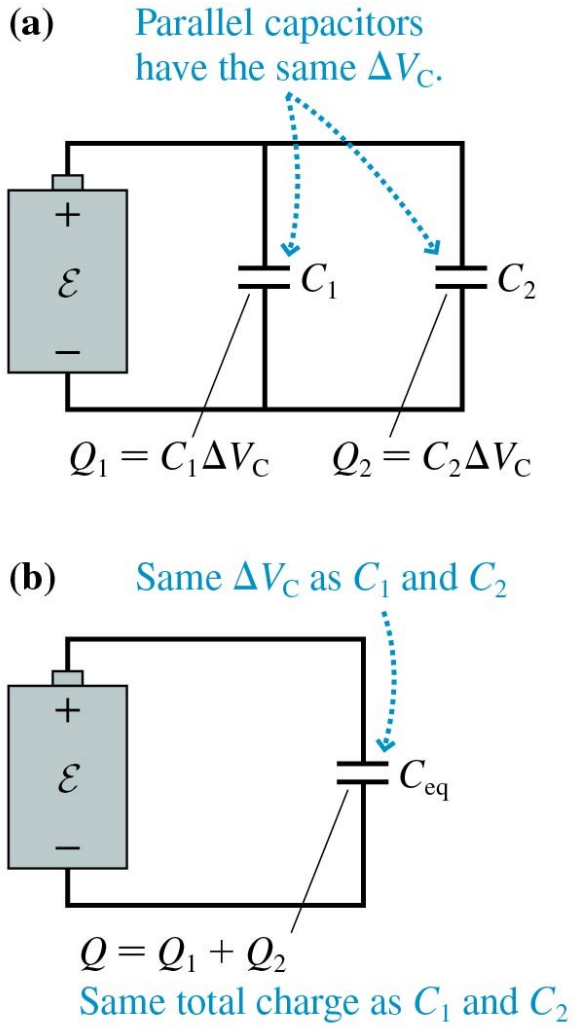
$$V_{\text{total}} = V_1 + V_2 + V_3$$

Capacitors in Combination

- Two capacitors C_1 and C_2 connected in parallel.
- The total charge drawn from the battery is $Q = Q_1 + Q_2$.
- We can replace the capacitors with a single "equivalent" capacitor:

$$C_{eq} = \frac{Q}{\Delta V_C} = \frac{Q_1 + Q_2}{\Delta V_C} = \frac{Q_1}{\Delta V_C} + \frac{Q_2}{\Delta V_C}$$

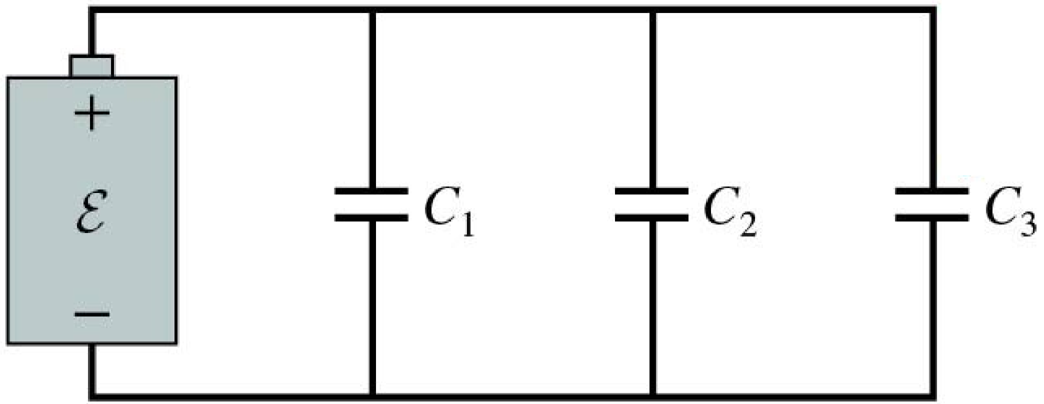
$$C_{eq} = C_1 + C_2$$



Capacitors in Combination

If capacitors C_1, C_2, C_3, … are in parallel, their equivalent capacitance is:

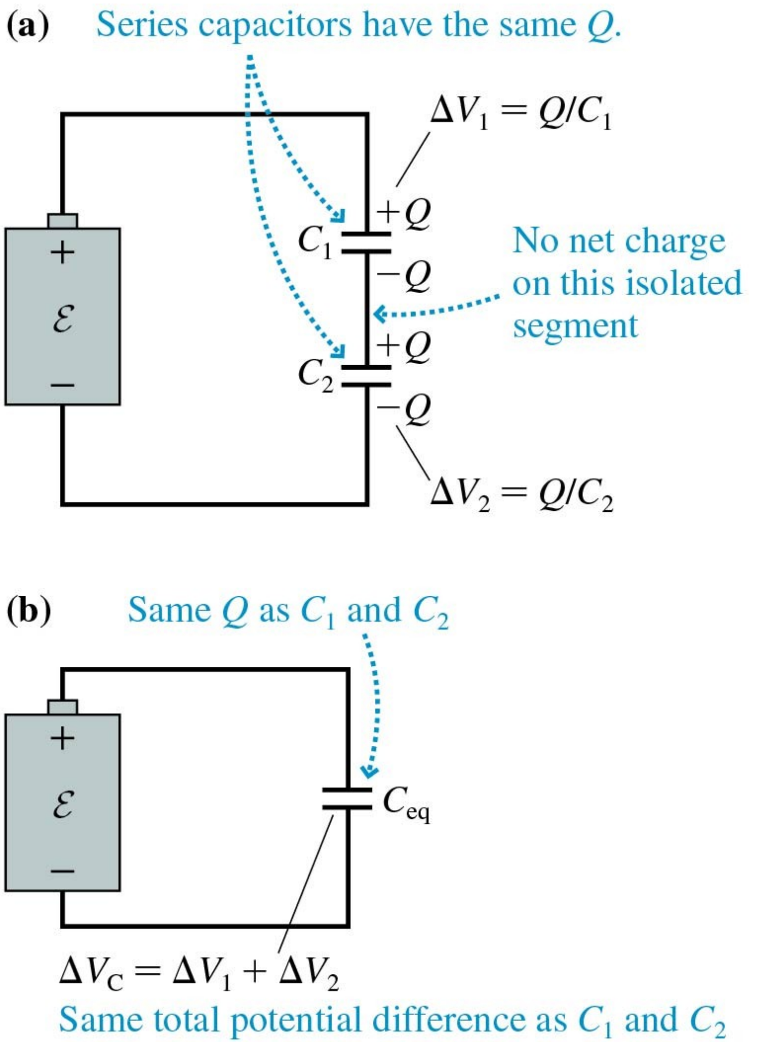
$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel capacitors)}$$



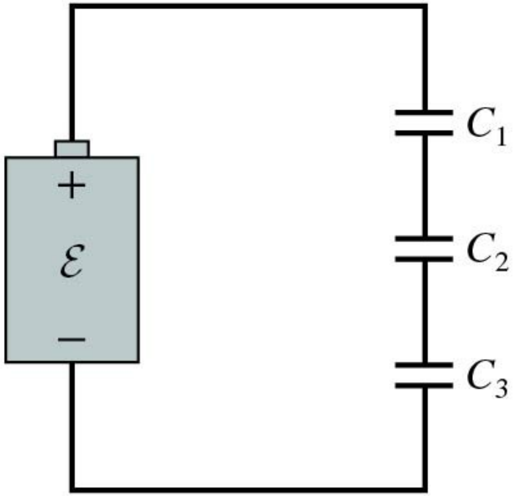
Capacitors in Combination

- two capacitors C_1 and C_2 connected in series.
- The total potential difference across both capacitors is $\Delta V_C = \Delta V_1 + \Delta V_2$.
- The inverse of the equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{\Delta V_C}{Q} = \frac{\Delta V_1 + \Delta V_2}{Q} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$



Capacitors in Combination

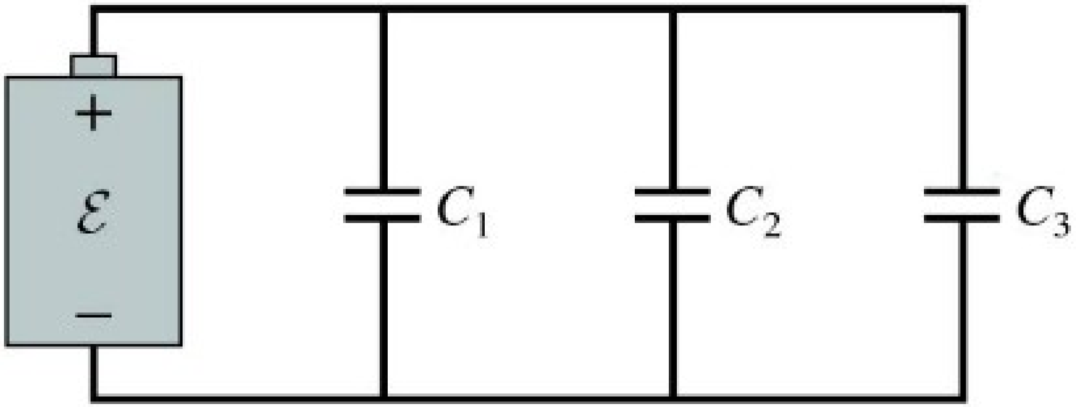


If capacitors C_1, C_2, C_3, … are in series, their equivalent capacitance is

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \right)^{-1} \quad \text{(series capacitors)}$$

Capacitors in Combination

Connected in parallel



$$C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots$$

$$Q_{\text{total}} = Q_1 + Q_2 + Q_3 + \cdots$$

$$V_{\text{total}} = V_1 = V_2 = V_3$$

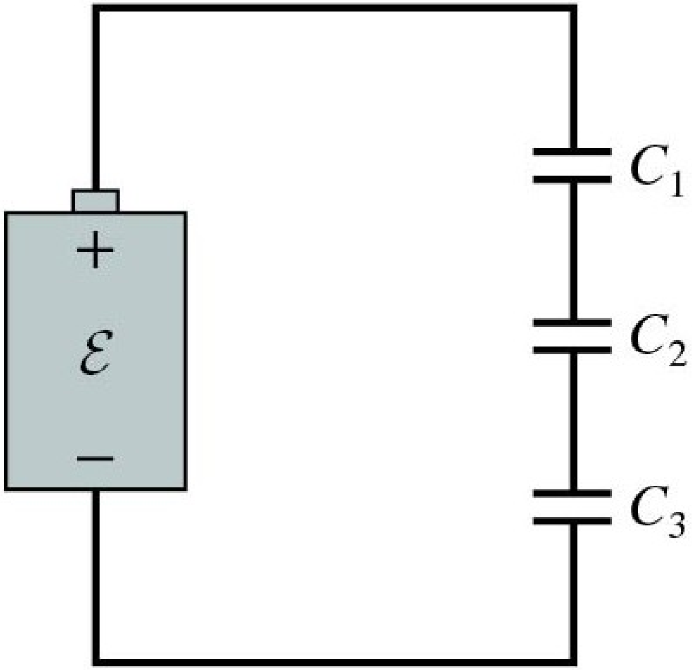
Capacitors in Combination

Connected in series

$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_3} + \frac{1}{C_3} + \cdots \right)^{-1}$$

$$Q_{\text{total}} = Q_1 = Q_2 = Q_3$$

$$V_{\text{total}} = V_1 + V_2 + V_3$$

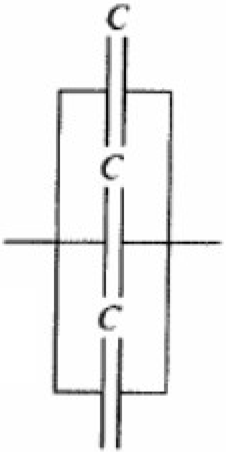


Capacitors in Combination

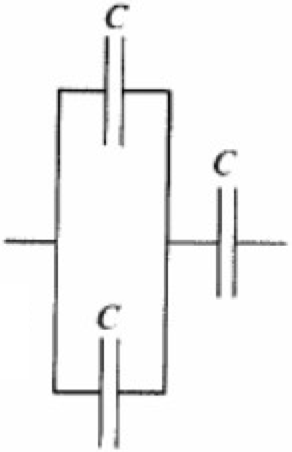
Which network has the greatest capacitance?

a.

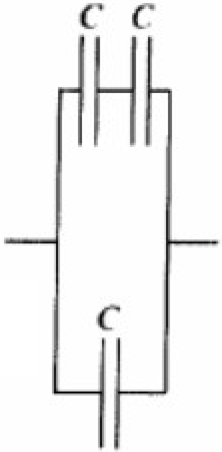
b.



c.



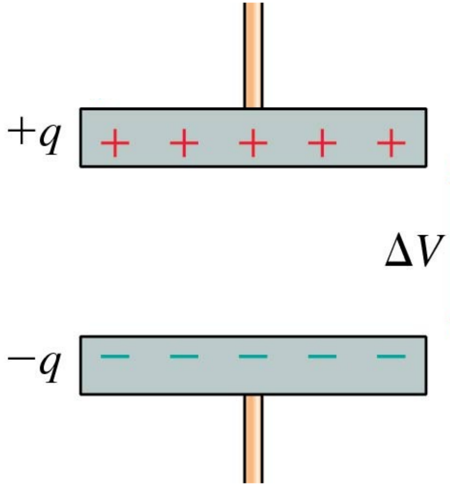
d.



Energy Stored in a Capacitor

- As a small charge *dq* is lifted to a higher potential, the potential energy of the capacitor increases by

$$dU = dq\,\Delta V = \frac{q\,dq}{C}$$

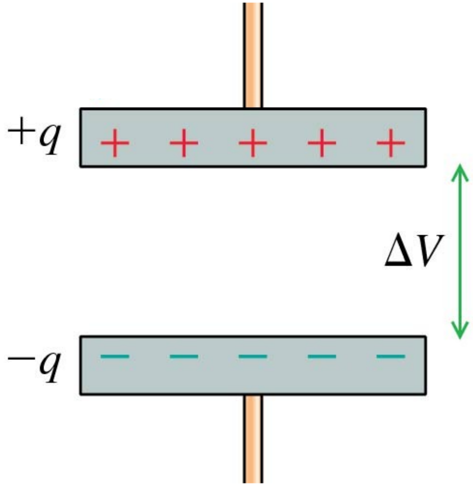


- The total energy transferred from the battery to the capacitor is

$$U_C = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

Energy Stored in a Capacitor

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V_C)^2$$



Energy Stored in a Capacitor

How much energy is stored in a 220 μF camera-flash capacitor that has been charged to 330 V? What is the average power dissipation if this capacitor is discharged in 1.0 ms?

$$U_C = \tfrac{1}{2}C(\Delta V_C)^2 = \tfrac{1}{2}(220 \times 10^{-6}\ \mathrm{F})(330\ \mathrm{V})^2 = 12\ \mathrm{J}$$

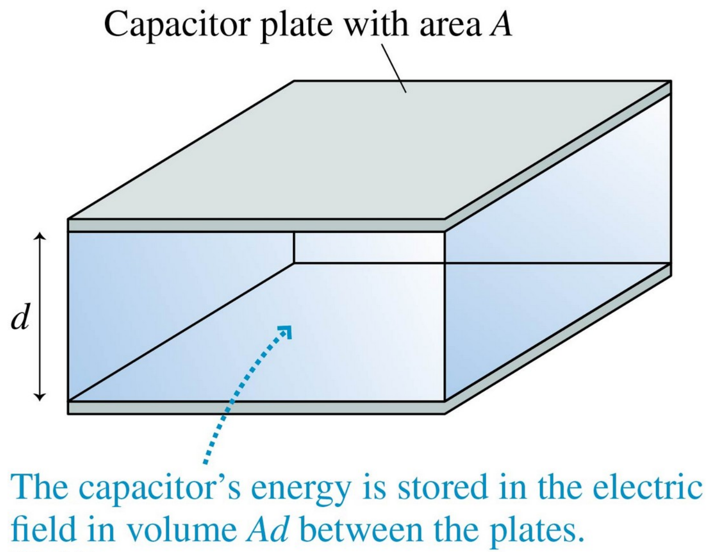
$$P = \frac{\Delta E}{\Delta t} = \frac{12\ \mathrm{J}}{1.0 \times 10^{-3}\ \mathrm{s}} = 12{,}000\ \mathrm{W}$$

Energy Stored in a Capacitor

- The **energy density** of an electric field, such as the one inside a capacitor, is:

$$u_E = \frac{\text{energy stored}}{\text{volume in which it is stored}} = \frac{U_C}{Ad} = \frac{\epsilon_0}{2}E^2$$

- units J/m^3.



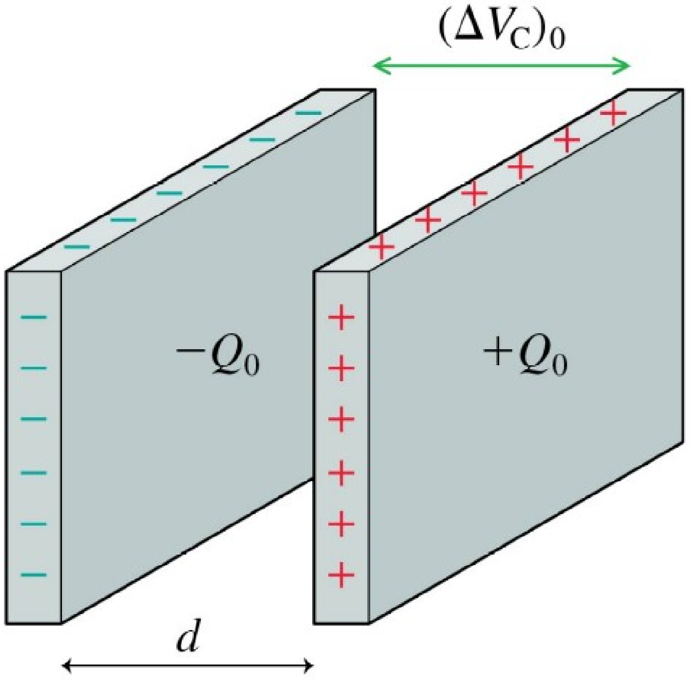
Energy Stored in a Capacitor

Capacitors are used in devices that need to store a lot of electrical energy for quick delivery

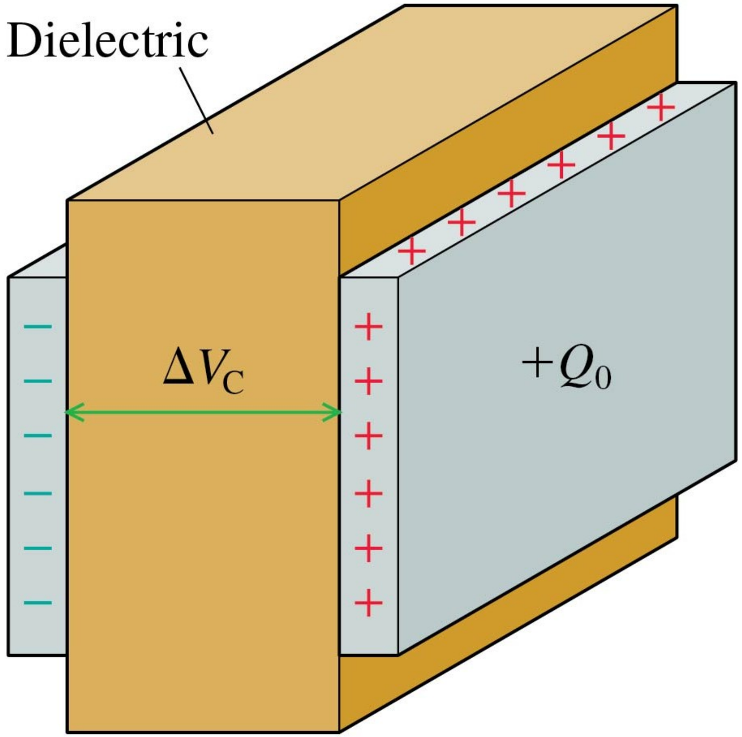
automated external defibrillator (AED)

camera flash

Dielectrics



Capacitance C_0 in vacuum



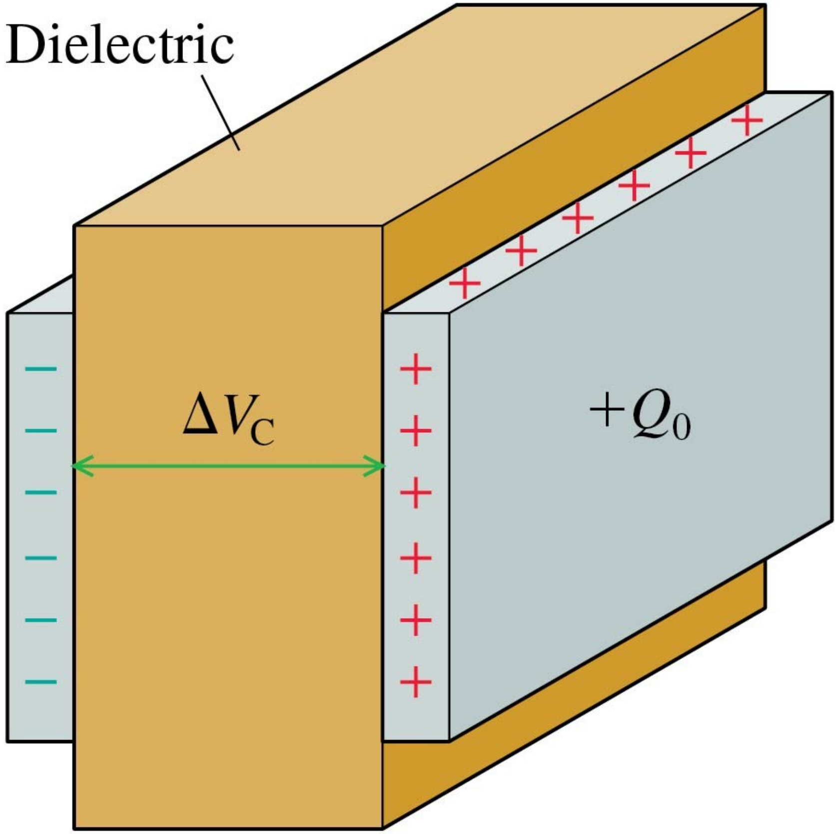
Capacitance $C > C_0$

$$C = \kappa C_0$$

Dielectrics

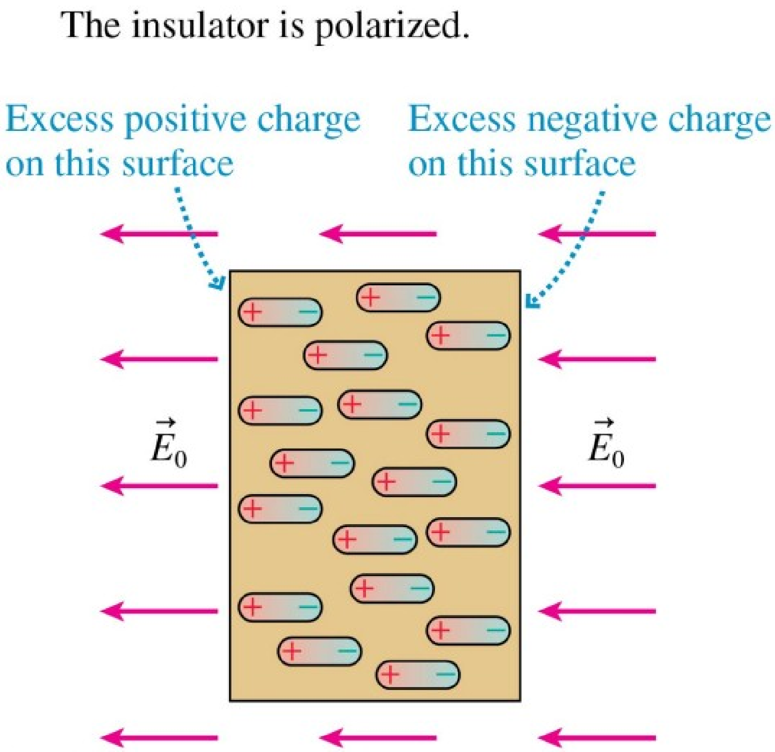
- The charge on the capacitor plates does not change ($Q = Q_0$).
- However, the voltage has *decreased*:

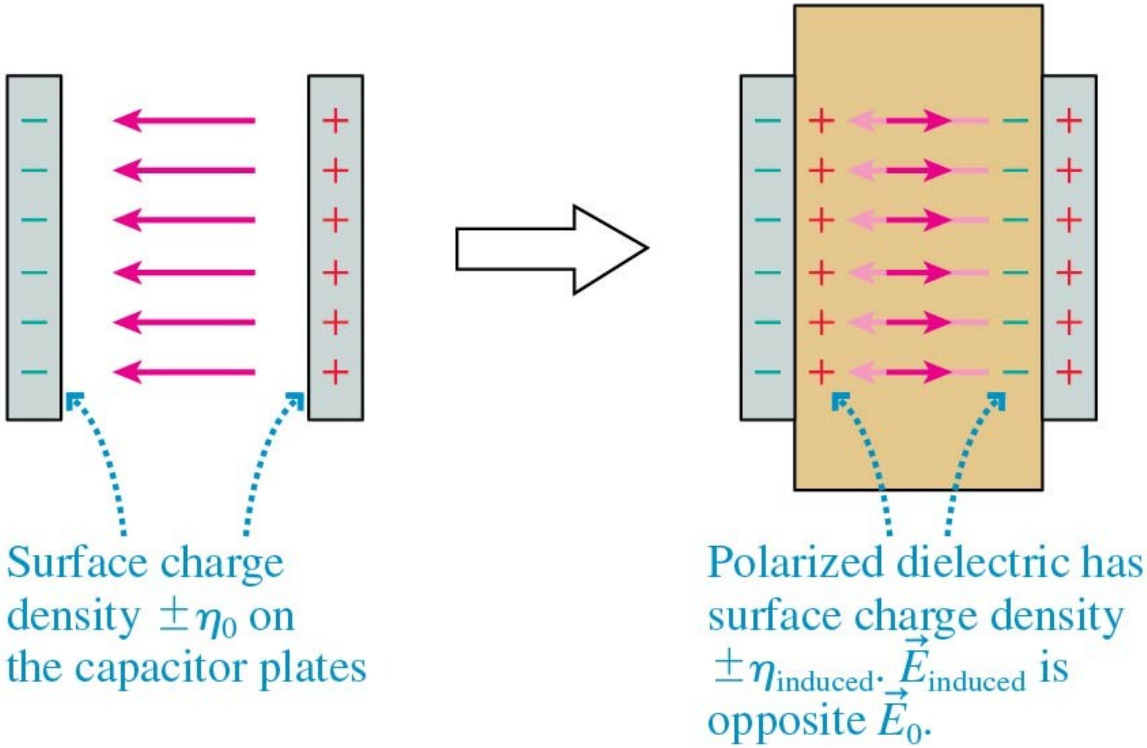
$$\Delta V_C < (\Delta V_C)_0$$



Capacitance $C > C_0$

Dielectrics





Dielectrics

- the **dielectric constant**: $$\kappa \equiv \frac{E_0}{E}$$
- Easily polarized materials have larger dielectric constants than materials not easily polarized.
- Vacuum has $\kappa = 1$ exactly.
- **Filling a capacitor with a dielectric increases the capacitance by a factor equal to the dielectric constant:**

$$C = \frac{Q}{\Delta V_C} = \frac{Q_0}{(\Delta V_C)_0/\kappa} = \kappa \frac{Q_0}{(\Delta V_C)_0} = \kappa C_0$$

Dielectrics

Material	Dielectric constant κ	Dielectric strength E_{max} (10^6 V/m)
Vacuum	1	—
Air (1 atm)	1.0006	3
Teflon	2.1	60
Polystyrene plastic	2.6	24
Mylar	3.1	7
Paper	3.7	16
Pyrex glass	4.7	14
Pure water (20°C)	80	—
Titanium dioxide	110	6
Strontium titanate	300	8

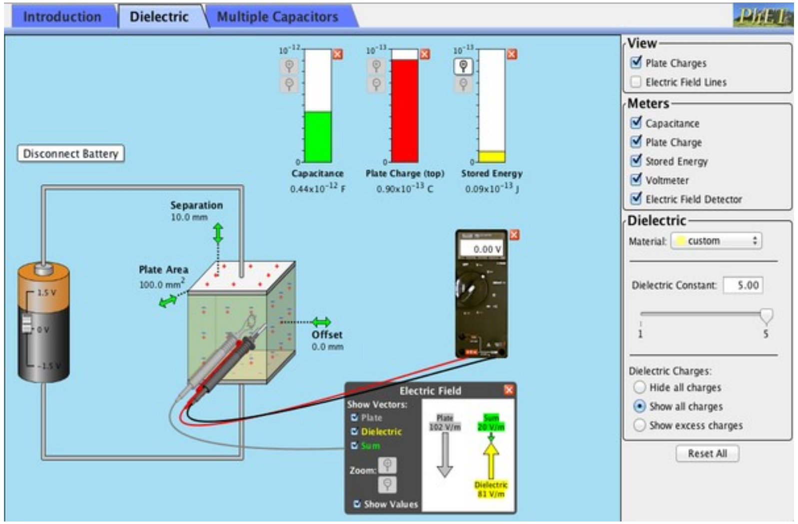
Dielectrics

With voltage source connected: V is constant, Q can change

With *no* voltage source connected: Q is constant, V can change

C only changes with a physical change to the capacitor.

See simulation:



Dielectrics

A 5.0 nF parallel-plate capacitor is charged to 160 V. It is then disconnected from the battery and immersed in distilled water. What are (a) the capacitance and voltage of the water-filled capacitor and (b) the energy stored in the capacitor before and after its immersion?

$$C = \kappa C_0 = 80 \times 5.0 \text{ nF} = 400 \text{ nF}$$

At the same time, the voltage decreases to

$$\Delta V_C = \frac{(\Delta V_C)_0}{\kappa} = \frac{160 \text{ V}}{80} = 2.0 \text{ V}$$

$$(U_C)_0 = \tfrac{1}{2} C_0 (\Delta V_C)_0^2 = \tfrac{1}{2}(5.0 \times 10^{-9} \text{ F})(160 \text{ V})^2 = 6.4 \times 10^{-5} \text{ J}$$

After being immersed, the stored energy is

$$U_C = \tfrac{1}{2} C (\Delta V_C)^2 = \tfrac{1}{2}(400 \times 10^{-9} \text{ F})(2.0 \text{ V})^2 = 8.0 \times 10^{-7} \text{ J}$$