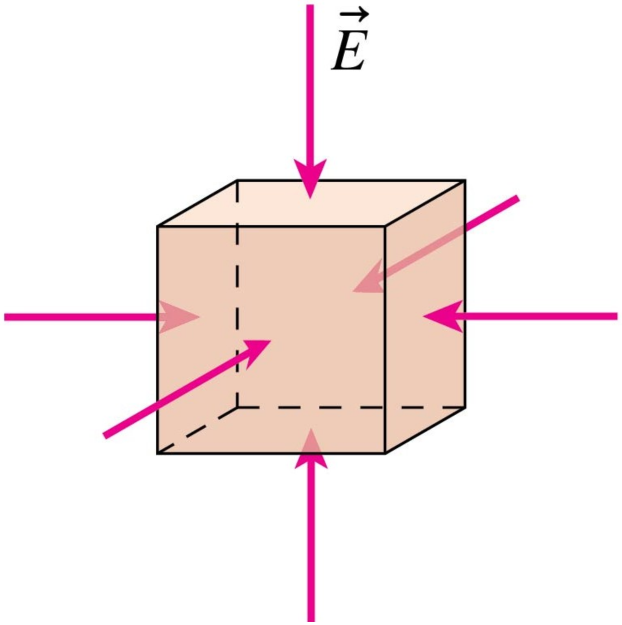
Field Flux

What must be the net charge inside this box?

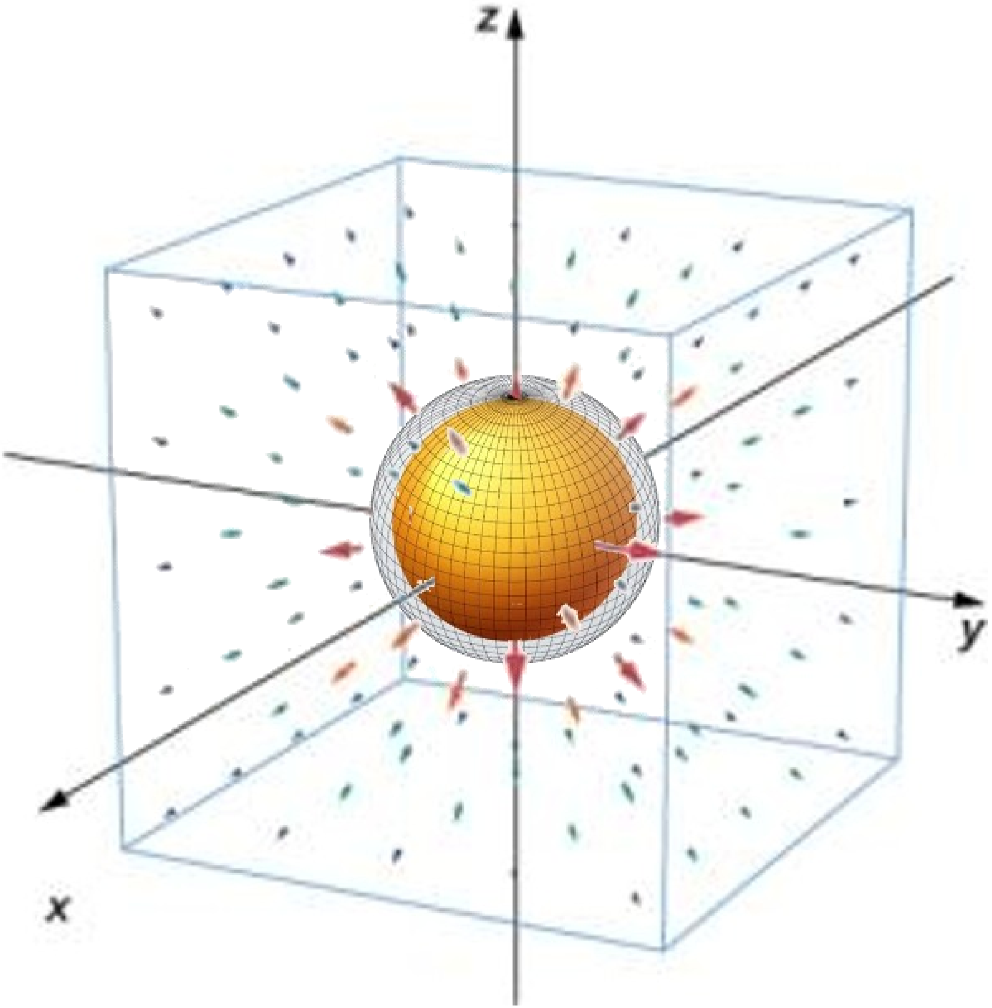
a. positive
b. negative ✔
c. zero
d. cannot tell



Chapter 6: Gauss’s Law

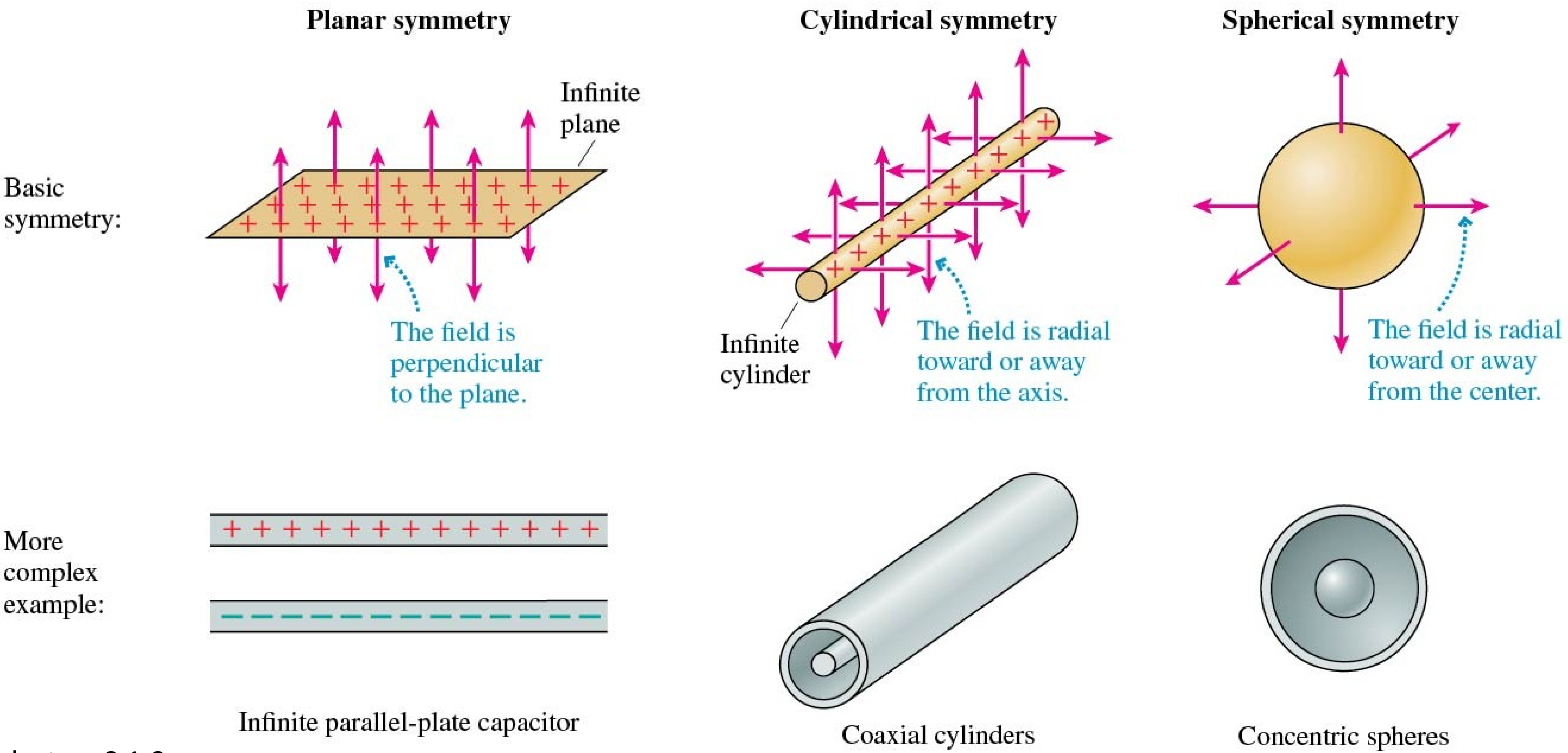
6.1 Electric Flux
6.2 Explaining Gauss's Law
6.3 Applying Gauss's Law
6.4 Conductors in Electrostatic

Carl Friedrich
Gauss
1777-1855



lecture 2.1.2

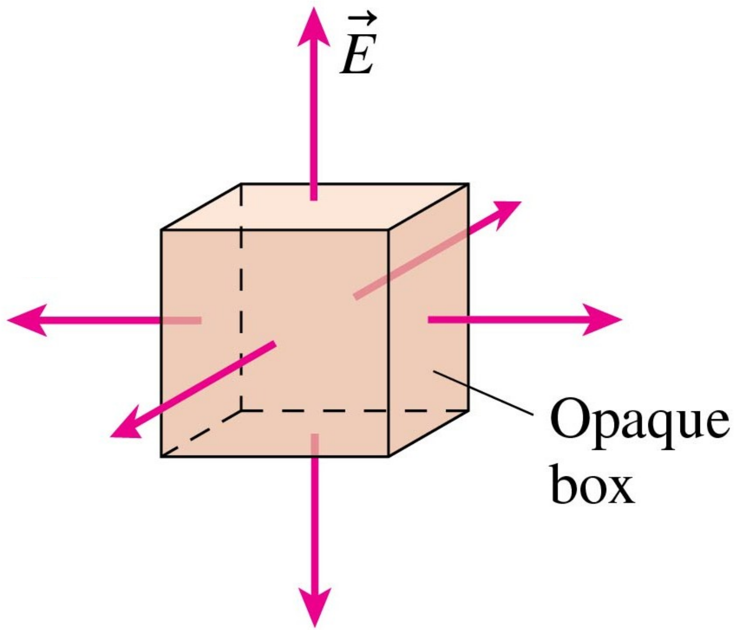
Symmetry of Charge Distributions



Field Flux

If you can’t see into this box, but there is an outward-pointing electric field passing through every surface...

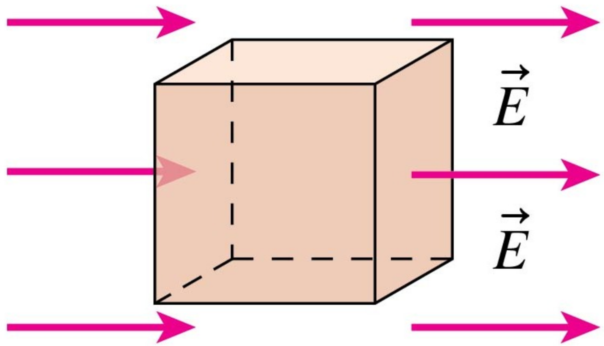
...it must contain a net positive charge.



Field Flux

What must be the net charge inside this box?

a. positive
b. negative
c. zero ✔
d. cannot tell



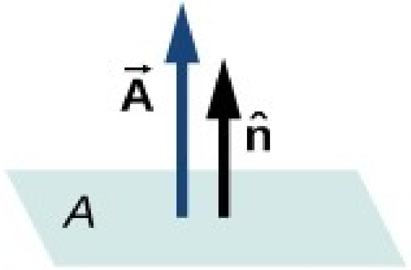
Field Flux

Field flux (Φ) is how much field (E) passes through an area (A):

$$\Phi = \vec{E} \cdot \vec{A}$$

The area vector $\vec{A}$:

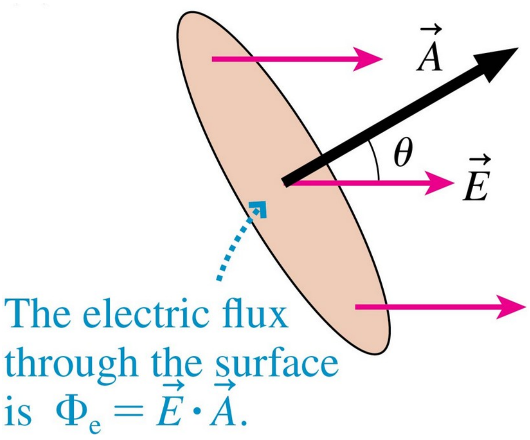
- direction is perpendicular to surface, $\hat{n}$
- magnitude equals the area

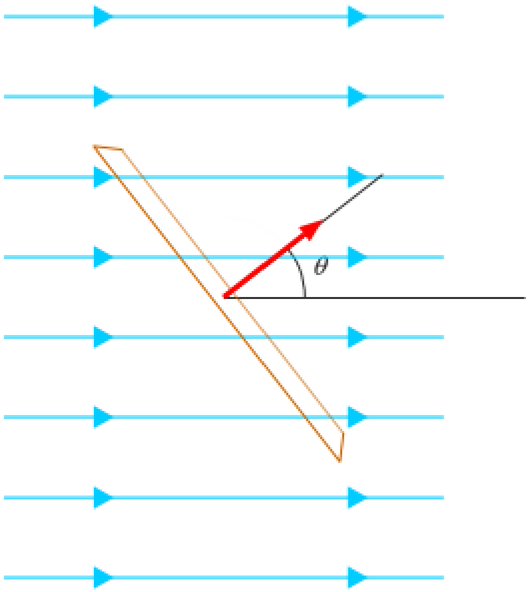


Field Flux

Field flux (Φ) is how much field (E) passes through an area (A):

$$\Phi = \vec{E} \cdot \vec{A} = EA\cos\theta$$

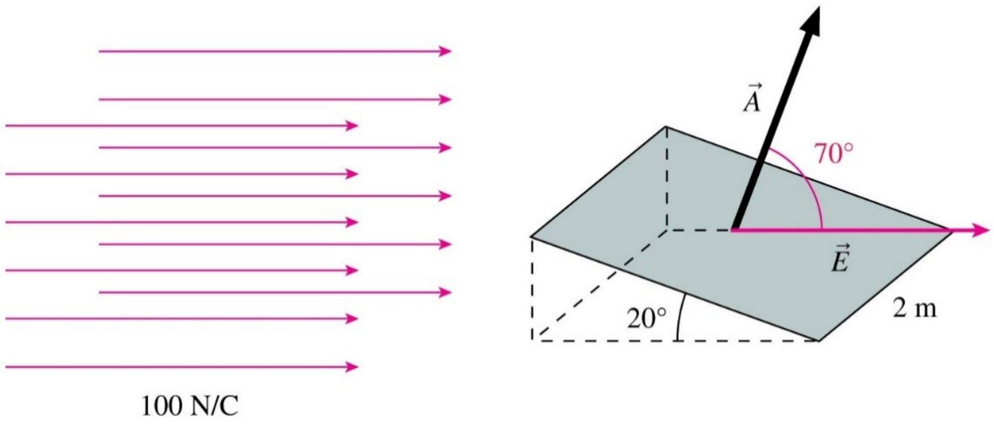




Field Flux

The electric flux through the shaded surface is

A. 0
B. 400cos20º N m²/C
✔ C. 400cos70º N m²/C
D. 400 N m²/C
E. Some other value.

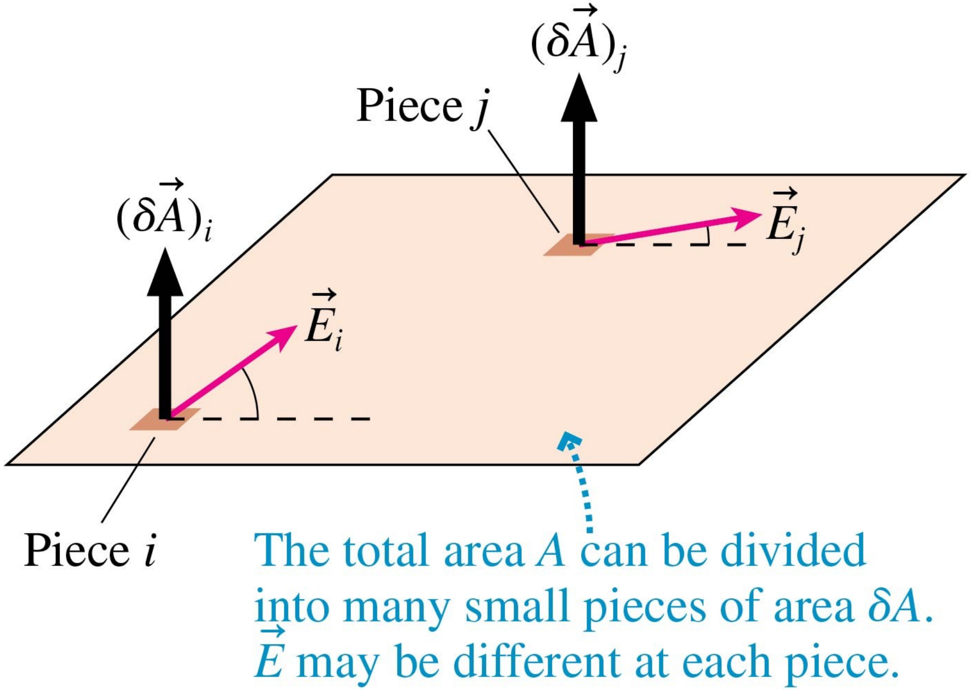


Field Flux

For a *non-uniform* field, divide the surface into many small pieces of area δA.

$$\delta\Phi_i = \vec{E}_i \cdot (\delta\vec{A})_i$$

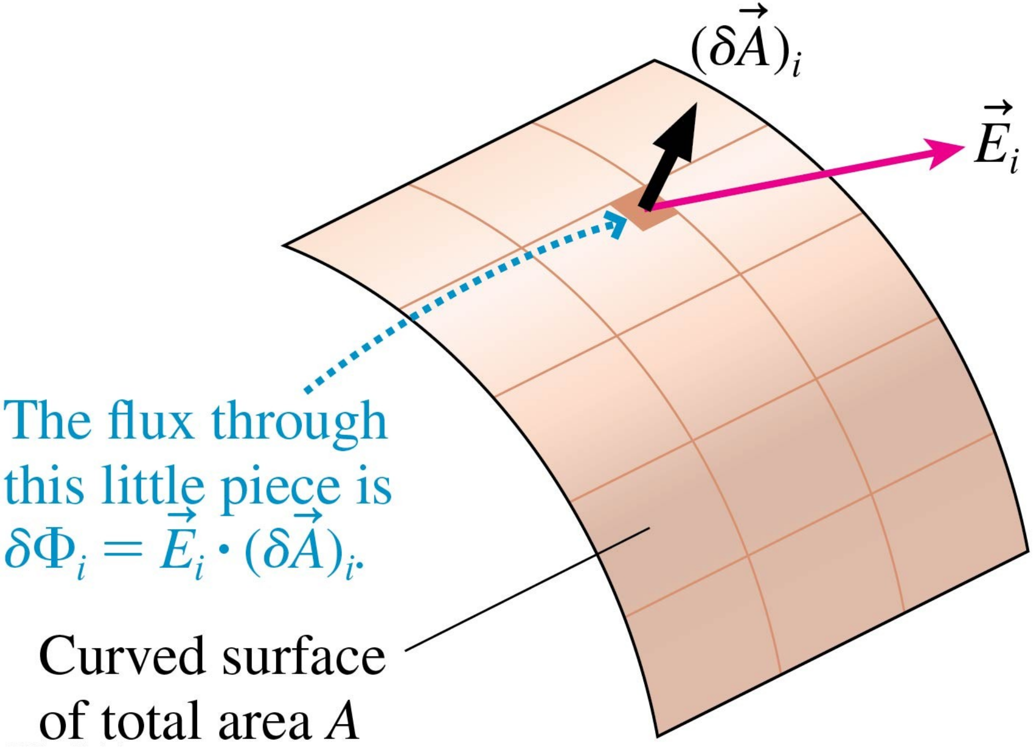
$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$



Field Flux

$$\delta\Phi_i = \vec{E}_i \cdot (\delta\vec{A})_i$$

$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

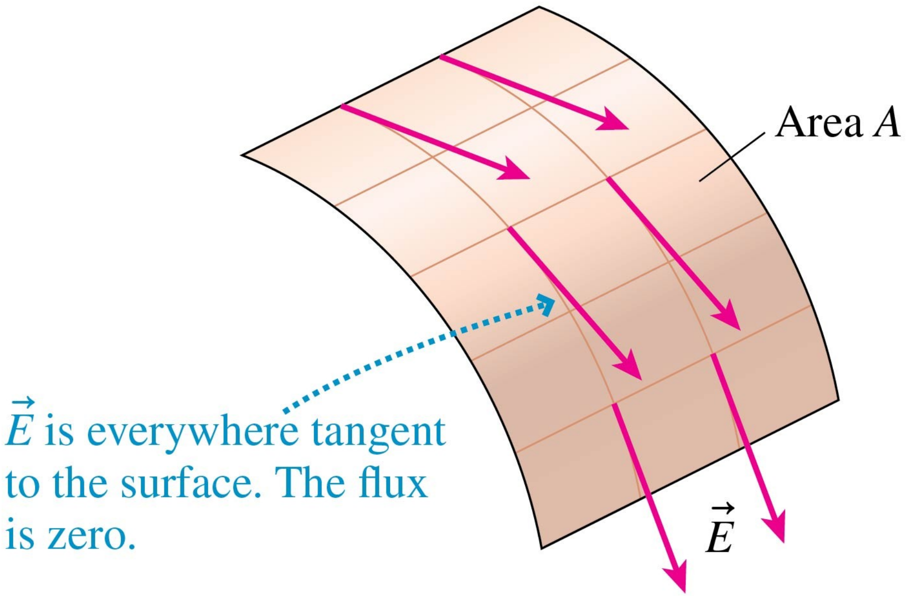


Field Flux

What is the flux through this surface?

$$\delta\Phi_i = \vec{E}_i \cdot (\delta\vec{A})_i$$

$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$



$$\Phi_e = 0$$

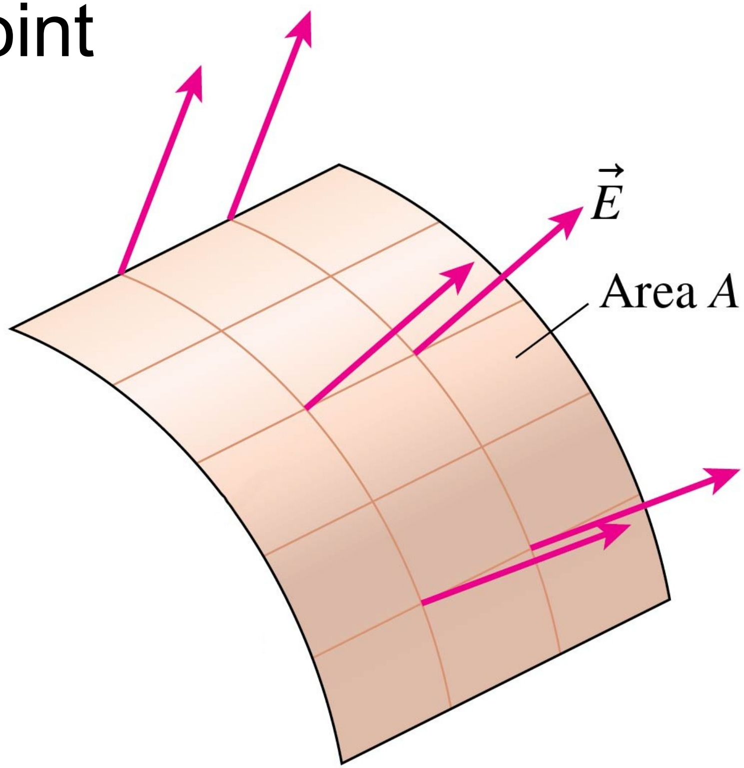
Field Flux

For an electric field that is

- everywhere perpendicular to the surface *and*
- has the same magnitude E at every point

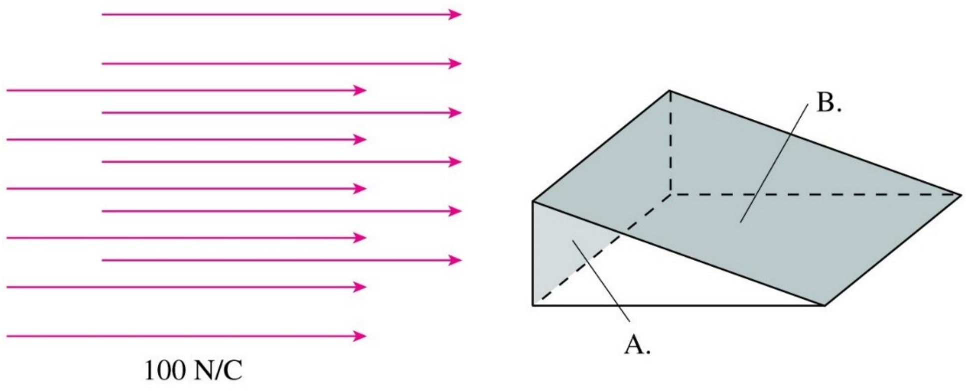
$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

$$= \int_{\text{surface}} E\,dA = E \int_{\text{surface}} dA = EA$$



Field Flux

Which surface, A or B, has the larger electric flux?



A. Surface A has more flux.
B. Surface B has more flux.
✔C. The fluxes are equal.

Field Flux

If you want total flux through a **closed surface**:

$$\Phi_e = \oint \vec{E} \cdot d\vec{A}$$

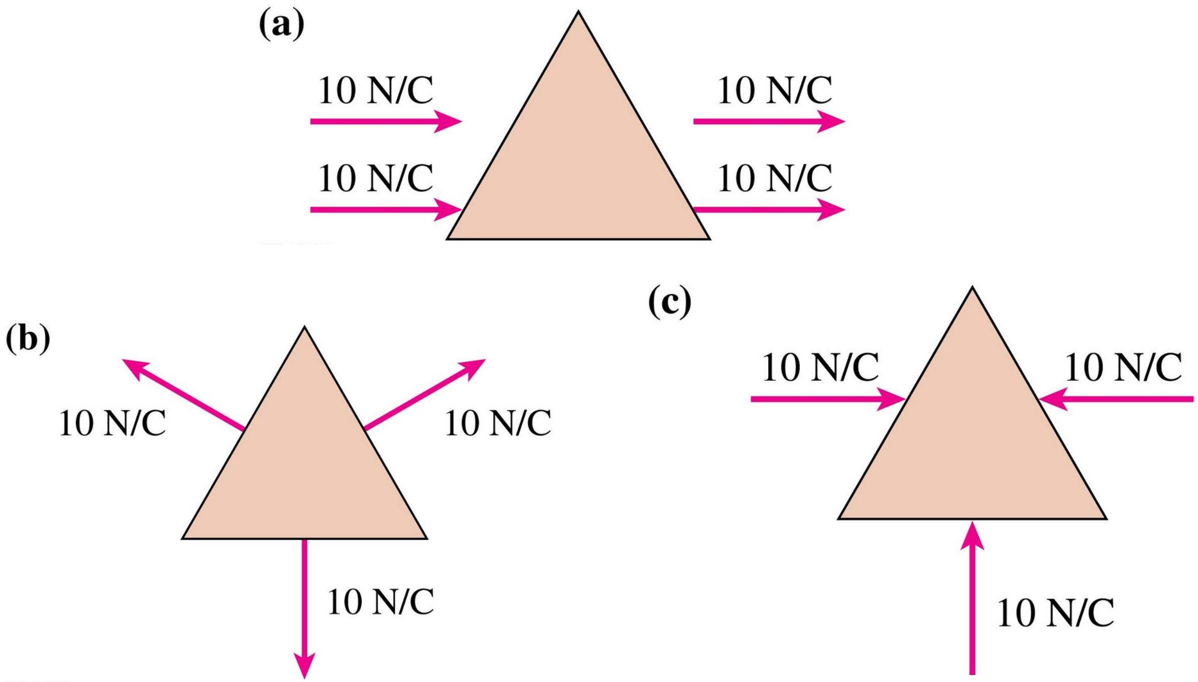
(we use the convention that the area vector dA is defined to always point *toward the outside*)

Such a surface is called a *Gaussian surface*.

Field Flux

These are cross sections of 3D closed surfaces. The top and bottom surfaces, which are flat, are in front of and behind the screen. The electric field is everywhere parallel to the screen. Which closed surface or surfaces have zero electric flux?

✔ A. Surface A
B. Surface B
C. Surface C
D. Surfaces B and C
E. All three surfaces



Field Flux

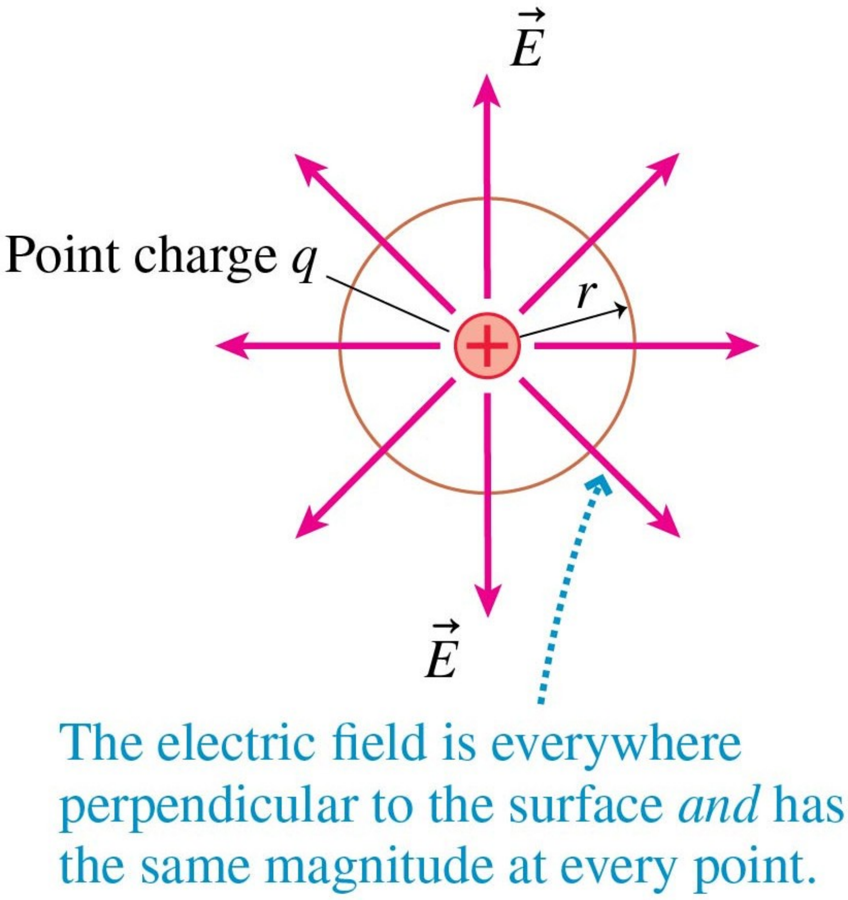
For a single point charge:

- consider a Gaussian surface centered on the charge

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = EA_{\text{sphere}}$$

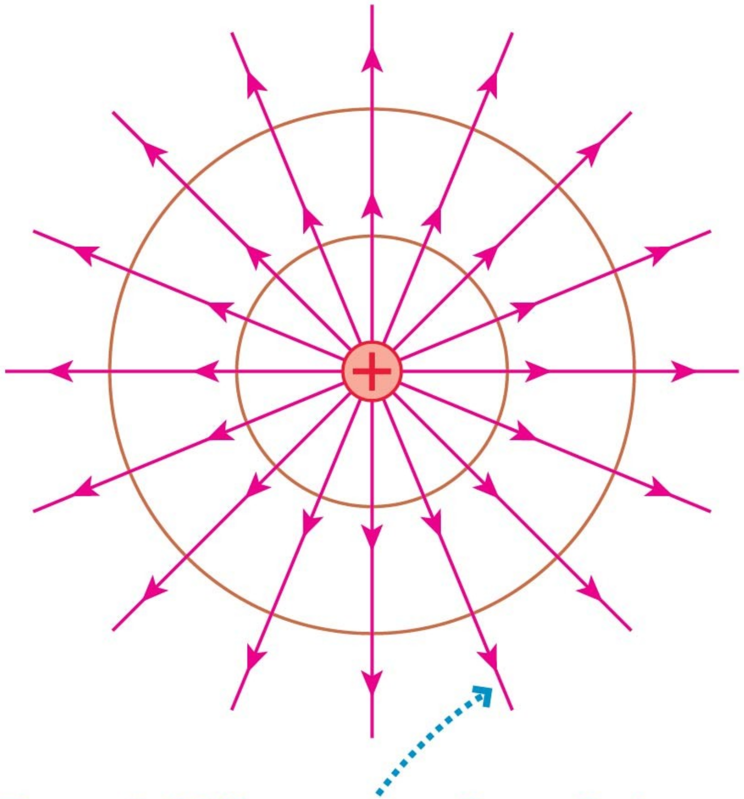
$$\Phi_e = \frac{q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{q}{\epsilon_0}$$

- depends on the amount of charge, but *not* on the radius



Field Flux

$$\Phi_e = \frac{q}{\epsilon_0}$$

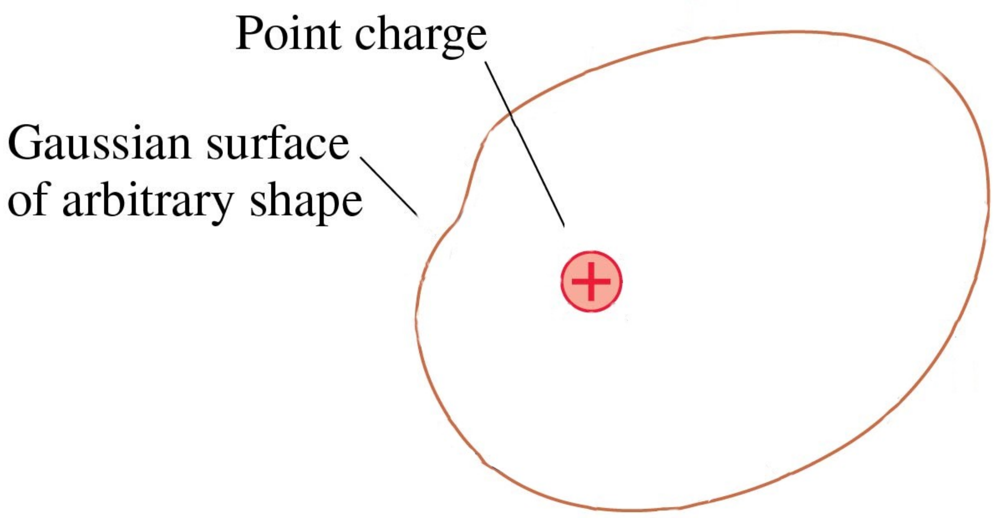


- depends on the amount of charge, but *not* on the radius

Every field line passes through the smaller *and* the larger sphere. The flux through the two spheres is the same.

Field Flux

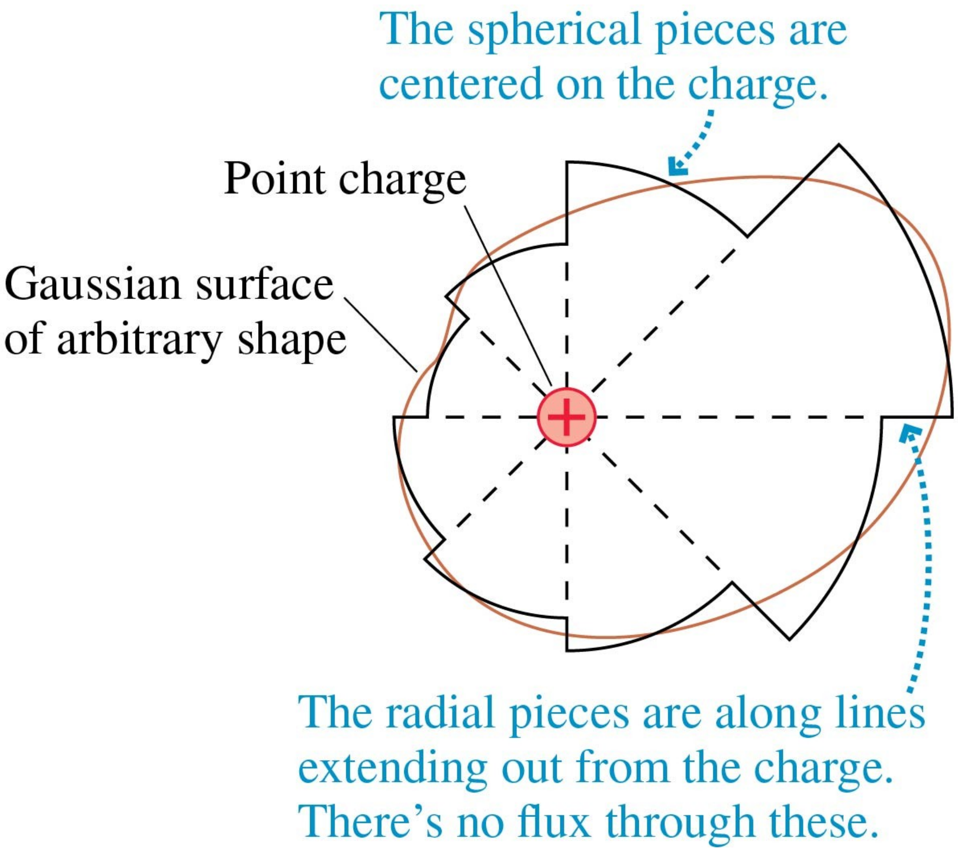
- The electric flux through any arbitrary closed surface surrounding a point charge q may be broken up into spherical and radial pieces.



Field Flux

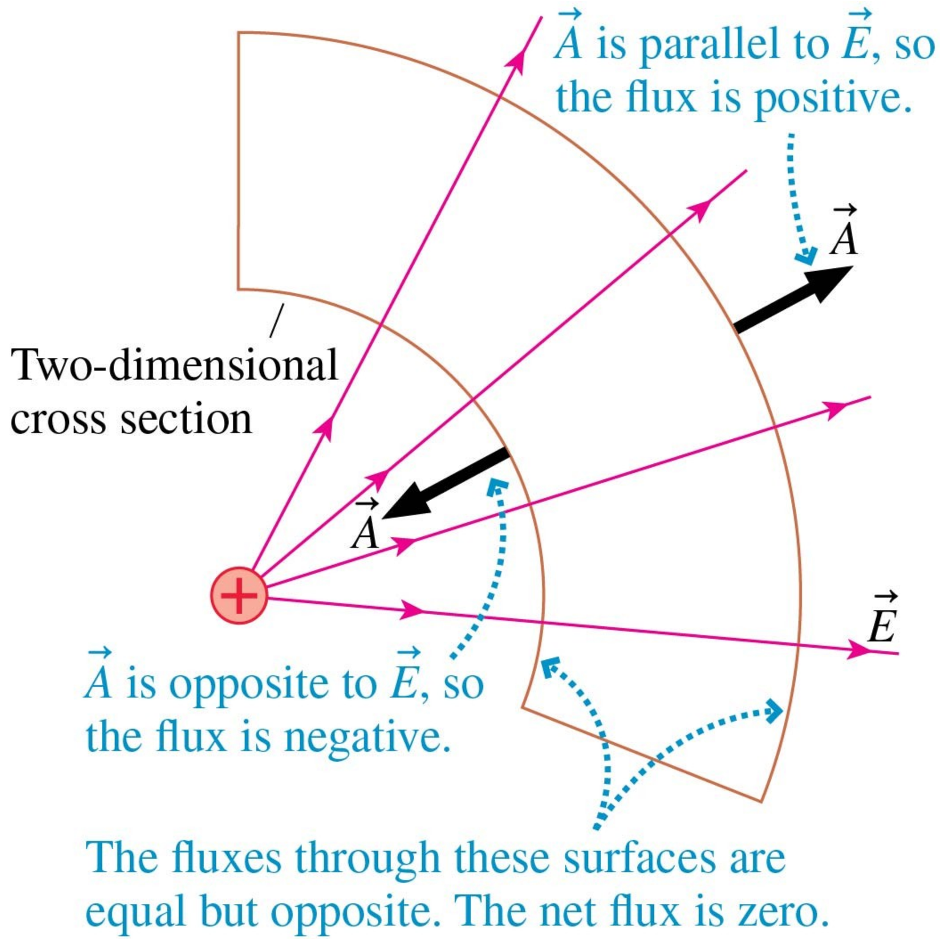
- The electric flux through any arbitrary closed surface surrounding a point charge q may be broken up into spherical and radial pieces.
- total flux same as through a single sphere:

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$



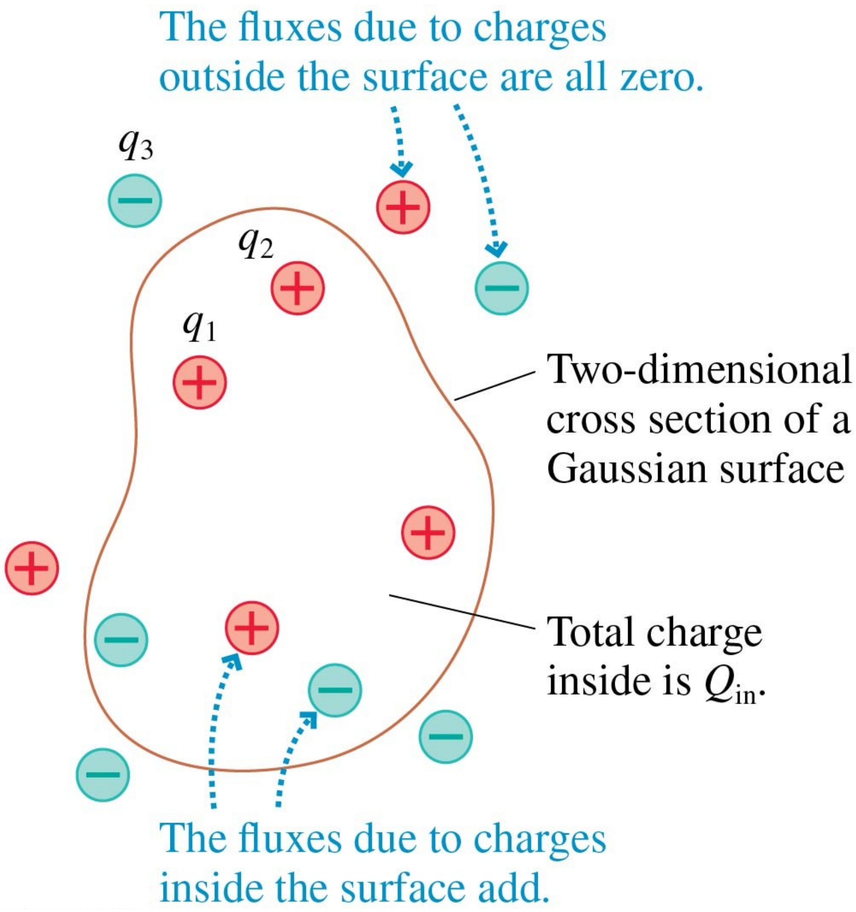
Field Flux

- The net electric flux is *zero* through a closed surface that does not contain any net charge.



Field Flux

- Consider an arbitrary Gaussian surface and a group of charges $q_1, q_2, q_3, \ldots$
- The contribution to the total flux for any charge q_i inside the surface is q_i/ϵ_0.
- The contribution for any charge outside the surface is zero.



Gauss's Law

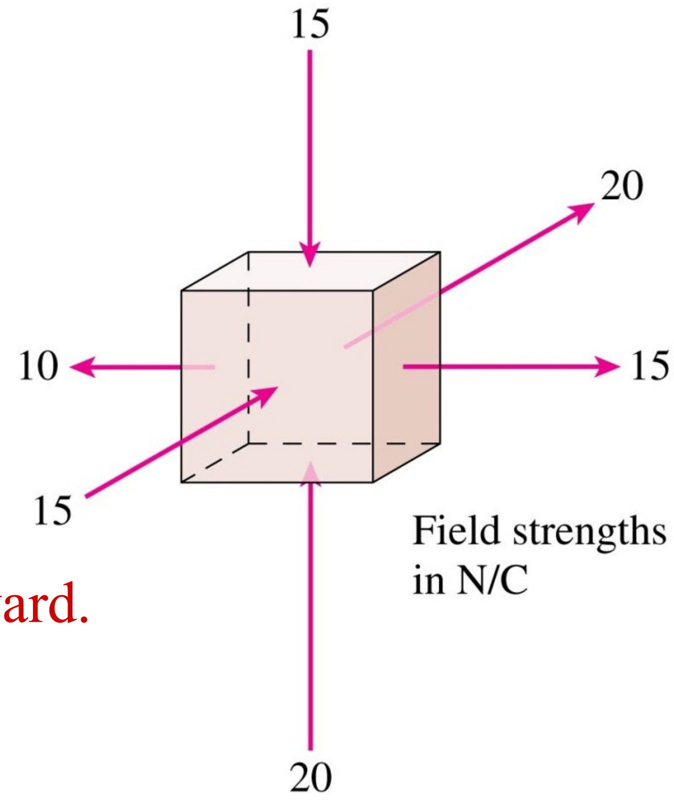
- For any *closed* surface enclosing total charge Q_{in}, the net electric flux through the surface is

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}$$

Gauss's Law

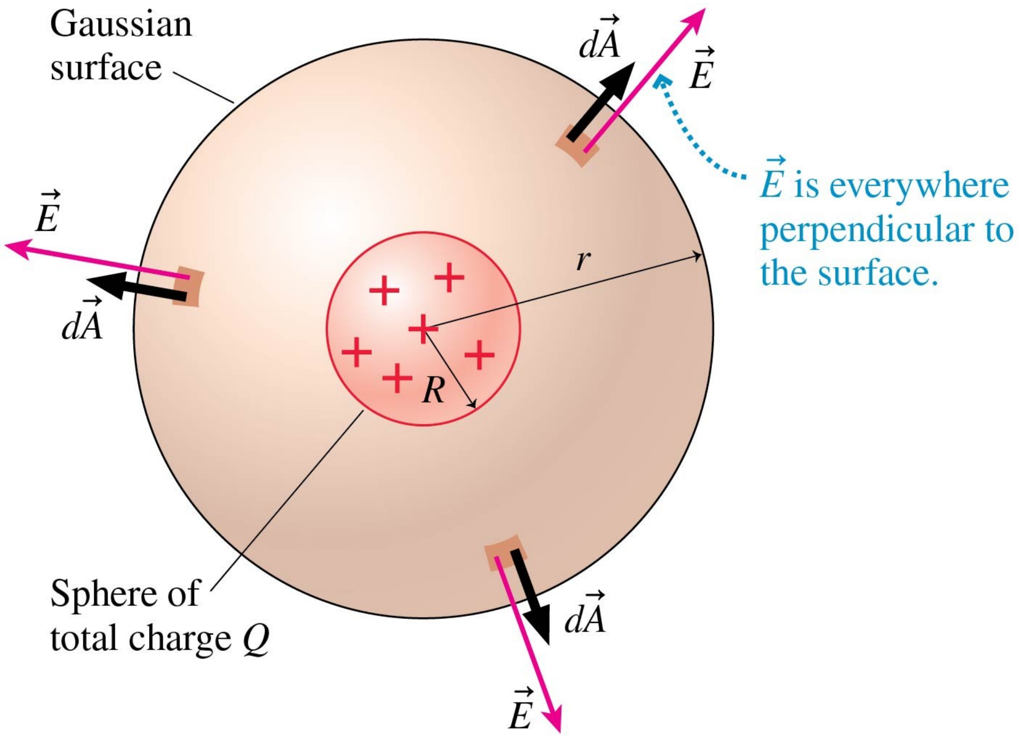
The electric field is constant over each face of the box. The box contains

A. Positive charge.
✔B. Negative charge. Net flux is inward.
C. No net charge.
D. Not enough information to tell.



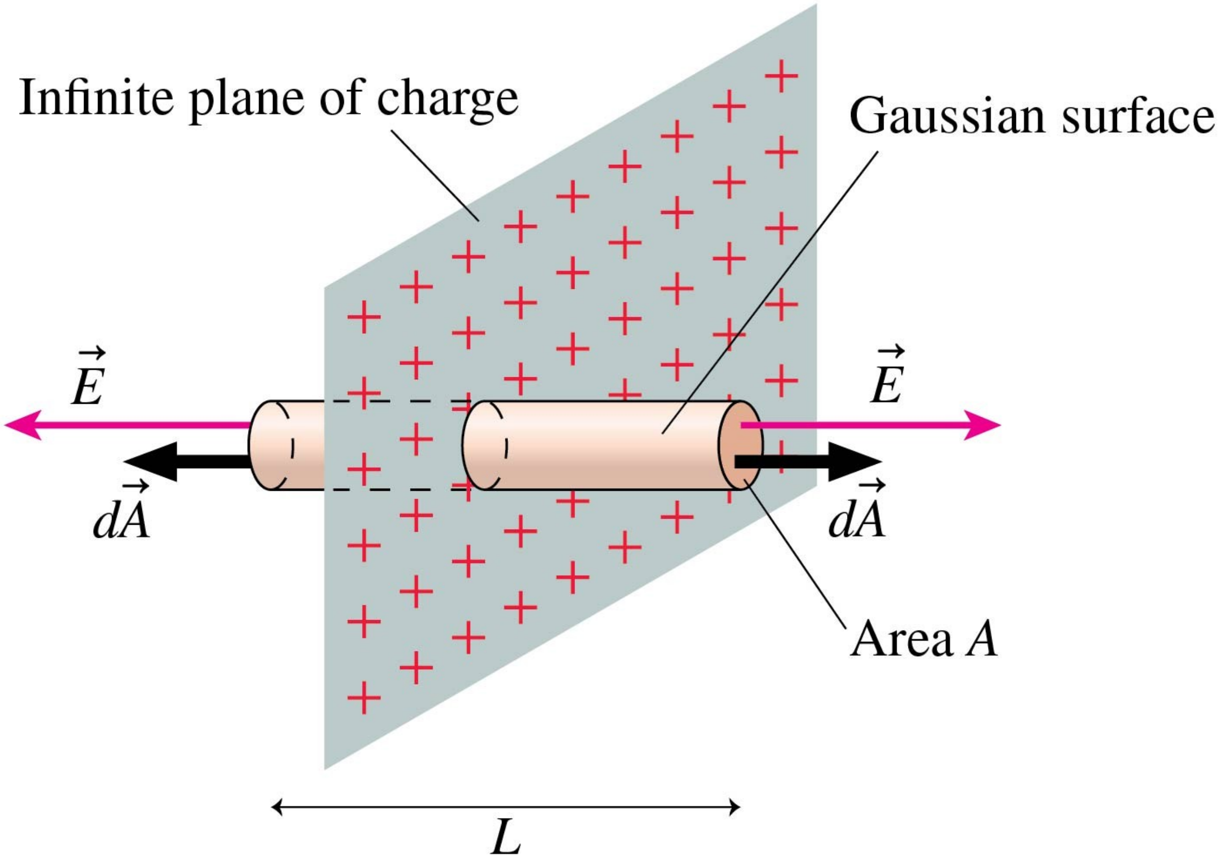
Gauss's Law

Use Gauss's Law to find the electric field outside of a spherical charge distribution.



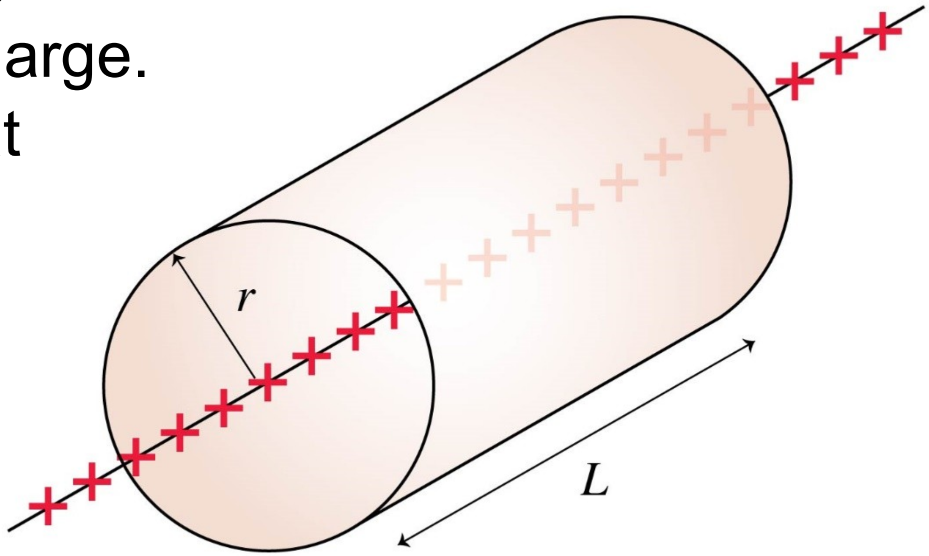
Gauss's Law

Use Gauss's Law to find the electric field near an infinite plane of charge with surface charge density σ (C/m^2).



Gauss's Law

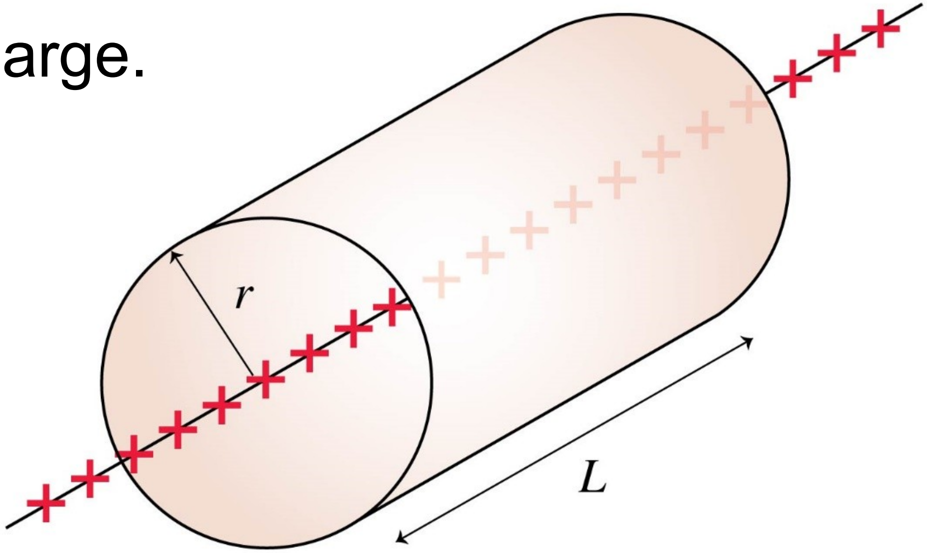
A cylindrical Gaussian surface surrounds an infinite line of charge. The flux Φ_e through the two flat ends of the cylinder is



A. 0 ✔
B. $2 \times 2\pi r E$
C. $2 \times \pi r^2 E$
D. $2 \times r L E$
E. It will require an integration to find out.

Gauss’s Law

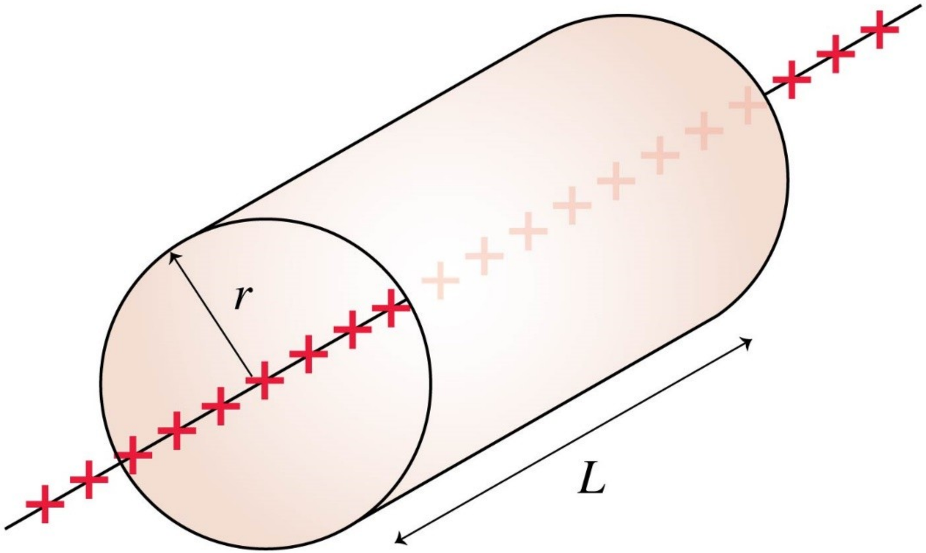
A cylindrical Gaussian surface surrounds an infinite line of charge. The flux Φ_e through the wall of the cylinder is



A. 0
B. $2\pi rLE$ ✔
C. $\pi r^2 LE$
D. rLE

Gauss's Law

Use Gauss's Law to find the electric field outside of a linear charge distribution (density λ C/m).



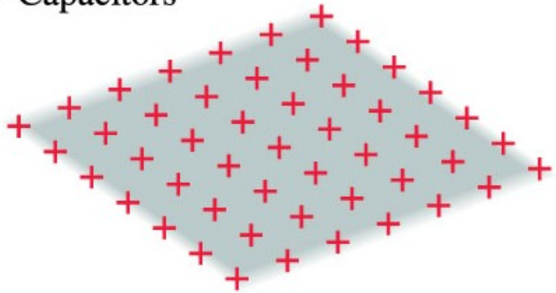
Electric Field of a Charge Distribution

Four key electric fields:

A point charge:
- Small charged objects

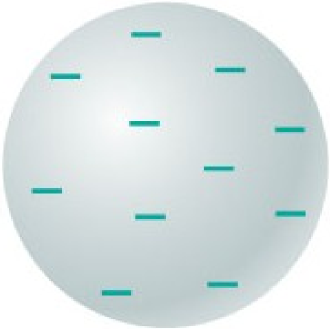
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

An infinitely wide plane of charge:
- Capacitors



$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}$$

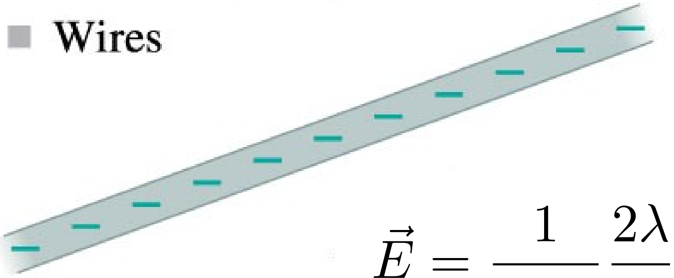
A sphere of charge:
- Electrodes



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

$$(r > R)$$

An infinitely long line of charge:
- Wires



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}\hat{r}$$

Gauss's Law

1. Gauss's law applies only to a *closed* surface, called a Gaussian surface.
2. A Gaussian surface is not a physical surface. It is an imaginary, mathematical surface that we define.
3. We can't find the electric field from Gauss's law alone. We need to apply Gauss's law in situations where, from symmetry and superposition, we already can guess the *shape* of the field.

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

Gauss's Law

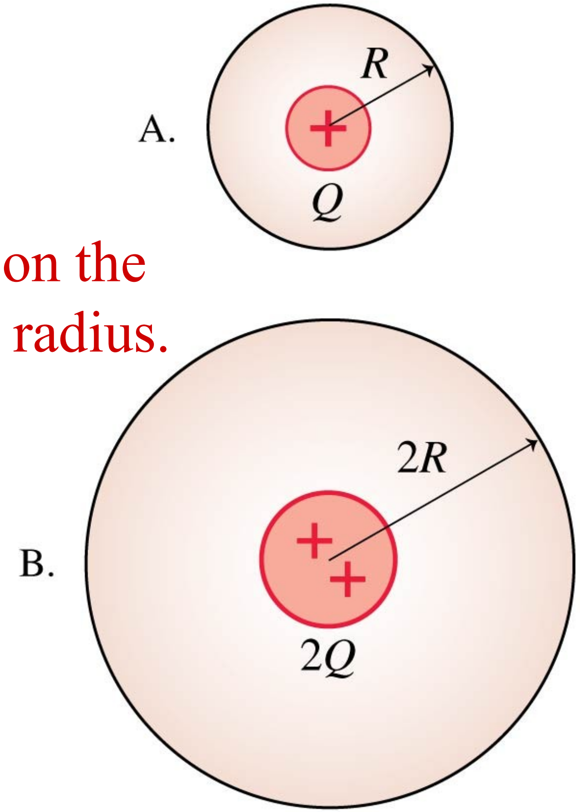
Which spherical Gaussian surface has the larger electric flux?

A. Surface A

✔B. Surface B

C. They have the same flux.

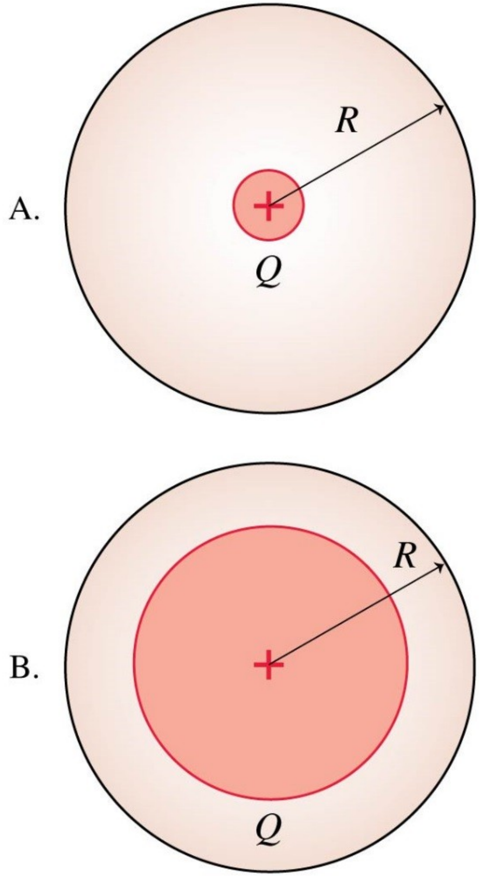
Total flux depends only on the enclosed charge, not the radius.



Gauss's Law

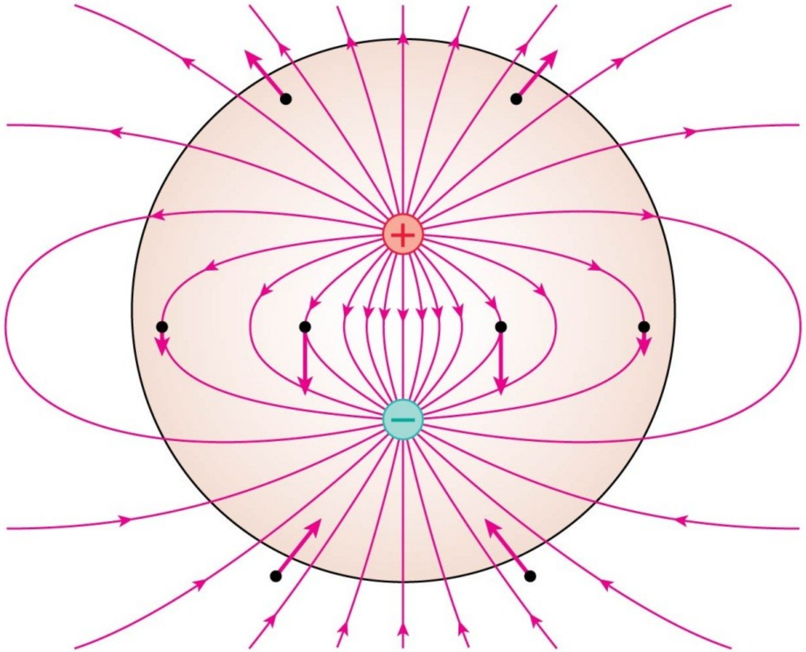
Spherical Gaussian surfaces of equal radius R surround two spheres of equal charge Q. Which Gaussian surface has the larger electric field?

A. Surface A
B. Surface B
✔ C. They have the same electric field.



Gauss's Law

A spherical Gaussian surface surrounds an electric dipole. The net enclosed charge is zero. Which is true?



A. The electric field is zero everywhere on the Gaussian surface.

✔B. The electric field is not zero everywhere on the Gaussian surface.

C. Whether or not the field is zero on the surface depends on where the dipole is inside the sphere.

The flux is zero, but that doesn't require the field to be zero.

Gauss's Law

The electric flux is shown through two Gaussian surfaces. In terms of q, what are charges q_1 and q_2?

A. $q_1 = 2q;\ q_2 = q$

B. $q_1 = q;\ q_2 = 2q$

✔ C. $q_1 = 2q;\ q_2 = -q$

D. $q_1 = 2q;\ q_2 = -2q$

E. $q_1 = q/2;\ q_2 = q/2$