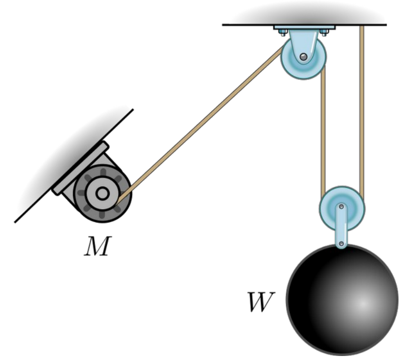
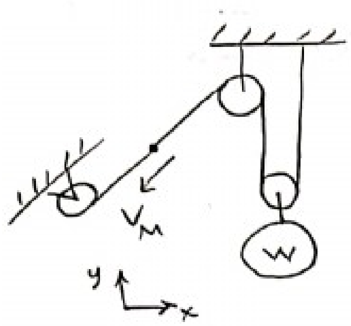
1. The motor M has efficiency $\varepsilon = 0.8$ and is used to lift the weight $W = 100$ lb. At the moment shown, the cable is being pulled into the motor at a speed of $v = 10$ ft/s, which is increasing at a rate of $a = 5$ ft/s^2. What power must be supplied to the motor at this instant?



answer:

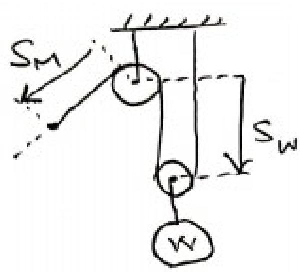
$P =$ 1.22 hp



$$P_{in} = \frac{1}{\epsilon} P_{out} = \frac{1}{\epsilon} T \underbrace{v_M}_{10\ \text{ft/s}}$$

find tension T:

$$\sum F_y = 2T - W$$
$$= m a_w$$



find a_w from cord length:

$$s_M + 2s_w + c = L$$

$$\dot{s}_M + 2\dot{s}_w = 0, \quad \underbrace{\ddot{s}_w}_{-a_w} = -\frac{1}{2}\underbrace{\ddot{s}_M}_{5\ \text{ft/s}^2}$$

$$a_w = \frac{5}{2}\ \text{ft/s}^2$$
$$= \frac{1}{2} a_M$$

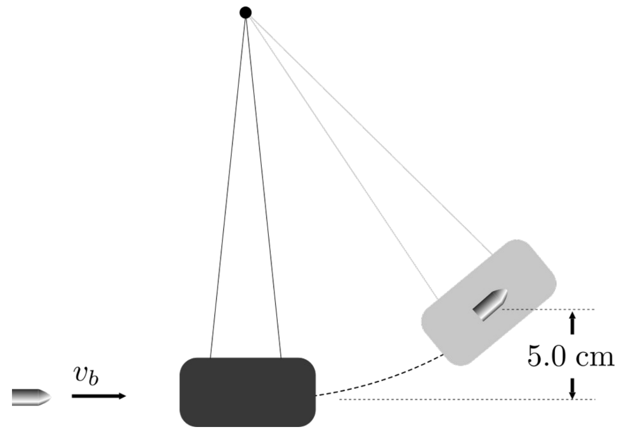
$$2T - W = \frac{W}{g}\cdot\frac{1}{2}a_M$$

$$T = \frac{1}{2}W\left(1 + \frac{a_M}{2g}\right)$$

$$P_{in} = \frac{1}{\epsilon} T v_M = \frac{v_M W}{2\epsilon}\left(1 + \frac{a_M}{2g}\right) = 673.52\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}$$

$$= 1.22\ \text{hp}$$

2. The device shown can be used to determine the speed of a bullet, v_b, as follows. The bullet (with mass 20 g) is fired into a clay block (mass 2.0 kg) suspended from strings, as shown. The bullet becomes embedded in the block and the combined mass swings up to a point where its center of mass is 5.0 cm above where it was before impact. Determine the velocity of the bullet before impact. *Hint*: you may use momentum conservation at impact, and energy conservation during the swing.



answer:
$v_b =$ 100.0 m/s

During swing: $\Delta T + \Delta V = 0$

$\Delta T = 0 - \frac{1}{2}(M+m)v_1^2$

$\Delta V = (M+m)gh$

$\rightarrow \quad v_1 = \sqrt{2gh}$

Impact: $\Delta(\text{momentum}) = 0$

$(M+m)v_1 = m v_b$

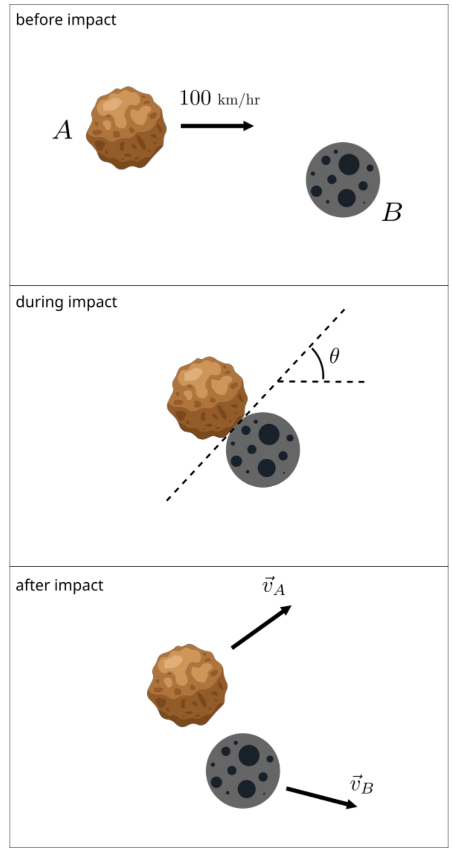
$\rightarrow \quad v_b = \frac{M+m}{m} v_1$

$\rightarrow \quad v_b = \frac{M+m}{m}\sqrt{2gh} = 100.0 \text{ m/s}$

3. Asteroid B is at rest in space when asteroid A approaches with a speed of 100 km/hr and collides with it. The plane of contact makes an angle of $\theta = 30°$ from the direction of the initial velocity of A.

The masses of the asteroids are $m_A = 1000$ kg and $m_B = 500$ kg. The collision has a coefficient of restitution $e = 0.5$.

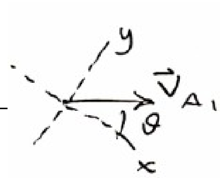
Find the velocity vectors of both asteroids after impact. Sketch the x and y axes you are using to report the vectors.



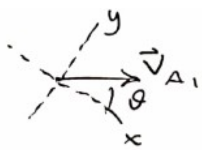
answers:

$\vec{v}_A =$ $(43.3\hat{\imath} + 50\hat{\jmath})$ km/h

$\vec{v}_B =$ $86.6\hat{\imath}$ km/h



$\vec{v}_{A1} = v_{A1}(\hat{\imath}\cos\theta + \hat{\jmath}\sin\theta)$



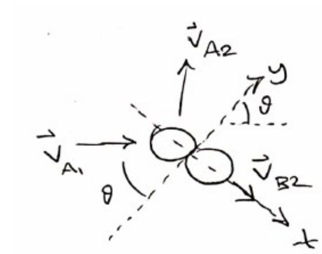
$\vec{v}_{B1} = 0$

$v_{B2y} = v_{B1y} = 0$

$v_{A2y} = v_{A1y} = v_{A1}\sin\theta$

$$e = -\frac{v_{B2x} - v_{A2x}}{0 - v_{A1x}} = \frac{v_{B2x} - v_{A2x}}{v_{A1}\cos\theta}$$

$\rightarrow v_{B2x} = v_{A2x} + e v_{A1}\cos\theta$



x-momentum: $m_A v_{A2x} = -m_B v_{B2x} + m_A v_{A1}\cos\theta$

$$(m_A + m_B) v_{A2x} = v_{A1}\cos\theta\,(m_A - e m_B)$$

$$\rightarrow v_{A2x} = \frac{m_A - e m_B}{m_A + m_B} v_{A1}\cos\theta$$

$\vec{v}_{A2} = (43.3\hat{\imath} + 50\hat{\jmath})$ km/h $\qquad \vec{v}_{B2} = 86.6\hat{\imath}$ km/h

4. A rocket on the launchpad has a mass of M, including its fuel. Determine the rocket's velocity as a function of time after the engines ignite and the rocket lifts off vertically from rest. The engines exhaust the fuel at a constant speed of u relative to the rocket, burning it at a constant rate $dm/dt = R$. You may neglect the change in gravitational strength with altitude and the drag resistance of the air, but not the force of gravity. Consider only time before all the fuel has been used. Your solution must include a free-body diagram.

The velocity should be in terms of time t, gravitational acceleration g, and the variables described above.

answer: $v(t) =$ $u \ln\left(\frac{M}{M-Rt}\right) - gt$

$$uR - mg = ma \longrightarrow a = \frac{uR}{m} - g$$

$$m(t) = M - Rt$$

$$a = \frac{dv}{dt} = \frac{uR}{M-Rt} - g$$

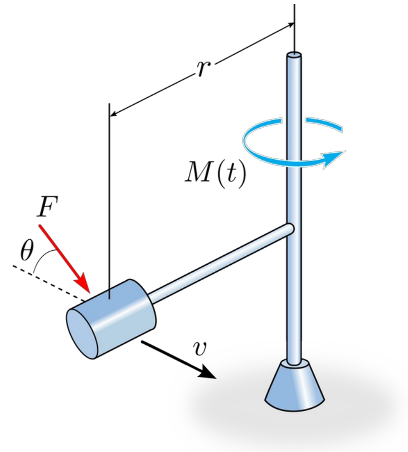
$$\int_0^v dv = \int_0^t \left(\frac{uR}{M-Rt} - g\right) dt$$

$$v(t) = uR\left(-\frac{1}{R}\right) \ln(M-Rt) \Big|_0^t - gt$$

$$= -u \ln \frac{M-Rt}{M} - gt$$

$$= u \ln \frac{M}{M-Rt} - gt$$

5. A 5 kg mass is attached to a frame of negligible mass that can rotate with the mass at distance $r = 0.5$ m from the center. At time $t = 0$, the mass has a speed of $v = 2.0$ m/s, and starts to be subjected to both a torque given by $M(t) = At^2 + Bt$ (where $A = 2$ N·m/s² and $B = 3$ N·m/s), and a constant force $F = 10$ N applied at angle $\theta = 30°$. As the mass rotates, the force maintains its direction relative to the mass.



Determine the speed of the mass at time $t = 3$ s.

answer: $v =$ 19.80 m/s

Angular impulse - momentum: $\int M_z dt = \Delta H_z$

$$M_z = At^2 + Bt + rF\cos\theta$$

$$\int_0^t M_z\, dt = \frac{1}{3}At^3 + \frac{1}{2}Bt^2 + trF\cos\theta$$

$$\Delta H_z = rmv - rmv_0$$

$$\rightarrow v(t) = v_0 + \frac{1}{rm}\left(\frac{1}{3}At^3 + \frac{1}{2}Bt^2 + trF\cos\theta\right)$$

at $t = 3s$, $v = 19.80$ m/s