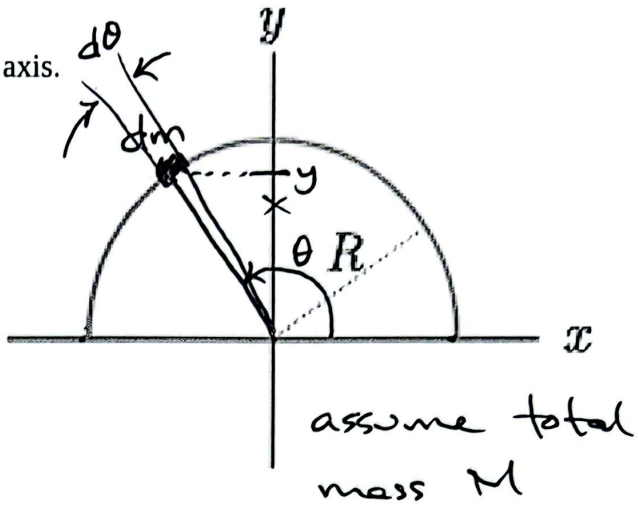
A semicircular wire of radius $R$ lies in the $xy$ plane.
The center is at the origin and the endpoints are on the $x$ axis.

Where is the center of mass of this object?
Give your answer in terms of $R$.

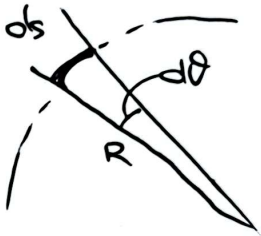



CM at $\bar{x}, \bar{y}$

expect $\bar{x} = 0$

$$\bar{y} = \frac{1}{M} \int y \, dm$$

$y = R \sin\theta$




$ds = R\,d\theta$

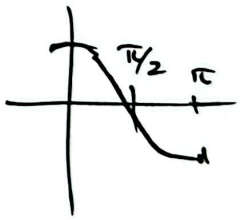
mass per unit length: $\frac{M}{\pi R}$

$$dm = \left(\frac{M}{\pi R}\right) ds = \frac{M}{\pi R} \cdot R\,d\theta$$

$$= \frac{M}{\pi} d\theta$$

$$\bar{y} = \frac{1}{M} \int R \sin\theta \cdot \frac{M}{\pi} d\theta$$

$$= \frac{M}{M} \cdot \frac{R}{\pi} \int_0^{\pi} \sin\theta \, d\theta = \frac{R}{\pi} \left[ -\cos\theta \right]_0^{\pi}$$




$$= -\frac{R}{\pi} (-1 - 1) = \frac{2R}{\pi}$$

**CM is at $\left(0, \frac{2R}{\pi}\right)$**