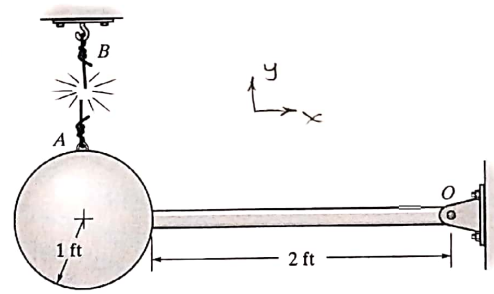
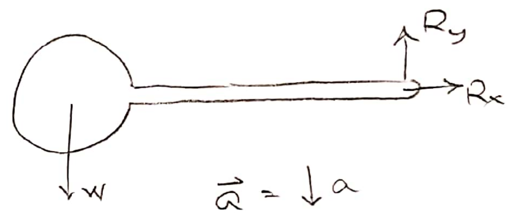
1. The pendulum shown consists of a 20-lb sphere and a massless rod. Compute the reaction at the pin O just after the cord AB is cut.



answer:

$\vec{R} =$ $(0.851 \text{ lb})\ \hat{j}$

$I_O = \frac{2}{5} m r^2 + m l^2$ $\quad l = 3 \text{ ft}$



$\sum F_x = 0 = R_x$

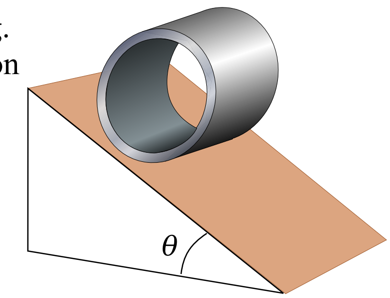
$\vec{a} = \downarrow a$

$\sum F_y = R_y - W = -ma$

$\sum M_O = I_O \alpha = Wl \longrightarrow \alpha = \frac{Wl}{I_O} = \frac{a}{l}$

$R_y - W = -\frac{mWl^2}{I_O}$ $\qquad R_y = W\left(1 - \frac{ml^2}{I_O}\right) = 0.851 \text{ lb}$

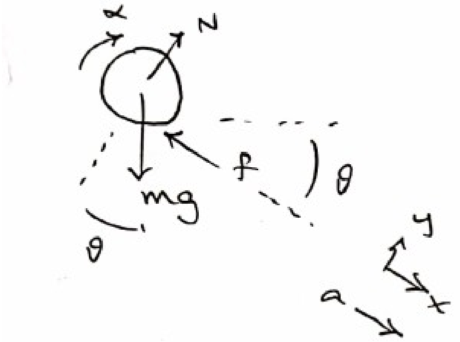
2. The cylinder rolls down the inclined plane ($\theta = 30°$) without slipping. Determine the angular acceleration of the cylinder and the acceleration of its mass center. The cylinder has an outer radius of 50.0 cm, a radius of gyration of 40.0 cm and mass 10.0 kg. Your solution must include a free-body diagram.



answers:

$\alpha =$ 5.98 rad/s²

$a =$ 2.99 m/s²



$$\sum F_x = mg\sin\theta - f = ma \quad (2)$$

$$\sum F_y = N - mg\cos\theta = 0$$

$$\sum M_G = -fr = -I\alpha, \quad I = mk^2 \quad (1)$$

rolling without slip: $a = \alpha r$

(1)

$$f = \frac{mk^2}{r}\frac{a}{r} = \left(\frac{k}{r}\right)^2 ma$$

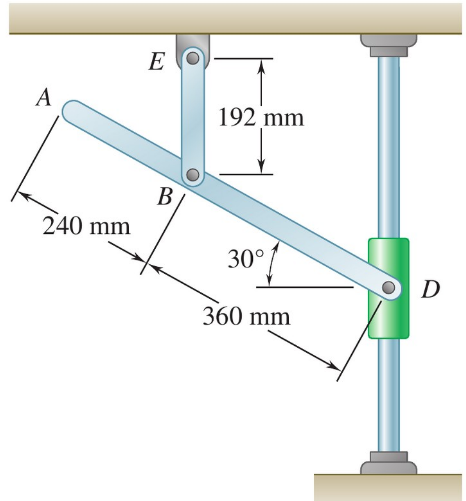
(2)

$$mg\sin\theta = ma + \left(\frac{k}{r}\right)^2 ma \longrightarrow a = \frac{g\sin\theta}{1+\left(\frac{k}{r}\right)^2}$$

$$a = \left(9.8\,\frac{m}{s^2}\right)\frac{1/2}{1+(4/5)^2} = 2.99\ m/s^2$$

$$\alpha = \frac{a}{r} = 5.98\ rad/s^2$$

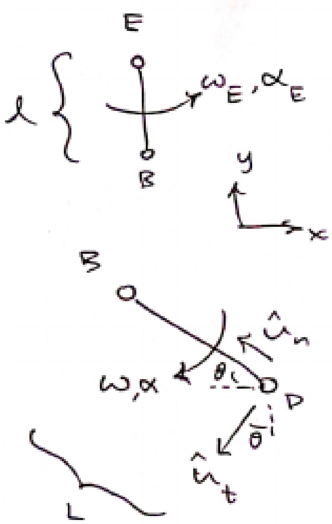
3. At the instant shown, rod *BE* has a counterclockwise angular velocity of 4.0 rad/s that is increasing at a rate of 1.0 rad/s^2. Determine the velocity and acceleration of the collar *D* (using positive for upward and negative for downward along the rod).



answers:

$v =$ -1.33 m/s

$a =$ $15.847\ \frac{m}{s^2}$



$\vec v_B = \omega_E l \hat i$ (1)

$\vec a_B = \alpha_E l \hat i + \omega_E^2 l \hat j$ (2)

(1)

$\vec v_D = \vec v_B + \vec v_{D/B}$

$-\hat i \sin\theta - \hat j \cos\theta$

$v\hat j = \omega_E l \hat i + \omega L \hat u_t$

$\rightarrow\ 0 = \omega_E l - \omega L \sin\theta \qquad \omega = \frac{l\,\omega_E}{L\sin\theta}$

$= 4.267\ \text{rad/s}$

$v = -\omega L\cos\theta = -1.33$ m/s

$a\hat j$ (2) $\qquad -\hat i\cos\theta + \hat j \sin\theta$

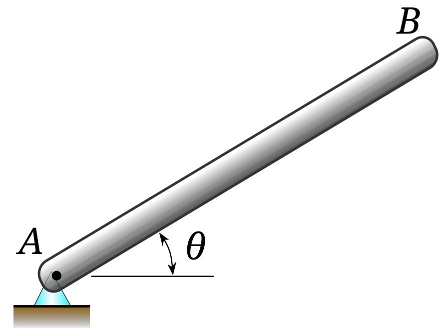
$\vec a_D = \vec a_B + \vec a_{D/B} = \alpha_E l \hat i - \omega_E^2 l \hat j + \alpha L \hat u_t + \omega^2 L \hat u_n$

$a\hat j = \alpha_E l \hat i + \omega_E^2 l \hat j - \alpha L(\hat i \sin\theta + \hat j\cos\theta) + \omega^2 L(-\hat i\cos\theta + \hat j \sin\theta)$

$\hat i \longrightarrow\ 0 = \alpha_E l - \alpha L \sin\theta - \omega^2 L\cos\theta \longrightarrow \alpha = -30.464\ \frac{\text{rad}}{s^2}$

$\hat j \longrightarrow\ a = \omega_E^2 l - \alpha L\cos\theta + \omega^2 L \sin\theta = 15.847\ \frac{m}{s^2}$

4. The uniform rod *AB* has a mass of 5.0 kg and length of 1.0 m. It is frictionlessly pinned at *A*. Determine the angular acceleration of the rod when released from rest at angle $\theta = 30°$. Also determine the reaction forces at *A* at this moment.

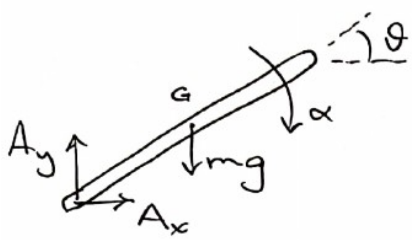


answers:

$\alpha =$ 12.7 rad/s²

$A_x =$ 15.93 N

$A_y =$ 21.46 N

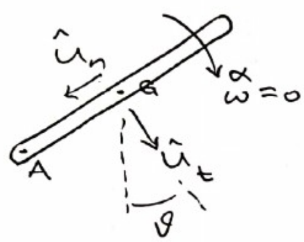


$$\sum M_A = -\tfrac{1}{2} L \cos\vartheta \cdot mg = -I_A \alpha$$

$$I_A = \tfrac{1}{3} m L^2$$

$$\rightarrow \tfrac{1}{2} m L g \cos\vartheta = \tfrac{1}{3} m L^2 \alpha$$

$$\alpha = \tfrac{3}{2}\tfrac{g}{L}\cos\vartheta = 12.7 \text{ rad/s}^2$$



$$\vec{a}_G = \tfrac{1}{2}\alpha L\,\hat{u}_t + 0\,\hat{u}_n$$

$$= \tfrac{1}{2}\alpha L(\hat{\imath}\sin\vartheta - \hat{\jmath}\cos\vartheta)$$

$$\sum \vec{F} = m\vec{a}_G$$

$$\rightarrow \vec{A} - mg\hat{\jmath} = \frac{m\alpha L}{2}(\hat{\imath}\sin\vartheta - \hat{\jmath}\cos\vartheta)$$

$$A_x = \tfrac{1}{2} m\alpha L \sin\vartheta = 15.93 \text{ N}$$

$$A_y = mg - \tfrac{1}{2} m\alpha L\cos\vartheta = 21.46 \text{ N}$$