17 Planar Kinetics of a Rigid Body

The dynamics of rotating bodies is analogous to the dynamics of translating bodies.

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{v}) \qquad \mathbf{M}_O = \frac{d}{dt}\mathbf{H}_O$$

$$\mathbf{F} = m\mathbf{a} \qquad \mathbf{M} = I\vec{\alpha}$$

$$T_{trans} = \tfrac{1}{2}mv^2 \qquad T_{rot} = \tfrac{1}{2}I\omega^2$$

17.1 Mass Moment of Inertia

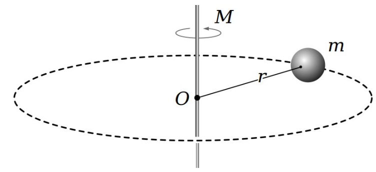
The dynamics of rotating bodies depends on their rotational inertia, I – a resistance to a change in their rotational motion.

I is also known as the **mass moment of inertia**.

$$\vec{M} = I\vec{\alpha}$$

For a point mass:

$$I = mr^2$$

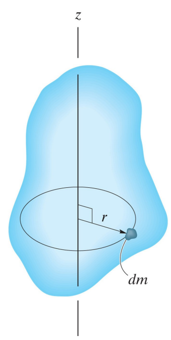


17.1 Mass Moment of Inertia

For a rigid body:

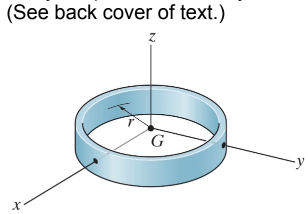
$$I = \int dI = \int_m r^2\,dm = \int_V r^2\rho\,dV$$

where r is the distance to the axis of rotation.



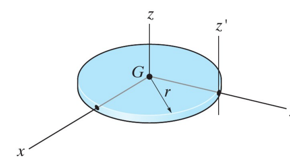
17.1 Mass Moment of Inertia

Many shapes have already been integrated to find their inertia. (See back cover of text.)



Thin ring

$$I_{xx} = I_{yy} = \frac{1}{2}mr^2 \qquad I_{zz} = mr^2$$

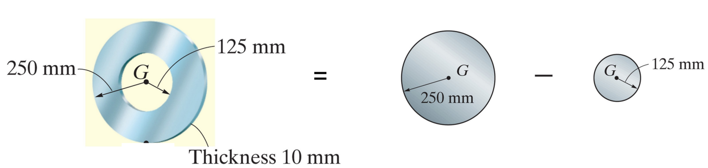


Thin Circular disk

$$I_{xx} = I_{yy} = \frac{1}{4}mr^2 \qquad I_{zz} = \frac{1}{2}mr^2 \qquad I_{z'z'} = \frac{3}{2}mr^2$$

17.1 Mass Moment of Inertia

If a body consists of a number of simple shapes (a **composite body**), the total inertia is the sum of the inertia of the parts.



$$I = \tfrac{1}{2}\left(\rho\pi(.25)^2 d\right)(.25)^2 - \tfrac{1}{2}\left(\rho\pi(.125)^2 d\right)(.125)^2$$

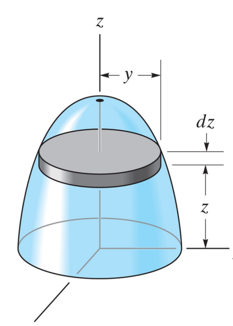
17.1 Mass Moment of Inertia

The inertia integral can often be written as a 1D integral using composite (infinitesimal) bodies.

$$I = \int dI = \int_y d(\tfrac{1}{2}my^2) = \tfrac{1}{2}\int_y y^2\,dm$$

this requires

- getting an expression for dm
- getting the integration limits

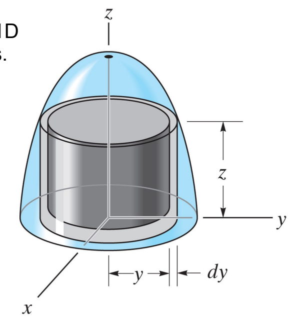


17.1 Mass Moment of Inertia

The inertia integral can often be written as a 1D integral using composite (infinitesimal) bodies.

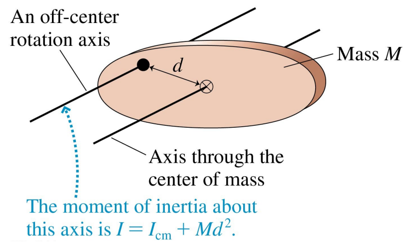
$$I = \int dI = \int_y d(my^2)$$

$$= \int_y y^2 \, dm$$



17.1 Mass Moment of Inertia

Parallel Axis Theorem:
The inertia about any axis is equal to the inertia about a parallel axis through the center of mass (CM) plus Md^2, where M is the body's mass and d is the distance between the parallel axes.



17.1 Mass Moment of Inertia

Radius of Gyration:
The inertia of a body is sometimes specified by giving its mass m and its radius of gyration, k.

$$I = mk^2$$

If all of the mass of a body were compressed to a point, how far from the rotating axis would it have to be to have the same inertia as the original body?

Answer: the radius of gyration, k