20.4 Relative Motion with Rotating Axes

Sections 20.1-20.3 develop relative motion analysis for two points on a rigid body:

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \quad \text{where} \quad \vec{v}_{B/A} = \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \quad \text{where} \quad \vec{a}_{B/A} = \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})$$

If A and B are ***not*** on the same rigid body, use an xyz coordinate system that is both translating and rotating.

The equations and analysis are identical to those of Ch 16, but now

- $\vec{\Omega}$ and $\dot{\vec{\Omega}}$ can point any direction (not just $\pm\hat{k}$)
- if there are two rotational axes be careful to include all the motion

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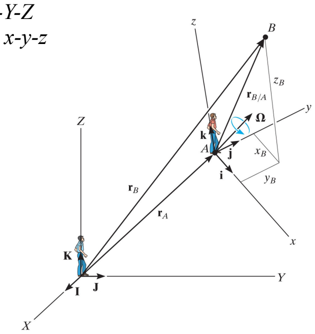
Establish (1) a fixed coordinate system X-Y-Z
and (2) a moving coordinate system x-y-z

Let A be the origin of the x-y-z system, which is rotating with angular velocity $\mathbf{\Omega}$, which is changing at rate $\dot{\mathbf{\Omega}}$.

We want to describe the absolute motion of point B.

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$
$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$



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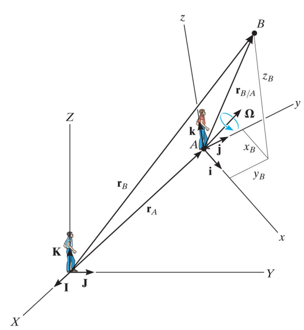
$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

motion of the origin

motion relative to origin

$$\vec{v}_B = \vec{v}_A + \underbrace{\vec{\Omega} \times \vec{r}_{B/A}}_{\text{motion due to } xyz \text{ rotation}} + \underbrace{(\vec{v}_{B/A})_{xyz}}_{\text{motion of } B \text{ as seen from the } xyz \text{ system}}$$

(see text for derivation)



20.4 Relative Motion with Rotating Axes

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

motion of the origin

motion relative to origin

$$\vec{a}_B = \vec{a}_A + \underbrace{\dot{\vec{\Omega}} \times \vec{r}_{B/A} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{B/A})}_{\text{motion due to } xyz \text{ rotation } (\alpha r \text{ term and } \omega^2 r \text{ term})} + \underbrace{2\vec{\Omega} \times (\vec{v}_{B/A})_{xyz}}_{\text{“Coriolis acceleration” due to moving while in rotating system (see video demo)}} + \underbrace{(\vec{a}_{B/A})_{xyz}}_{\text{motion of } B \text{ as seen from the } xyz \text{ system}}$$

(see text for derivation)

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Procedure for Analysis:

- establish coordinate systems X-Y-Z and x-y-z
  (Careful that the unit vectors in each system are not the same if the axes are not parallel)
- determine motion of the origin A
- determine $\mathbf{\Omega}$ and $\dot{\mathbf{\Omega}}$
- apply the relative motion equations

$$\vec{v}_B = \vec{v}_A + \vec{\Omega} \times \vec{r}_{B/A} + (\vec{v}_{B/A})_{xyz}$$
$$\vec{a}_B = \vec{a}_A + \dot{\vec{\Omega}} \times \vec{r}_{B/A} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{B/A}) + 2\vec{\Omega} \times (\vec{v}_{B/A})_{xyz} + (\vec{a}_{B/A})_{xyz}$$

- to find $(\vec{v}_{B/A})_{xyz}$ and $(\vec{a}_{B/A})_{xyz}$ you may need to repeat, establishing new x'-y'-z' coordinate axes.