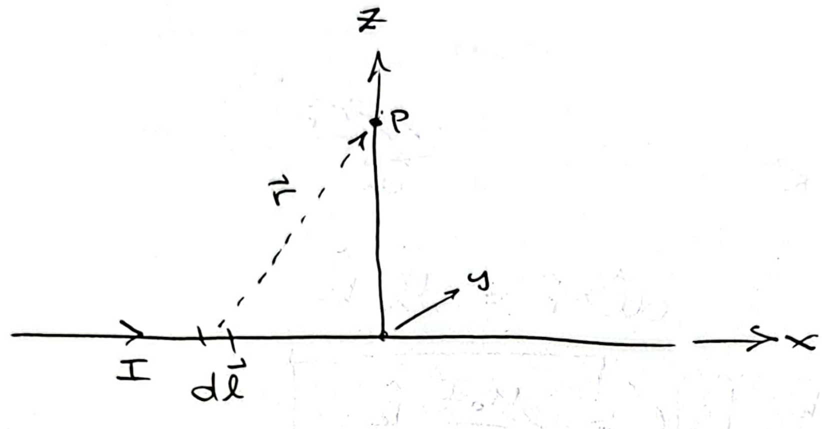
Use the Biot-Savart Law to find the field a distance $z$ away from a long straight wire carrying a current $I$.




$$d\vec{l} = \hat{\imath}\, dx$$

$$\vec{r} = -x\hat{\imath} + z\hat{k}$$

$$r = |\vec{r}| = \sqrt{x^2+z^2}$$

$$\hat{r} = \frac{\vec{r}}{r}$$

$$d\vec{l} \times \hat{r} = \frac{d\vec{l} \times \vec{r}}{r} = \frac{z\,dx\,(-\hat{\jmath})}{\sqrt{x^2+z^2}}$$

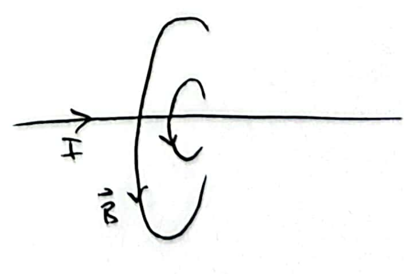
$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2} = -\frac{\mu_0 I \hat{\jmath}}{4\pi} \int \frac{z\,dx}{r^3}$$

$$= -\frac{\mu_0 I \hat{\jmath}}{4\pi}\, z \int \frac{dx}{(x^2+z^2)^{3/2}}$$

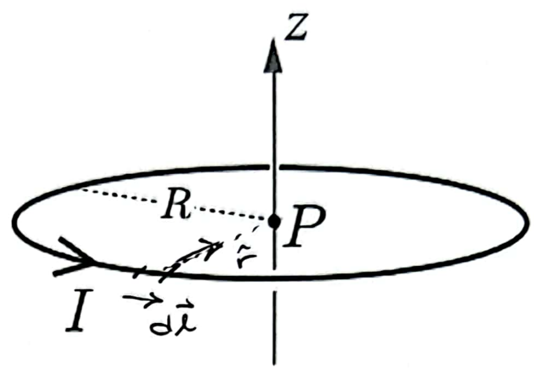
$$= -\frac{\mu_0 I z}{4\pi}\, \hat{\jmath} \left[ \frac{x}{z^2\sqrt{x^2+z^2}} \right]_{-\infty}^{\infty}$$

$$= -\frac{\mu_0 I z}{4\pi}\, \hat{\jmath}\, \frac{1}{z^2} \left[ 1 - (-1) \right] = \boxed{\frac{\mu_0 I}{2\pi z} (-\hat{\jmath})}$$

Generally: $\vec{B} = \frac{\mu_0 I}{2\pi z} \hat{\theta}$



Find the magnetic field at the center of a loop of current $I$.

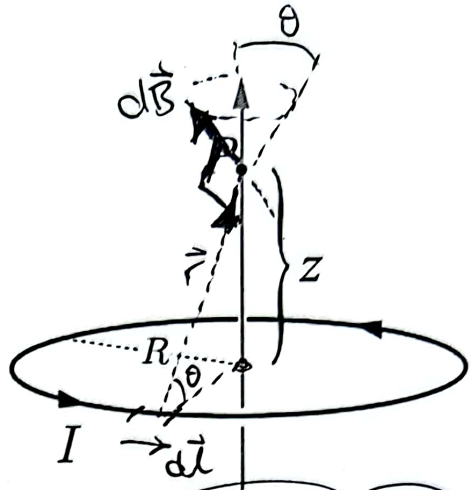



$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{d\ell} \times \hat{r}}{\underbrace{r^2}_{R^2}}$$

$$\vec{d\ell} \times \hat{r} = d\ell\, \hat{k}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \frac{\hat{k}}{R^2} \underbrace{\int d\ell}_{2\pi R} = \boxed{\frac{\mu_0 I}{2R} \hat{k}}$$

Find the magnetic field at a point on the axis of the loop.




$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2} \qquad \hat{r} = \frac{\vec{r}}{r}$$

$$r = \sqrt{z^2 + R^2}$$

$$d\vec{l} \times \vec{r}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \left(\hat{k} \cos\theta\right) \frac{dl}{z^2 + R^2}$$

$$\cos\theta = \frac{R}{\sqrt{z^2 + R^2}}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \hat{k} \frac{R}{(z^2 + R^2)^{3/2}} \underbrace{\int dl}_{2\pi R} =$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi} \frac{2\pi R^2}{(z^2 + R^2)^{3/2}} \hat{k}}$$