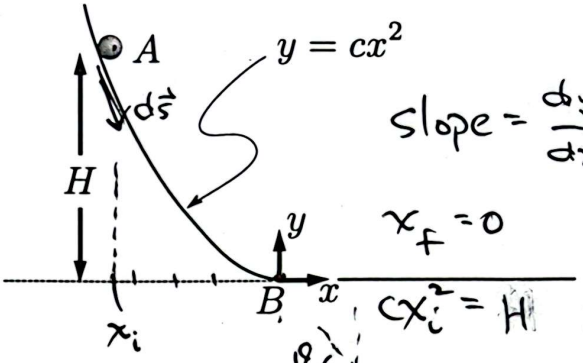
An object of mass m slides frictionlessly down a parabolic ramp. What is the work done by gravity in moving from point A to point B?

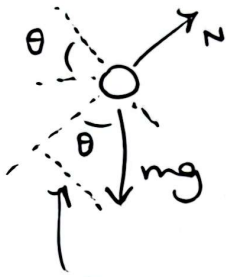


$$\text{slope} = \frac{dy}{dx} = 2cx$$

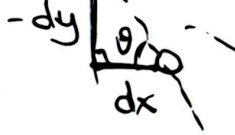
$x_f = 0$

$cx_i^2 = H \quad x_i = \sqrt{\frac{H}{c}}$

$$W = \int_A^B F_s \cdot ds$$



$F_s = mg\sin\theta$

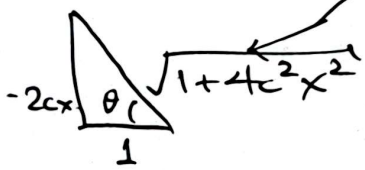


$$\tan\theta = -\frac{dy}{dx} = -2cx$$

$$W = \int_A^B mg\sin\theta \, ds = mg \int_A^B \sin\theta \, ds$$

in terms of x?

similar triangle:



$$\sin\theta = \frac{-2cx}{\sqrt{1+4c^2x^2}}$$

get ds in terms of dx:

$$ds = \sqrt{(dx)^2 + (dy)^2}$$

$$= \sqrt{(dx)^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]} = dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$= dx\sqrt{1+4c^2x^2}$$

$$W = mg \int \frac{-2cx}{\sqrt{1+4c^2x^2}} \cdot dx \sqrt{1+4c^2x^2}$$

$$= mg \int_{-\sqrt{H/c}}^{0} -2cx \, dx = -2cmg \left(\tfrac{1}{2}x^2\right)\Big|_{-\sqrt{H/c}}^{0}$$

$$= -2cmg \cdot \tfrac{1}{2}\left(0 - \tfrac{H}{c}\right)$$

$$= mgH$$