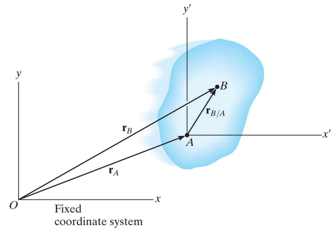
16.8 Relative Motion with Rotating Axes

We have so far considered the relative motion of two points **on a rigid body**.

- We used an x'-y' coordinate system that moves with point *A*, while *translating only.*

We now relax that requirement for a more general approach.

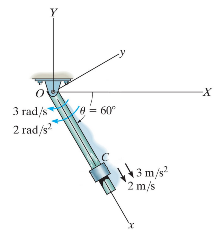
- Use an x'-y' coordinate system that is both *translating and rotating*.

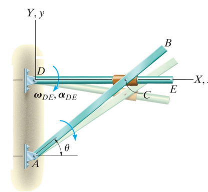


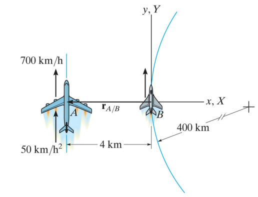
$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$
$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

16.8 Relative Motion with Rotating Axes

For relative motion between two points ***not*** on the same rigid body, establish (1) a fixed coordinate system X-Y-Z
and (2) a moving coordinate system x-y-z







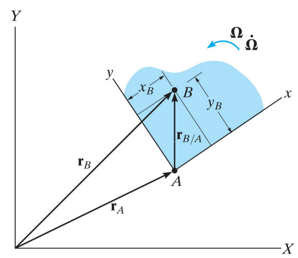
16.8 Relative Motion with Rotating Axes

Establish (1) a fixed coordinate system X-Y-Z
and (2) a moving coordinate system x-y-z

Let A be the origin of the x-y-z system, which is rotating with angular velocity $\boldsymbol{\Omega}$, and angular acceleration $\dot{\boldsymbol{\Omega}}$.

We want to describe the absolute motion of point B.

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$
$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$



16.8 Relative Motion with Rotating Axes

example: suppose *B* is an ant crawling around on a moving rigid body

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

motion of the origin

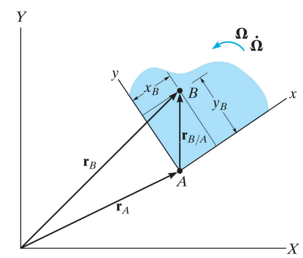
motion relative to origin

$$\vec{v}_B = \vec{v}_A + \vec{\Omega} \times \vec{r}_{B/A} + (\vec{v}_{B/A})_{xyz}$$

motion due to xyz rotation

motion of B as seen from the xyz system

(see text for derivation)



16.8 Relative Motion with Rotating Axes

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

motion of the origin

motion relative to origin

$$\vec{a}_B = \vec{a}_A + \dot{\vec{\Omega}} \times \vec{r}_{B/A} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{B/A}) + 2\vec{\Omega} \times (\vec{v}_{B/A})_{xyz} + (\vec{a}_{B/A})_{xyz}$$

motion due to xyz rotation (αr term and $\omega^2 r$ term)

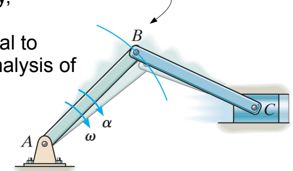
"Coriolis acceleration" due to moving while in rotating system (see video demo)

motion of B as seen from the xyz system

(see text for derivation)

16.8 Relative Motion with Rotating Axes

Note: if A and B **are** on the same rigid body, this analysis still works. In that case, we have $(\vec{v}_{B/A})_{xyz}$ and $(\vec{a}_{B/A})_{xyz}$ equal to zero, and this is exactly the same as the analysis of section 16.7 (with $\Omega = \omega$ and $\dot{\Omega} = \alpha$).



$$\Omega r \hat{u}_t$$

$$\vec{v}_B = \vec{v}_A + \vec{\Omega} \times \vec{r}_{B/A} + (\vec{v}_{B/A})_{xyz}$$

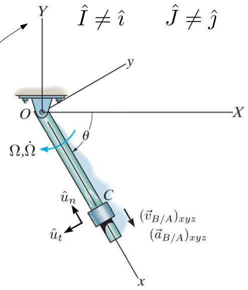
$$\vec{a}_B = \vec{a}_A + \dot{\vec{\Omega}} \times \vec{r}_{B/A} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_{B/A}) + 2\vec{\Omega} \times (\vec{v}_{B/A})_{xyz} + (\vec{a}_{B/A})_{xyz}$$

$$\dot{\Omega} r \hat{u}_t \qquad \Omega^2 r \hat{u}_n$$

16.8 Relative Motion with Rotating Axes

$\hat{I} \neq \hat{\imath}$ $\hat{J} \neq \hat{\jmath}$

- Good choice for coordinate systems X-Y-Z and x-y-z can save a lot of work.
- Unit vectors in each system are not the same if the axes are not parallel.
- If X-Y-Z and x-y-z are defined so they are momentarily aligned, they will have the same unit vectors.



$$\vec{v}_B = \vec{v}_A + \Omega r \hat{u}_t + (\vec{v}_{B/A})_{xyz}$$

$$\vec{a}_B = \vec{a}_A + \dot{\Omega} r \hat{u}_t + \Omega^2 r \hat{u}_n + 2\vec{\Omega} \times (\vec{v}_{B/A})_{xyz} + (\vec{a}_{B/A})_{xyz}$$

16.8 Relative Motion with Rotating Axes

Procedure for Analysis:

- establish coordinate systems X-Y-Z and x-y-z
  - Careful that the unit vectors in each system are not the same if the axes are not parallel
  - a good choice for coordinate system can save a lot of work
- determine motion of the origin A
- determine $\mathbf{\Omega}$ and $\dot{\mathbf{\Omega}}$
- apply the relative motion equations

$$\vec{v}_B = \vec{v}_A + \Omega r \hat{u}_t + (\vec{v}_{B/A})_{xyz}$$
$$\vec{a}_B = \vec{a}_A + \dot{\Omega} r \hat{u}_t + \Omega^2 r \hat{u}_n + 2\vec{\Omega} \times (\vec{v}_{B/A})_{xyz} + (\vec{a}_{B/A})_{xyz}$$

- use any known constraints on velocity and acceleration