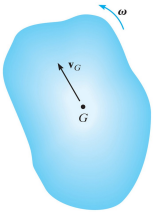
18 Planar Kinetics Work & Energy

The total kinetic energy of a body is the sum of its translational and rotational kinetic energy.

$$T = \tfrac{1}{2} m v_G^2 + \tfrac{1}{2} I_G \omega^2$$



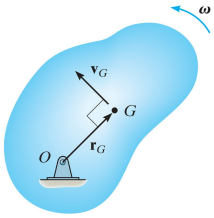
18 Planar Kinetics Work & Energy

For rotation about a fixed axis, simplify by using the parallel axis theorem:

$$T = \tfrac{1}{2} m v_G^2 + \tfrac{1}{2} I_G \omega^2$$
$$= \tfrac{1}{2} m (\omega r_G)^2 + \tfrac{1}{2} I_G \omega^2 = \tfrac{1}{2} \omega^2 \left(m r_G^2 + I_G \right)$$

$$T = \tfrac{1}{2} I_O \omega^2$$

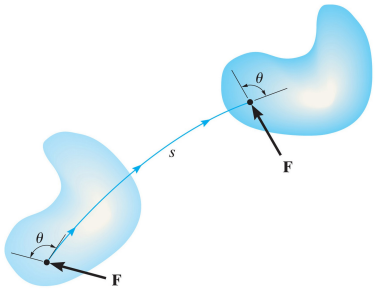
Note: Point O could also be the IC for general planar motion.



18.2 Work by a Force

The work done by a force over a given path is:

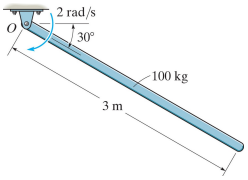
$$U = \int \mathbf{F} \cdot d\mathbf{r} = \int_s F \cos\theta \, ds$$

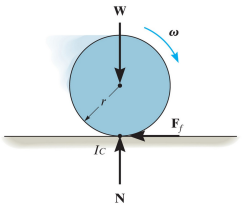


18.2 Work by a Force

Zero work is done by forces at fixed locations.

$$U = \int \mathbf{F} \cdot d\mathbf{r} = \int_s F \cos\theta \, ds$$



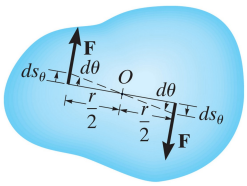


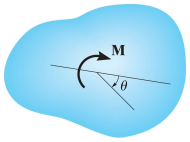
18.3 Work by a Moment

Consider the work done by the forces in a couple moment.

$$U = \int F_1 \, ds_1 + \int F_2 \, ds_2$$
$$= \int F \frac{r}{2} \, d\theta + \int F \frac{r}{2} \, d\theta = \int F r \, d\theta$$

$$U = \int M \, d\theta$$





18.4 Work-Energy Principle

The total work done on a system in bringing it from state 1 to state 2 equals its change in kinetic energy.

$$\sum U_{1-2} = \Delta T$$

$$U = \int \mathbf{F} \cdot d\mathbf{r} = \int_s F \cos\theta \, ds$$

$$U = \int M \, d\theta$$

$$T = \tfrac{1}{2} m v_G^2 + \tfrac{1}{2} I_G \omega^2$$