

Mathematical Expressions

Quadratic Formula

$$\text{If } ax^2 + bx + c = 0, \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}, \cosh x = \frac{e^x + e^{-x}}{2}, \tanh x = \frac{\sinh x}{\cosh x}$$

Trigonometric Identities

$$\sin \theta = \frac{A}{C}, \csc \theta = \frac{C}{A}$$

$$\cos \theta = \frac{B}{C}, \sec \theta = \frac{C}{B}$$

$$\tan \theta = \frac{A}{B}, \cot \theta = \frac{B}{A}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin(\theta \pm \phi) = \sin \theta \cos \phi \pm \cos \theta \sin \phi$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

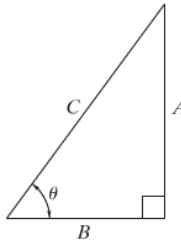
$$\cos(\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\cos \theta = \pm \sqrt{\frac{1 + \cos 2\theta}{2}}, \sin \theta = \pm \sqrt{\frac{1 - \cos 2\theta}{2}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$1 + \tan^2 \theta = \sec^2 \theta \quad 1 + \cot^2 \theta = \csc^2 \theta$$



Power-Series Expansions

$$\sin x = x - \frac{x^3}{3!} + \dots \quad \sinh x = x + \frac{x^3}{3!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \dots \quad \cosh x = 1 + \frac{x^2}{2!} + \dots$$

Derivatives

$$\frac{d}{dx}(u^n) = nu^{n-1} \frac{du}{dx}$$

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{d}{dx}(\cot u) = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \tan u \sec u \frac{du}{dx}$$

$$\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\sinh u) = \cosh u \frac{du}{dx}$$

$$\frac{d}{dx}(\cosh u) = \sinh u \frac{du}{dx}$$

Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$\int \frac{dx}{a+bx} = \frac{1}{b} \ln(a+bx) + C$$

$$\int \frac{dx}{a+bx^2} = \frac{1}{2\sqrt{-ba}} \ln \left[\frac{a+x\sqrt{-ab}}{a-x\sqrt{-ab}} \right] + C, ab < 0$$

$$\int \frac{x dx}{a+bx^2} = \frac{1}{2b} \ln(bx^2+a) + C$$

$$\int \frac{x^2 dx}{a+bx^2} = \frac{x}{b} - \frac{a}{b\sqrt{ab}} \tan^{-1} \frac{x\sqrt{ab}}{a} + C, ab > 0$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left[\frac{a+x}{a-x} \right] + C, a^2 > x^2$$

$$\int \sqrt{a+bx} dx = \frac{2}{3b} \sqrt{(a+bx)^3} + C$$

$$\int x\sqrt{a+bx} dx = \frac{-2(2a-3bx)\sqrt{(a+bx)^3}}{15b^2} + C$$

$$\int x^2\sqrt{a+bx} dx = \frac{2(8a^2-12abx+15b^2x^2)\sqrt{(a+bx)^3}}{105b^3} + C$$

$$\int \sqrt{a^2-x^2} dx = \frac{1}{2} \left[x\sqrt{a^2-x^2} + a^2 \sin^{-1} \frac{x}{a} \right] + C, a > 0$$

$$\int x\sqrt{x^2 \pm a^2} dx = \frac{1}{3} \sqrt{(x^2 \pm a^2)^3} + C$$

$$\int x^2\sqrt{a^2-x^2} dx = -\frac{x}{4}\sqrt{(a^2-x^2)^3} + \frac{a^2}{8} \left(x\sqrt{a^2-x^2} + a^2 \sin^{-1} \frac{x}{a} \right) + C, a > 0$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2}) \right] + C$$

$$\int x\sqrt{a^2-x^2} dx = -\frac{1}{3}\sqrt{(a^2-x^2)^3} + C$$

$$\int x^2\sqrt{x^2 \pm a^2} dx = \frac{x}{4}\sqrt{(x^2 \pm a^2)^3} \mp \frac{a^2}{8}x\sqrt{x^2 \pm a^2} - \frac{a^4}{8} \ln(x + \sqrt{x^2 \pm a^2}) + C$$

$$\int \frac{dx}{\sqrt{a+bx}} = \frac{2\sqrt{a+bx}}{b} + C$$

$$\int \frac{x dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2} + C$$

$$\int \frac{dx}{\sqrt{a+bx+cx^2}} = \frac{1}{\sqrt{c}} \ln \left[\sqrt{a+bx+cx^2} + x\sqrt{c} + \frac{b}{2\sqrt{c}} \right] + C, c > 0$$

$$= \frac{1}{\sqrt{-c}} \sin^{-1} \left(\frac{-2cx-b}{\sqrt{b^2-4ac}} \right) + C, c < 0$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int x \cos(ax) dx = \frac{1}{a^2} \cos(ax) + \frac{x}{a} \sin(ax) + C$$

$$\int x^2 \cos(ax) dx = \frac{2x}{a^2} \cos(ax) + \frac{a^2x^2-2}{a^3} \sin(ax) + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax-1) + C$$

$$\int \sinh x dx = \cosh x + C$$

$$\int \cosh x dx = \sinh x + C$$