

# Electric Potential Energy

A positive and a negative charge are released from rest in vacuum. They move toward each other. As they do,



- A. A positive potential energy becomes more positive.
- B. A positive potential energy becomes less positive.
- C. A negative potential energy becomes more negative.
- D. A negative potential energy becomes less negative.
- E. A positive potential energy becomes a negative potential energy.

# Electric Potential Energy

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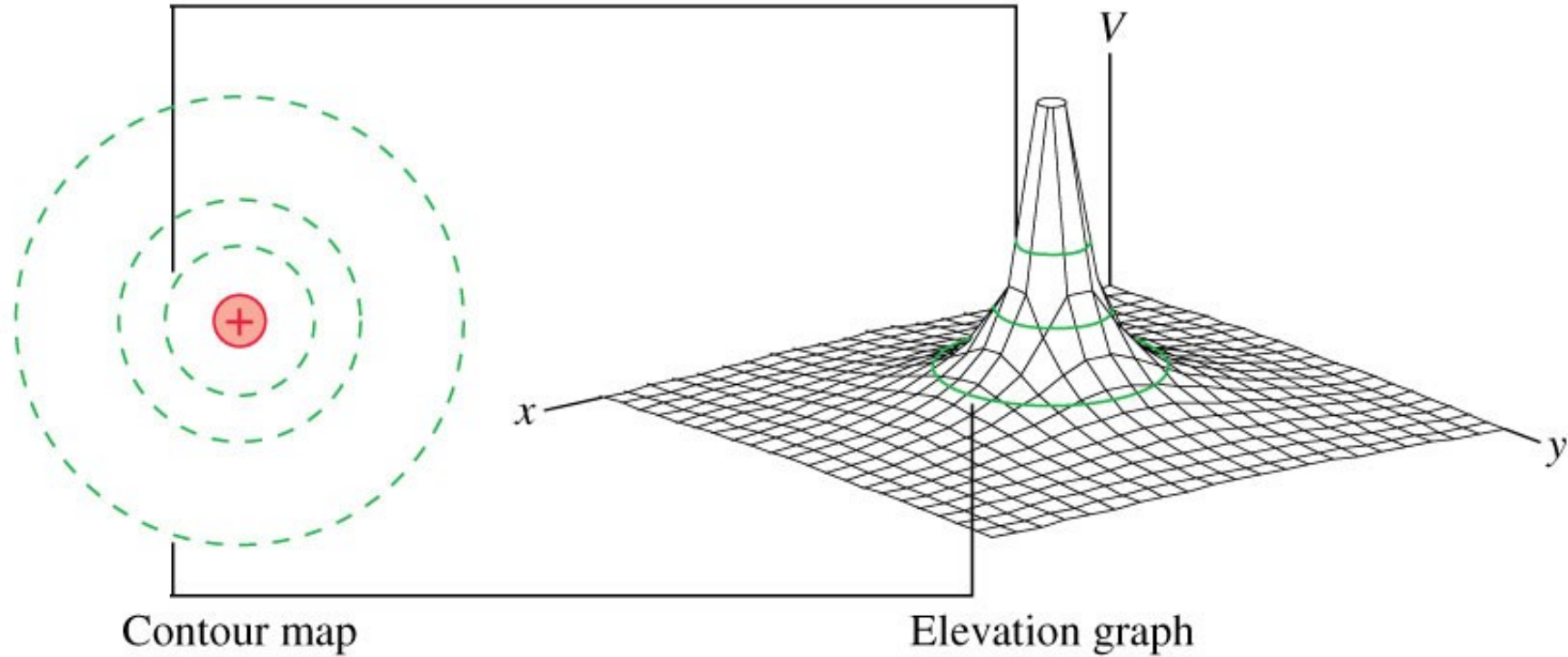
- A. A positive potential energy becomes more positive.
- B. A positive potential energy becomes less positive.
- ✓ C. **A negative potential energy becomes more negative.**
- D. A negative potential energy becomes less negative.
- E. A positive potential energy becomes a negative potential energy.

$$U_{\text{elec}} = \frac{Kq_1q_2}{r}$$

Opposite signs, so  $U$  is Negative.

$U$  increases in magnitude as  $r$  decreases.

# Electric Potential (Voltage)



# Electric Potential (Voltage)

- Outside a uniformly charged sphere of radius  $R$ , the electric potential is identical to that of a point charge  $Q$  at the center:

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

where  $r \geq R$ .

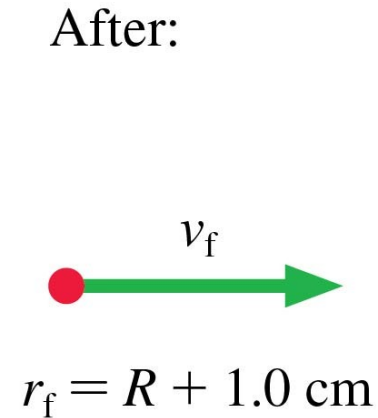
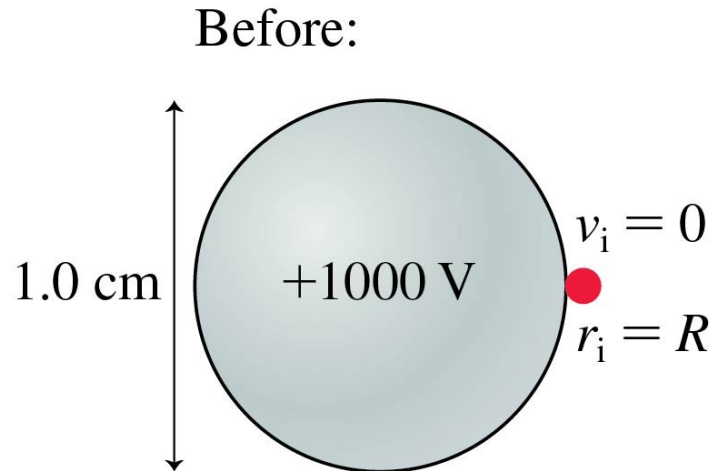
- If the potential at the surface  $V_0$  is known, then the potential at  $r \geq R$  is

$$V = \frac{R}{r} V_0$$

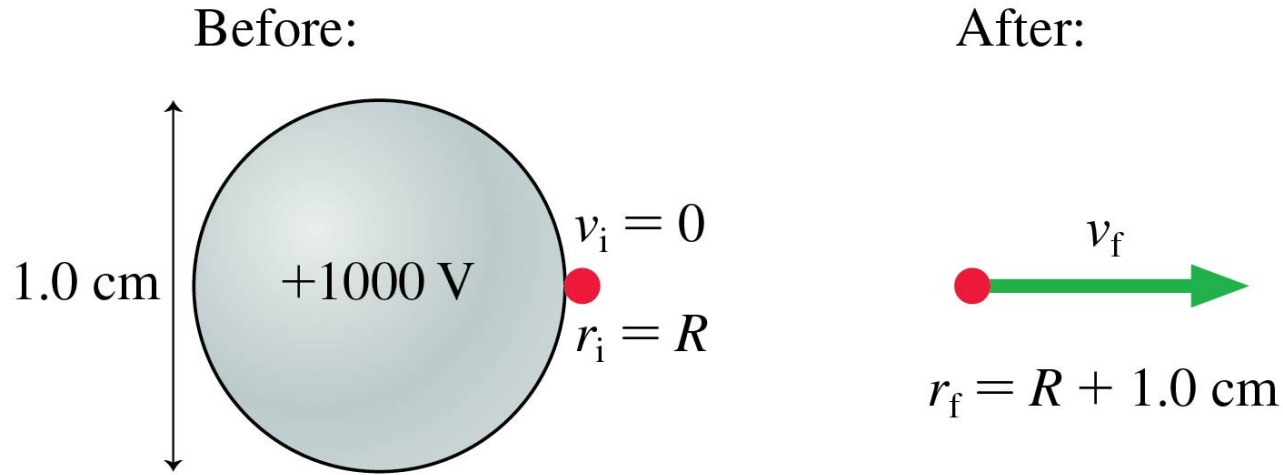
# Electric Potential (Voltage)

A proton is released from rest at the surface of a 1.0-cm-diameter sphere that has been charged to +1000 V.

- What is the charge of the sphere?
- What is the proton's speed at 1.0 cm from the sphere?



# Electric Potential (Voltage)



The charge of the sphere is

$$Q = 4\pi\epsilon_0 R V_0 = 0.56 \times 10^{-9} \text{ C} = 0.56 \text{ nC}$$

# Electric Potential (Voltage)

$$\frac{1}{2}mv_f^2 + \frac{eR}{r_f}V_0 = \frac{1}{2}mv_i^2 + \frac{eR}{r_i}V_0$$

The proton starts from the surface of the sphere,  $r_i = R$ , with  $v_i = 0$ . When the proton is 1.0 cm from the *surface* of the sphere, it has  $r_f = 1.0 \text{ cm} + R = 1.5 \text{ cm}$ . Using these, we can solve for  $v_f$ :

$$v_f = \sqrt{\frac{2eV_0}{m} \left( 1 - \frac{R}{r_f} \right)} = 3.6 \times 10^5 \text{ m/s}$$

# Electric Potential of Many Charges

## The principle of superposition.

The electric potential  $V$  at a point in space is the sum of the potentials due to each charge:

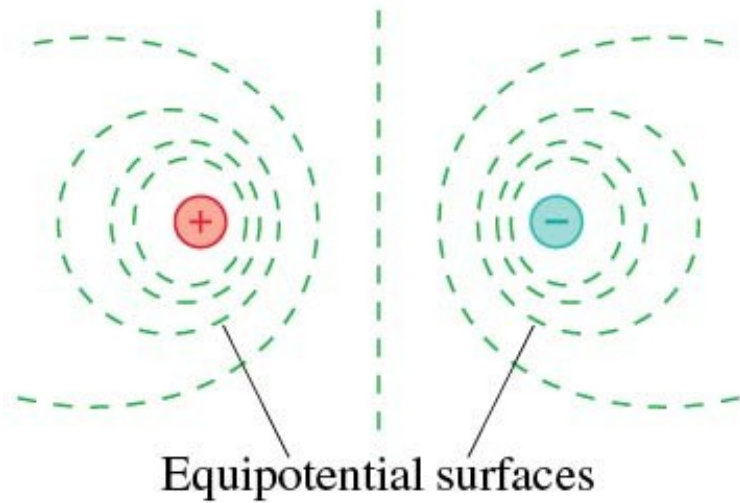
$$V = \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i}$$

where  $r_i$  is the distance from charge  $q_i$  to the point in space where the potential is being calculated.

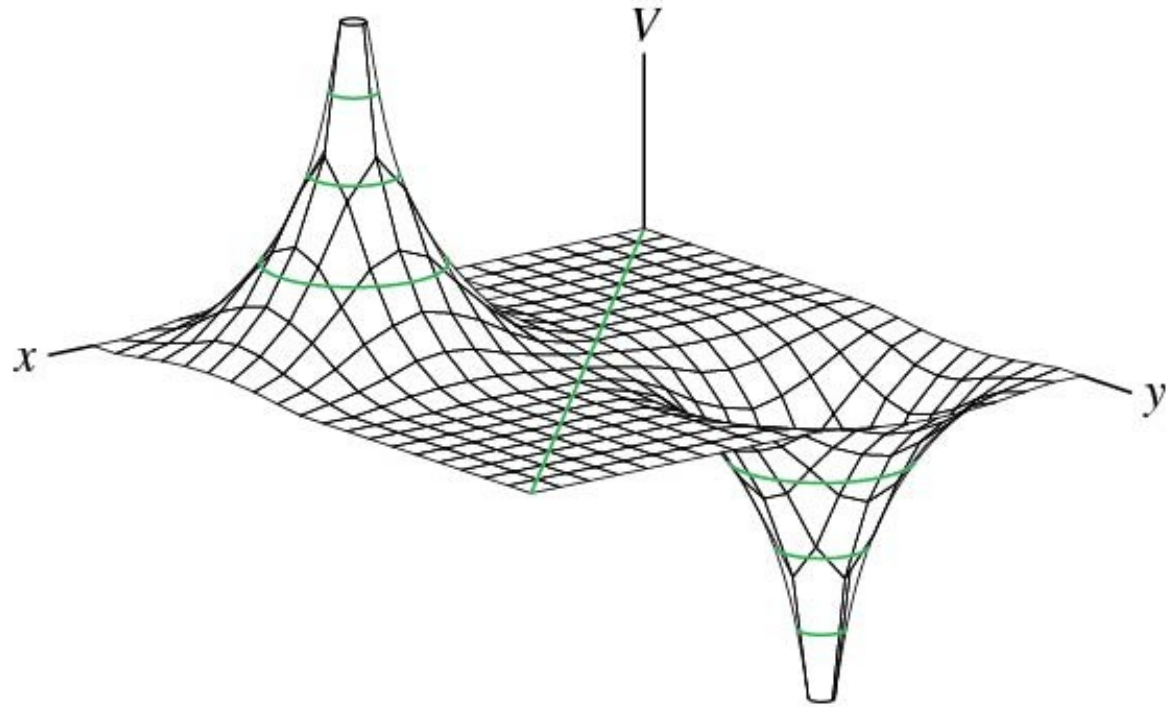


# Electric Potential of Many Charges

(a) Contour map

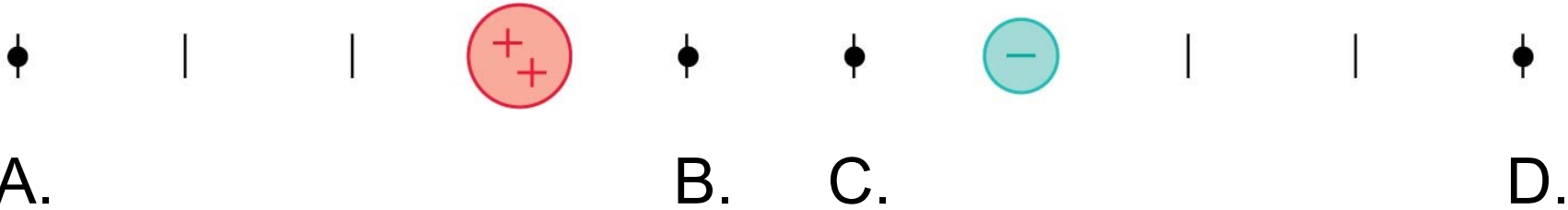


(b) Elevation graph



# Electric Potential of Many Charges

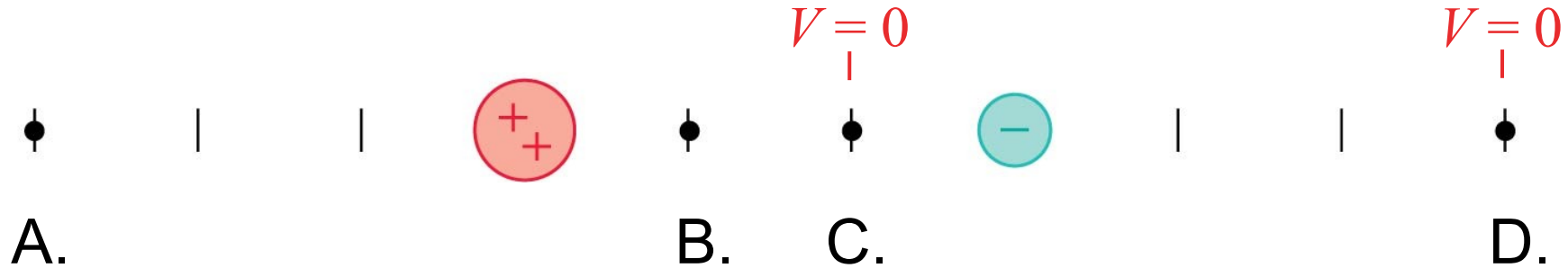
At which point or points is the electric potential zero?



E. More than one of these.

# Electric Potential of Many Charges

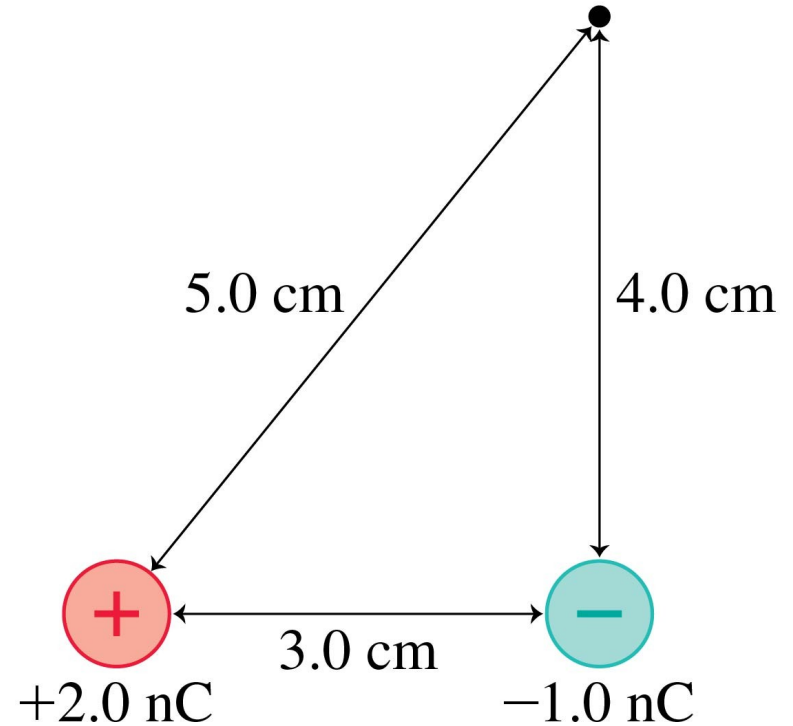
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# Electric Potential of Many Charges

Find the potential at the point.

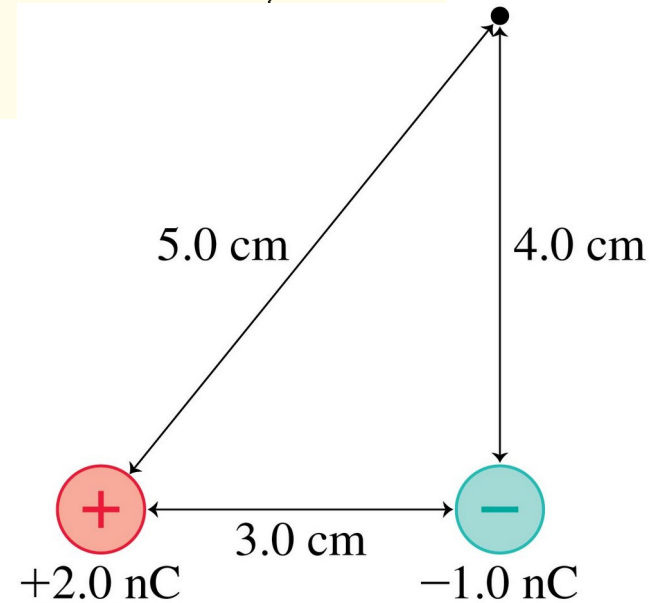


# Electric Potential of Many Charges

$$V = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2}$$

$$= (9.0 \times 10^9 \text{ N m}^2/\text{C}^2) \left( \frac{2.0 \times 10^{-9} \text{ C}}{0.050 \text{ m}} + \frac{-1.0 \times 10^{-9} \text{ C}}{0.040 \text{ m}} \right)$$

$$= 135 \text{ V}$$



# Electric Potential and Field

Force and energy are related by

$$\Delta U = - \int_i^f \vec{F} \cdot d\vec{s} \qquad \vec{F} = - \frac{dU}{ds}$$

Force per charge is field:

$$\vec{E} = \frac{\vec{F}}{q}$$

Energy per charge is voltage:

$$\Delta V = \frac{\Delta U}{q}$$

# Electric Potential and Field

Force and energy are related by

$$\Delta U = - \int_i^f \vec{F} \cdot d\vec{s} \qquad F_s = - \frac{dU}{ds}$$

Electric field and potential are related by

$$\Delta V = - \int_i^f \vec{E} \cdot d\vec{s} \qquad E_s = - \frac{dV}{ds}$$

# Electric Potential and Field

The potential difference between two points in space is

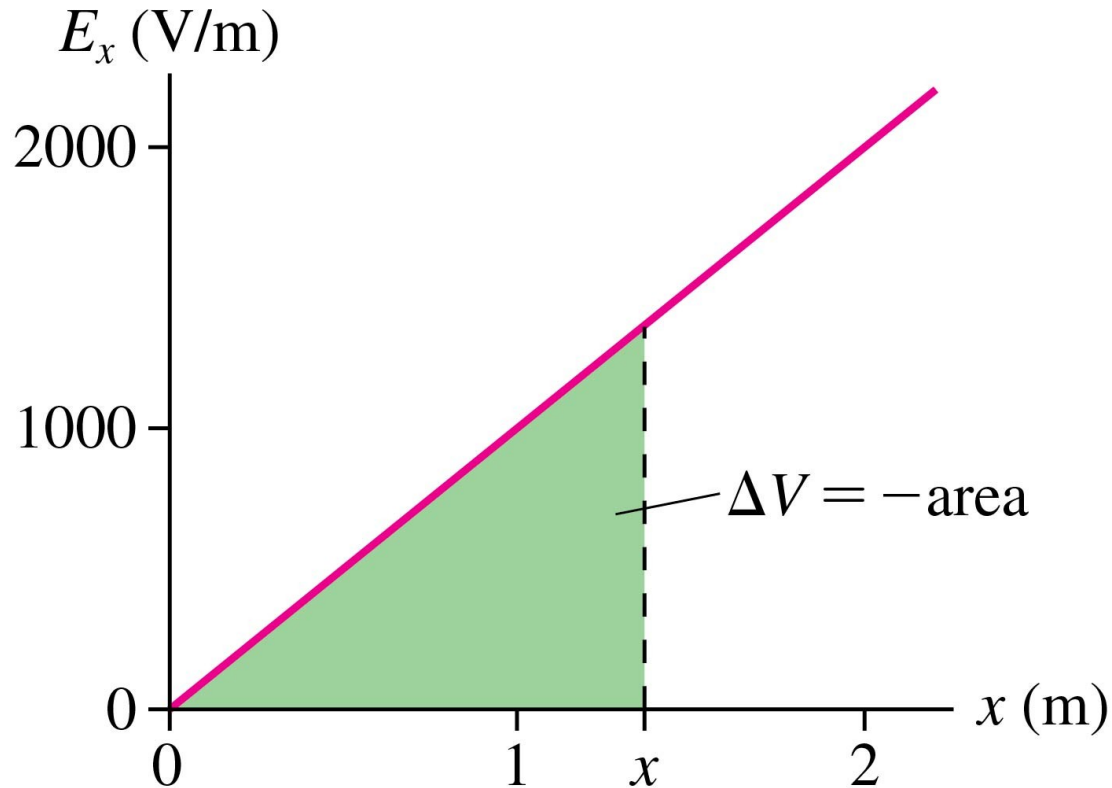
$$\Delta V = V_f - V_i = - \int_{s_i}^{s_f} E_s ds = - \int_i^f \vec{E} \cdot d\vec{s}$$

where  $s$  is the position along a line from point  $i$  to point  $f$ .



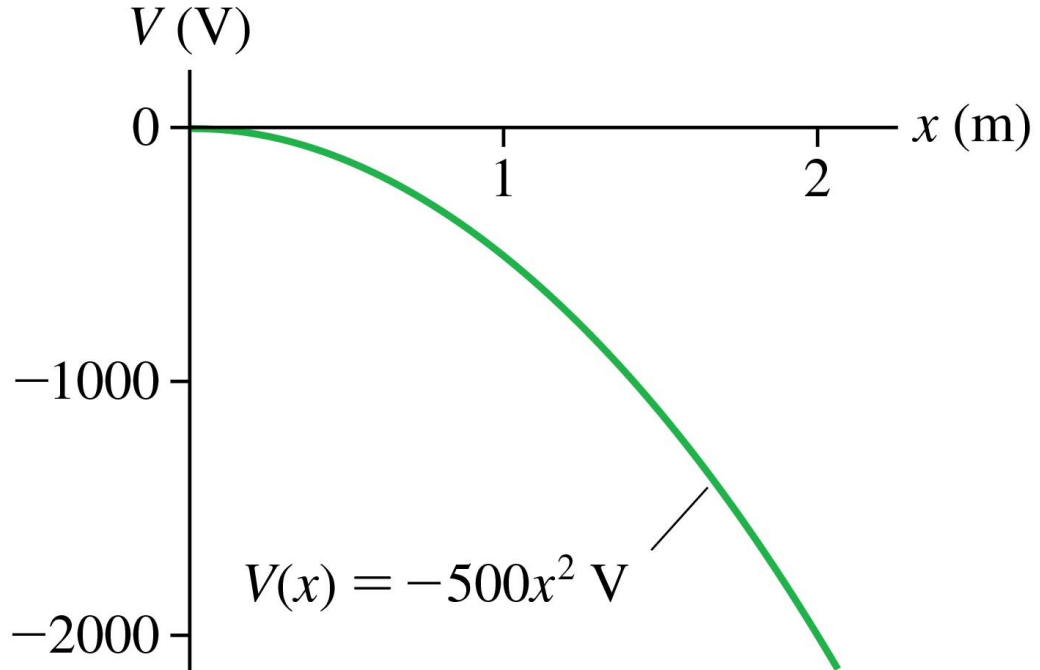
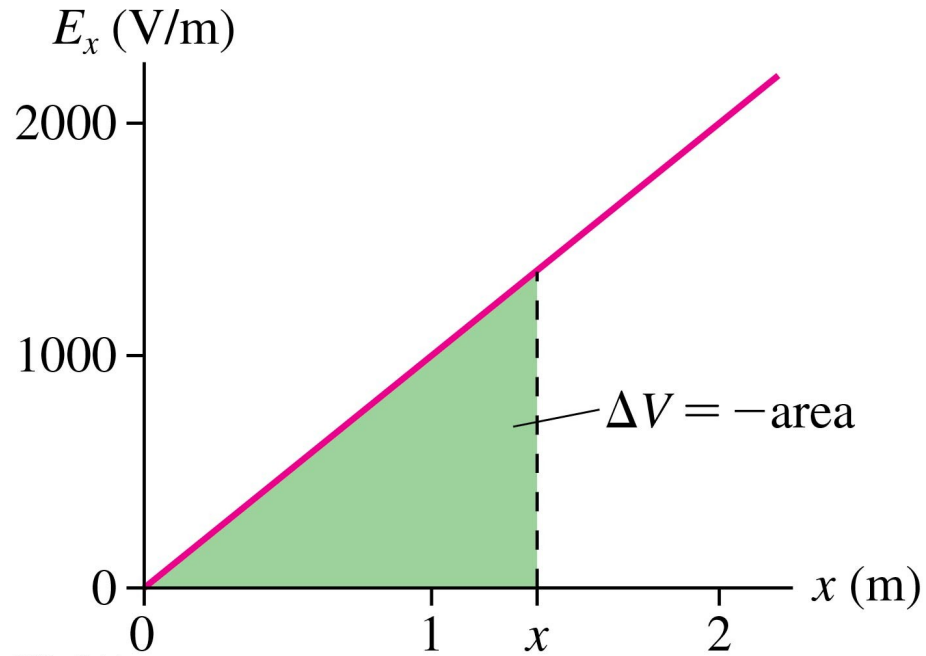
# Electric Potential and Field

Given the electric field shown, what is the graph of the voltage?



$$\Delta V = - \int_i^f \vec{E} \cdot d\vec{s}$$

# Electric Potential and Field



# The Electric Potential

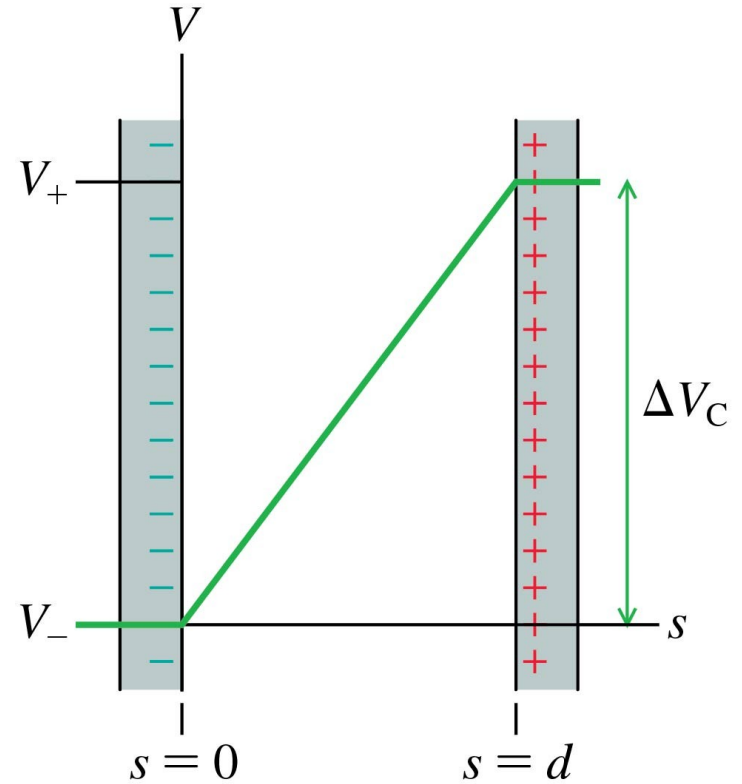
- The electric potential inside a parallel-plate capacitor is

$$V = Es \quad (\text{electric potential inside a parallel-plate capacitor})$$

where  $s$  is the distance from the *negative* electrode.

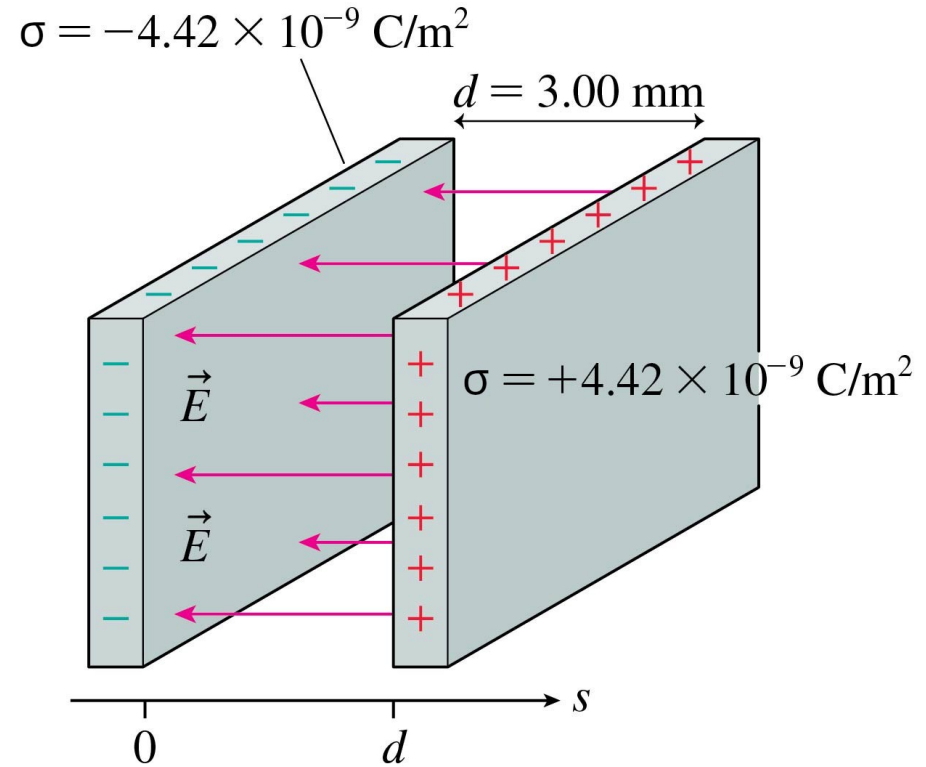
- The *potential difference*  $\Delta V_C$ , or “voltage” between the two capacitor plates is

$$\Delta V_C = V_+ - V_- = Ed$$



# The Electric Field Inside a Parallel-Plate Capacitor

$$\vec{E} = \frac{\sigma}{\epsilon_0} \hat{n}$$
$$= (500 \text{ N/C, from right to left})$$



# Units of Electric Field

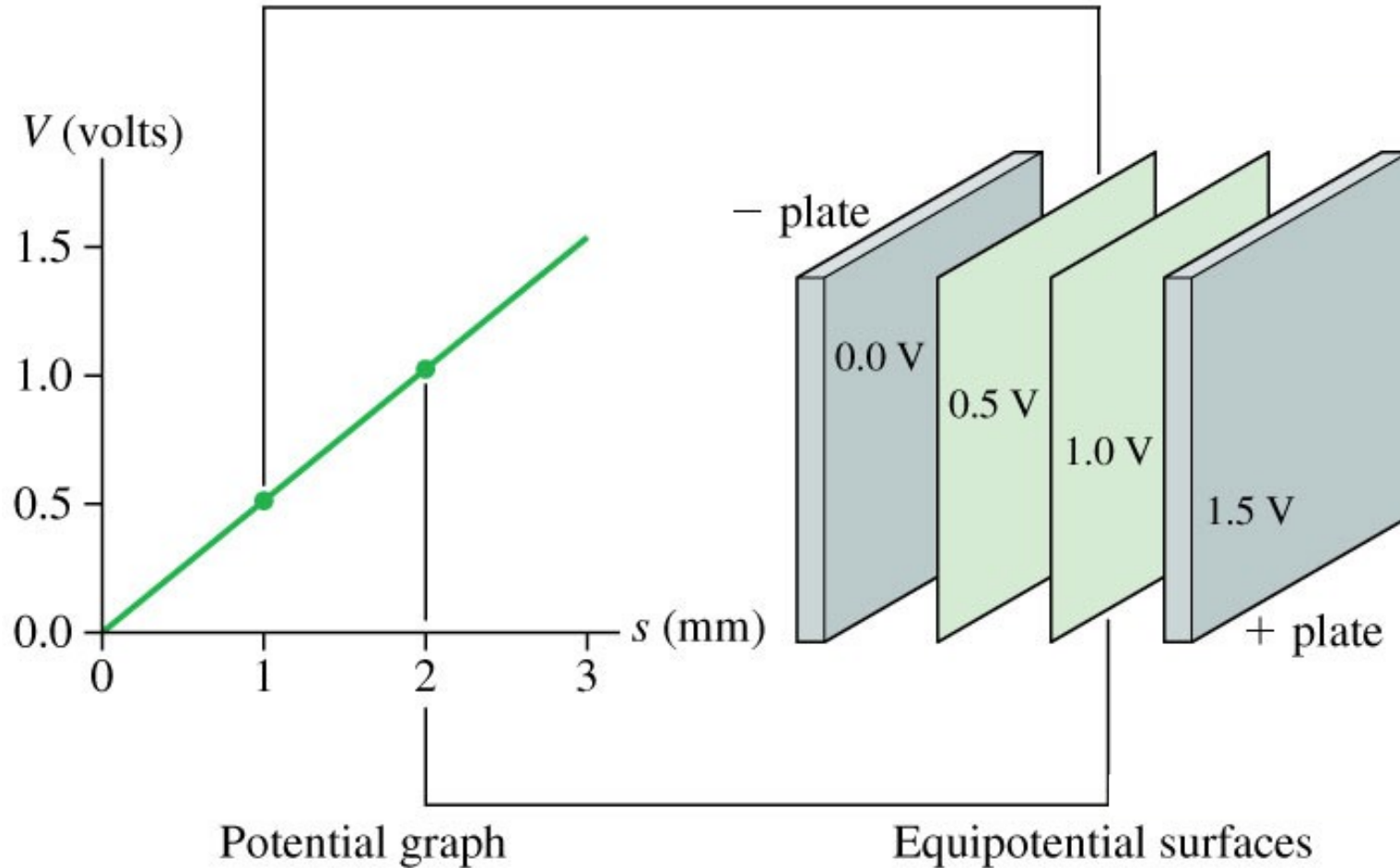
- If we know a capacitor's voltage  $\Delta V$  and the distance between the plates  $d$ , then the electric field strength within the capacitor is

$$E = \frac{\Delta V_C}{d}$$

- The units of electric field are volts per meter, or V/m.

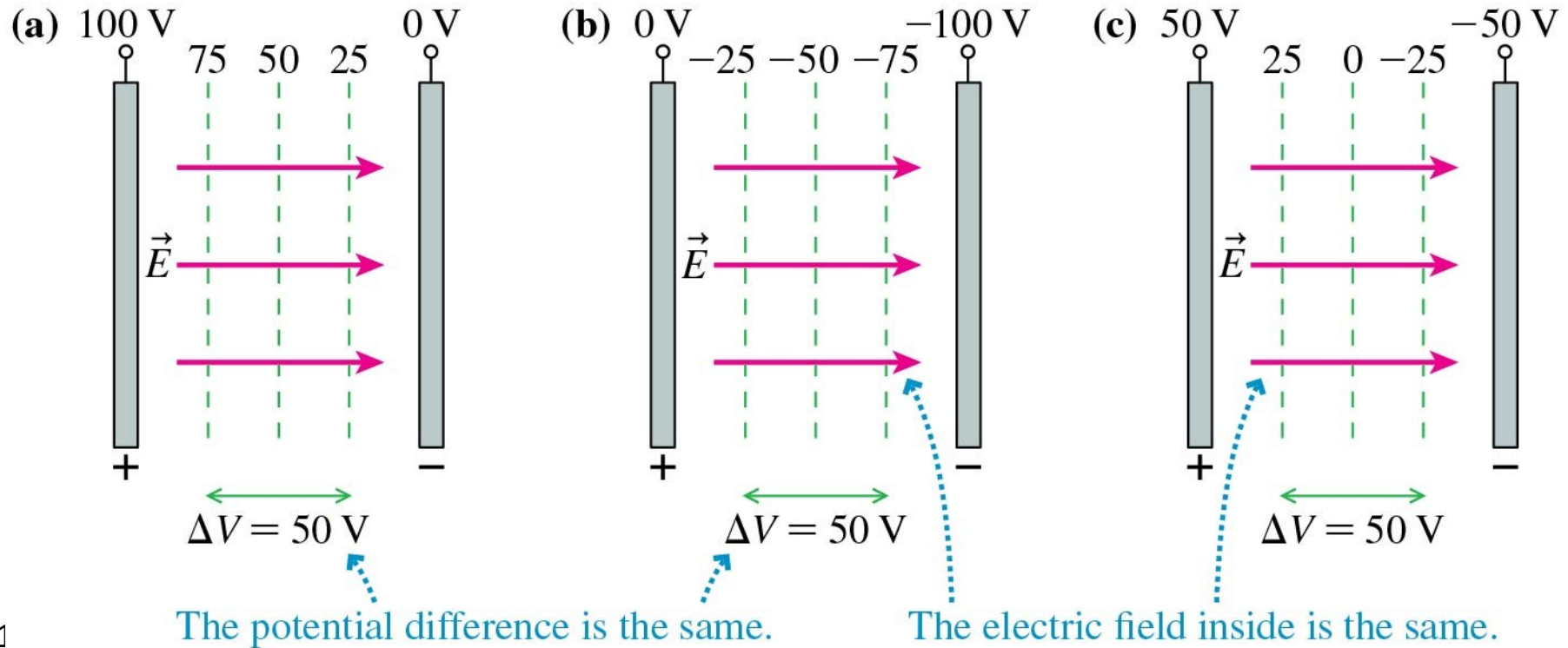
$$1 \text{ N/C} = 1 \text{ V/m}$$

# The Electric Potential



# The Zero Point of Electric Potential

The three contour maps below represent the *same physical situation*.

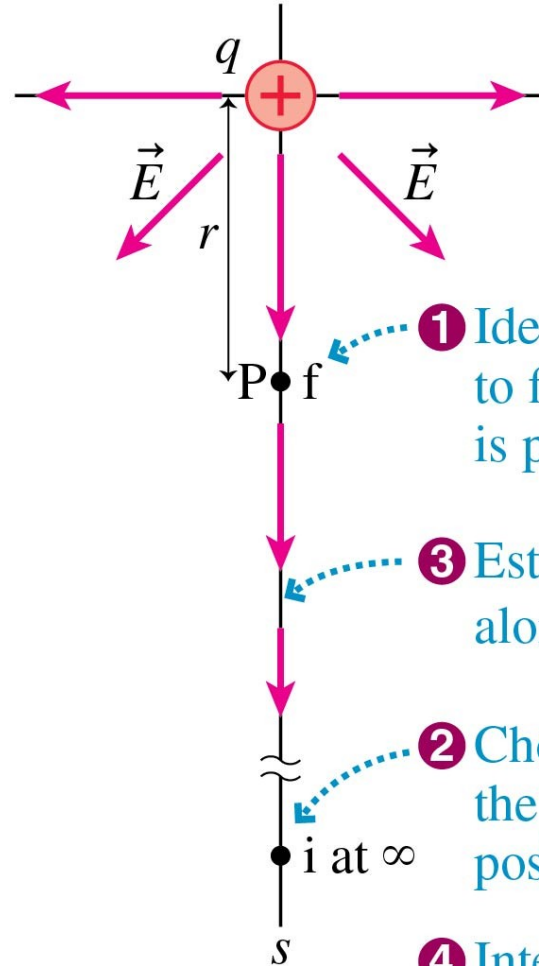


# Electric Potential and Field

$$E_s = \frac{1}{4\pi\epsilon_0} \frac{q}{s^2}$$

$$V(r) = V(\infty) + \frac{q}{4\pi\epsilon_0} \int_r^\infty \frac{ds}{s^2}$$

$$V_{\text{point charge}} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$



**1** Identify the point at which to find the potential. This is position  $f$  at  $s_f = r$ .

**3** Establish a coordinate axis along which  $\vec{E}$  is known.

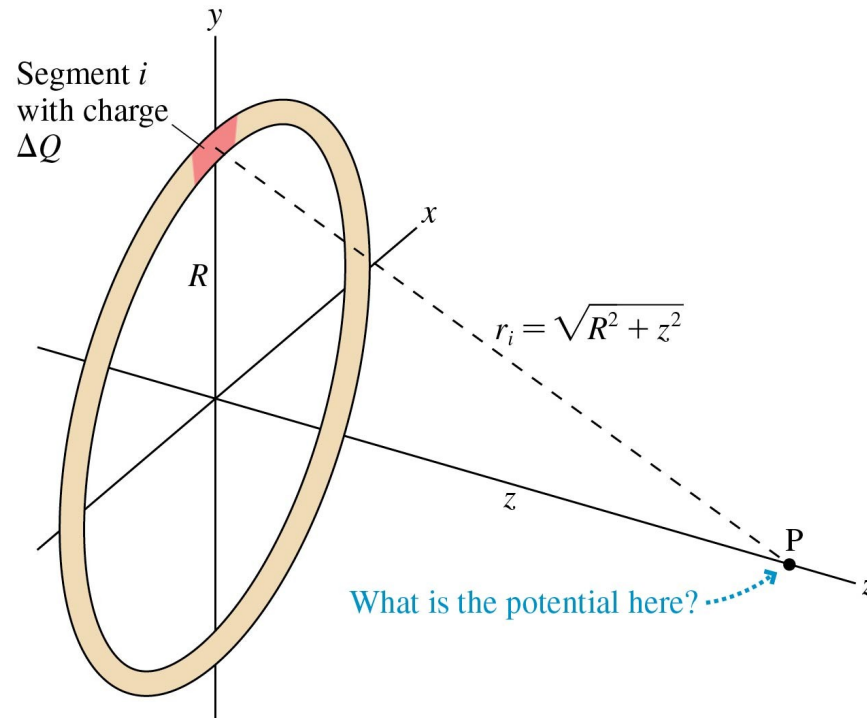
**2** Choose a zero point of the potential. In this case, position  $i$  is at  $s_i = \infty$ .

**4** Integrate along the  $s$ -axis.



# Electric Potential of a Charge Distribution

A thin, uniformly charged ring of radius  $R$  has total charge  $Q$ . Find the potential at distance  $z$  on the axis of the ring.

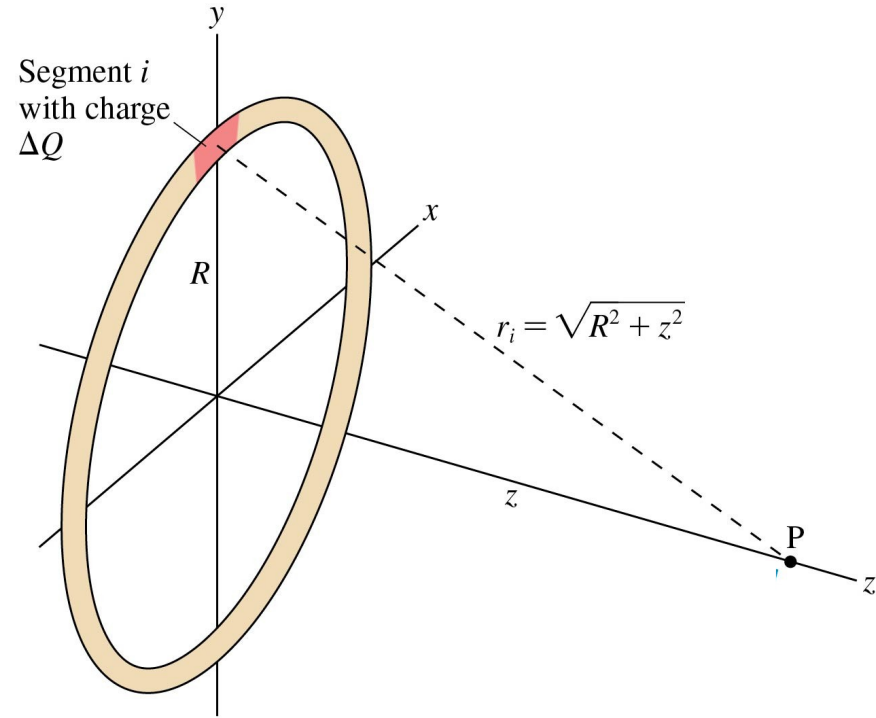


# Electric Potential and Field

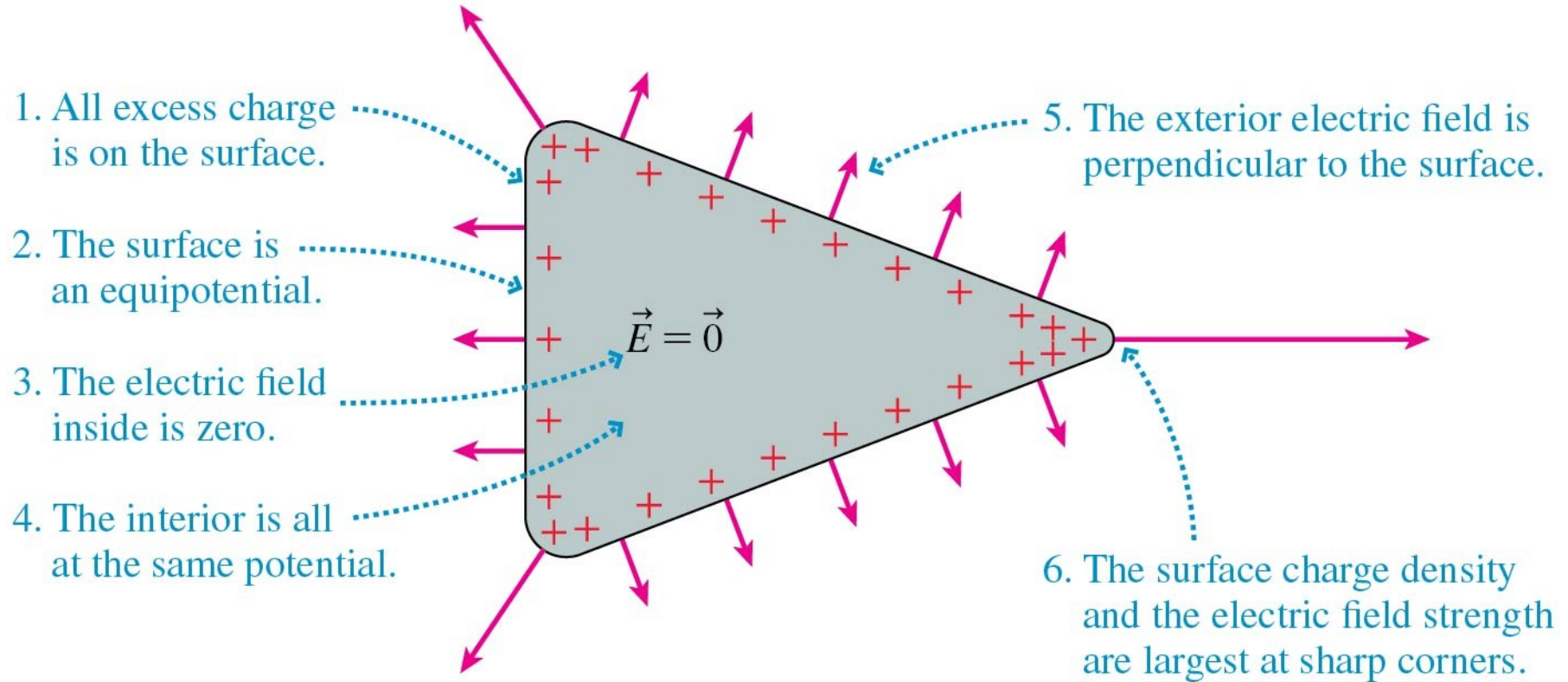
What is the electric field on the axis of a ring of charge?

$$V_{\text{ring}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{z^2 + R^2}}$$

$$\begin{aligned} E_z &= -\frac{dV}{dz} = -\frac{d}{dz} \left( \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{z^2 + R^2}} \right) \\ &= \frac{1}{4\pi\epsilon_0} \frac{zQ}{(z^2 + R^2)^{3/2}} \end{aligned}$$

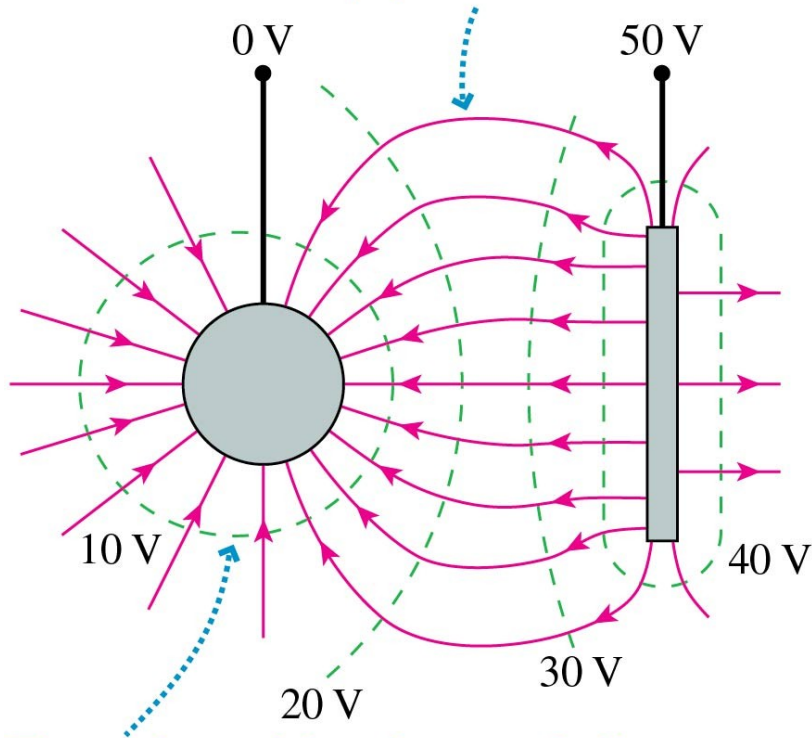


# Electric Potential and Conductors



# Electric Potential and Conductors

The field lines are perpendicular to the equipotential surfaces.



The equipotential surfaces gradually change from the shape of one electrode to that of the other.

Since a conductor surface must be an equipotential, the equipotential surfaces close to each electrode roughly match the shape of the electrode.

# Electric Potential and Conductors

Metal wires are attached to the terminals of a 3 V battery. What is the potential difference between points 1 and 2?

- A. 6 V
- ✓ B. 3 V
- C. 0 V
- D. Undefined.
- E. Not enough information to tell.

