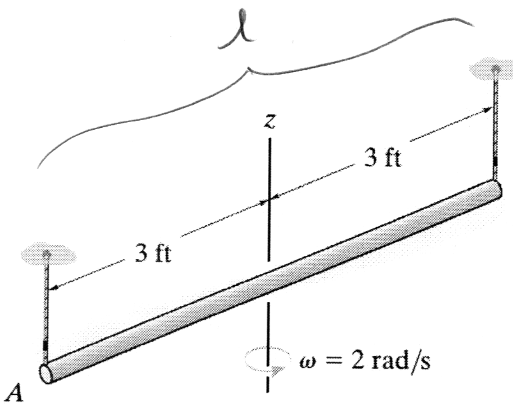
21–30.

The rod weighs 3 lb/ft and is suspended from parallel cords at A and B. If the rod has an angular velocity of 2 rad/s about the z axis at the instant shown, determine how high the center of the rod rises at the instant the rod momentarily stops swinging.



$$\Delta T + \Delta V = 0$$

$$\Delta T = 0 - T_i = -\frac{1}{2} I_G \omega^2 = -\frac{1}{2}\frac{1}{12} m l^2 \omega^2$$

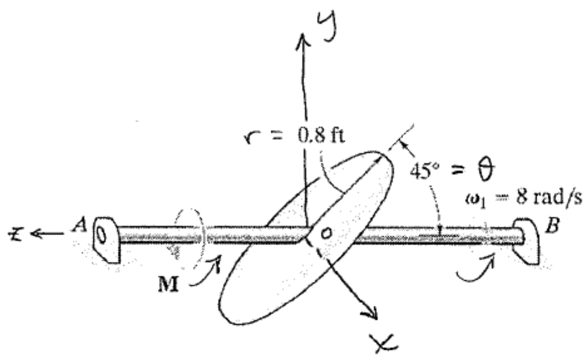
$$\Delta V = mgh - 0$$

$$\longrightarrow \quad \frac{1}{24} \cancel{m} l^2 \omega^2 = \cancel{m} g h$$

$$h = \frac{1}{24g} l^2 \omega^2 = \boxed{0.186 \text{ ft}}$$

21–27.

The circular disk has a weight of 15 lb and is mounted on the shaft AB at an angle of 45° with the horizontal. Determine the angular velocity of the shaft when $t = 2$ s if a torque $M = (4e^{0.1t})$ lb · ft, where t is in seconds, is applied to the shaft. The shaft is originally spinning at $\omega_1 = 8$ rad/s when the torque is applied.



$$\int \vec{M}_o \, dt = \Delta \vec{H}_o$$

(1) Find $\Delta \vec{H}_o$ $\quad \vec{H}_o = \{I\}_o \vec{\omega}$ for fixed point O.

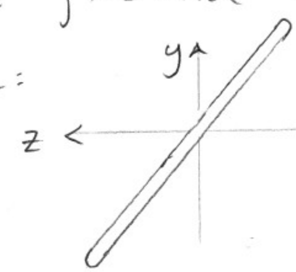
$\vec{\omega} = \omega_z \hat{k} \quad (\omega_x = \omega_y = 0)$

$\vec{H}_o = \{I\}_o \vec{\omega} = -I_{xz}\omega_z \hat{\imath} - I_{yz}\omega_z \hat{\jmath} + I_{zz}\omega_z \hat{k}$

Find these three components of inertia tensor.

(a) $I_{xz} = 0$ by symmetry. Disk is symmetric about the $x=0$ plane so $I_{xz} = \int xz \, dm = 0$.

(b) Consider I_{yz}. View from side:



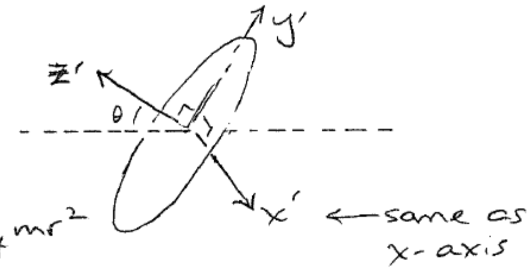
If $y>0$ then $z<0$,

if $y<0$ then $z>0$.

So $I_{yz} = \int yz \, dm < 0$

We won't need its value to solve the problem.

(c) Find I_{zz}



first consider $x'y'z'$ axes:

$$I_{z'} = \frac{1}{2}mr^2 \qquad I_{x'} = I_{y'} = \frac{1}{4}mr^2$$

$$I_{x'y'} = I_{x'z'} = I_{y'z'} = 0$$

rotate to find I_{zz} ($\hat{u}$ in direction of z-axis)

$$\hat{u} = \cos\theta\,\hat{k} - \sin\theta\,\hat{j}$$

products of inertia zero.

$$I_{zz} = I_{x'}u_{x'}^2 + I_{y'}u_{y'}^2 + I_{z'}u_{z'}^2 + 0$$

$$= 0 + \frac{1}{4}mr^2\sin^2\theta + \frac{1}{2}mr^2\cos^2\theta$$

for $\theta = 45°$ $\quad \sin^2\theta = \cos^2\theta = \frac{1}{2}$

$$I_{zz} = \left(\frac{1}{8} + \frac{1}{4}\right)mr^2 = \frac{3}{8}mr^2$$

$$\vec{H}_o = -I_{yz}\omega_z\,\hat{j} + \frac{3}{8}mr^2\omega_z\,\hat{k}$$

initially $\omega_z = \omega_1$ $\qquad$ finally $\omega_z = \omega_f$

So: $\quad \Delta\vec{H}_o = -I_{yz}(\omega_f - \omega_1)\,\hat{j} + \frac{3}{8}mr^2(\omega_f - \omega_1)\,\hat{k}$

(2) Use impulse-momentum principal: $\int \vec{M}_o dt = \Delta\vec{H}_o$

There is angular impulse in the $\hat{j}$ direction (at the instant shown) — this creates a reaction moment in bearings at A & B.

Impulse in the $\hat{k}$ direction is provided by the applied torque:

$$\int_{t_1}^{t_2} M(t)\,dt = \frac{3}{8}mr^2(\omega_f - \omega_i)$$