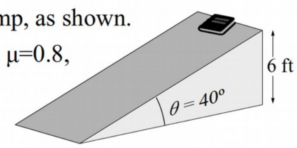
Problem Solving

Using symbols rather than numbers.

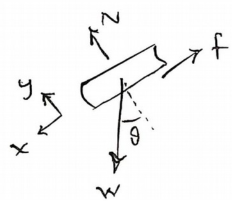
There are a lot of complicated problems to solve in this course, so students sensibly use the most practical and familiar methods to do it. This often involves carrying numbers throughout a long solution. The following scenario illustrates the advantages of using symbols (variables) rather than numbers.

Alice and Bob, both good students, have different ways of solving Dynamics problems. Let's compare their approaches as they confront the problem shown at the right.

A 2.0-lb book slides down a large ramp, as shown. If the coefficient of kinetic friction is μ=0.8, what is its speed at the bottom of the ramp?



They both begin with a free body diagram and Newton's 2nd Law.



$$\sum F_y = 0 = N - W\cos\theta$$

$$\sum F_x = ma = W\sin\theta - f$$

But from this point Alice proceeds symbolically while Bob sticks with numerical values:

Alice

$W = 2.0 \text{ lb} \quad \theta = 40^\circ \quad h = 6 \text{ ft}$

$N = W\cos\theta = mg\cos\theta$

$f = \mu N = \mu mg\cos\theta$

$ma = mg\sin\theta - \mu mg\cos\theta$

$a = g(\sin\theta - \mu\cos\theta)$

$v^2 = 2as \qquad \sin\theta = \frac{h}{s}$

$v^2 = \frac{2ah}{\sin\theta} = 2gh\left(1 - \frac{\mu\cos\theta}{\sin\theta}\right)$

$v = \sqrt{2gh\left(1 - \frac{\mu}{\tan\theta}\right)}$

$v = 4.243 \text{ ft/s}$

Bob

$W = 2.0 \text{ lb}$

$N = (2.0 \text{ lb})\cos 40^\circ = 1.532 \text{ lb}$

$f = (0.8)(1.532 \text{ lb}) = 1.226 \text{ lb}$

$ma = (2.0 \text{ lb})(\sin 40^\circ) - 1.226 \text{ lb}$

$= 0.5958 \text{ lb}$

$m = \frac{2.0 \text{ lb}}{32.2 \text{ lb/slug}} = 0.06211 \text{ slug}$

$a = \frac{0.5958 \text{ lb}}{0.06211 \text{ slug}} = 0.9593 \text{ ft/s}^2$

$s \cdot \sin 40^\circ = 6 \text{ ft} \longrightarrow s = 9.334 \text{ ft}$

$v^2 = 2as = 2\left(0.9593 \frac{\text{ft}}{\text{s}^2}\right)(9.334 \text{ ft})$

$= 17.91 \text{ ft}^2/\text{s}^2$

$v = \sqrt{17.91 \text{ ft}^2/\text{s}^2} = 4.232 \text{ ft/s}$

They both got the right answer, but note the following differences.

1. Alice is done sooner. She had to write less and spent far less time typing into her calculator.
2. Alice is less likely to make a mistake. On each line it's easy to check her algebra. She may also use dimensional analysis as an additional quick check (for example when she finds acceleration, it reasonably appears to be some dimensionless factor times g). Bob had to carefully copy each number from his calculator, potentially introducing a hidden error, and he cannot check his result without re-doing it.
3. Alice understands more. She sees that it doesn't matter that the book is 2.0 lb – the answer applies to any weight. She sees that if $\mu = 0$ (frictionless ramp) then $v = \sqrt{2gh}$, which she might recognize as the velocity after free-fall. She notices the interesting fact that the analysis fails when $\tan\theta < \mu$. She can see relationships like the fact that a ramp of twice the height would yield a velocity only $\sqrt{2}$ times faster. If she wanted she could produce a graph of final speed vs. ramp angle. Bob, unfortunately, can do none of these things.
4. If the final answer were incorrect, Alice would have a much easier time locating the error. Someone helping her could quickly scan the solution for misapplied theory or algebra mistake, but would have to walk through every step of Bob's work with a calculator.
5. Alice doesn't have to worry much about units. Units only appear on the first line when she defines her symbols, and on the last as she goes to the calculator. Bob has to make sure his units are right at every step.
6. Alice can go back later and understand what she did. All her equations are there to see. She might be able to re-use one of her intermediate results in a later problem.
7. Even though Bob kept 4 significant digits, round-off error has accumulated in his solution. His answer is incorrect in the third digit. This would be greater problem if he only kept 3 digits, or if the problem had more steps (as many do), or if an intermediate result is squared or exponentiated, which compounds the error.
8. If Alice wants to compare solutions with another student who was given μ=0.7, she can easily do so. Bob would have to re-do the entire problem.
9. Alice's algebra skills get stronger with each problem. Poor Bob develops tendinitis in his calculator finger.

Of course the method you use is up to you. Some combination of numerical and symbolic steps may suit you best.

If you use the symbolic approach, you might consider these tips:

- Assign symbols to input data, including units.
- If it gets complicated, define intermediate values for expressions that keep being repeated. It helps if the values you define are dimensionless; this makes your equations easier to check and their meaning more clear.
- As you work, keep in mind which of the many variables have known (or knowable) values and which are still unknown.
- For a complete record of the problem and solution, consider copy-pasting the problem statement and figure into a Jupyter Notebook, include a scan of your hand-written symbolic solution, and then do the final calculation in computer code. For examples, see our course resources.