16.7 Relative Motion – Acceleration

We return to *relative motion analysis*, now including acceleration.

For points A and B located on a single rigid body, B moves relative to A ($\vec{v}_{B/A}$ and $\vec{a}_{B/A}$) in pure rotation.

(assuming A, B and O are in the plane of motion)

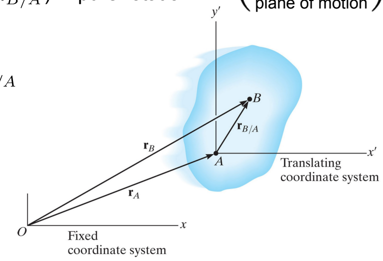
$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$
$$= \vec{a}_A + \left(\vec{a}_{B/A}\right)_t + \left(\vec{a}_{B/A}\right)_n$$

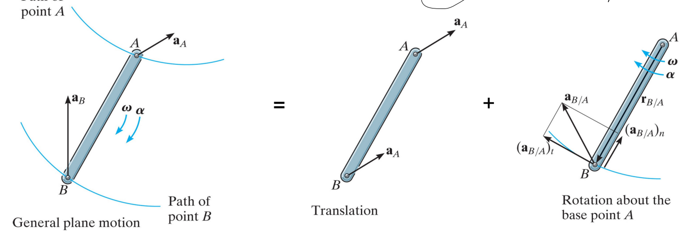
magnitude αr

magnitude $\omega^2 r$



16.7 Relative Motion – Acceleration

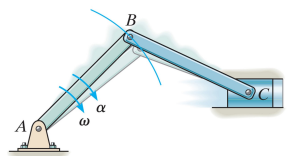
$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_t + \left(\vec{a}_{B/A}\right)_n$$
$$= \vec{a}_A + \alpha r \hat{u}_t + \omega^2 r \hat{u}_n \qquad \hat{u}_n = \left(-\hat{r}_{B/A}\right)$$



16.7 Relative Motion – Acceleration

For multiple linked points, work your way out using the relative motion equations:

- Start at fixed point A, then write expressions for $\mathbf{v}_B$ and $\mathbf{a}_B$
  $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad \vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$
- From point B, write expressions for $\mathbf{v}_C$ and $\mathbf{a}_C$
  $\vec{v}_C = \vec{v}_B + \vec{v}_{C/B} \qquad \vec{a}_C = \vec{a}_B + \vec{a}_{C/B}$
- The final point will have constrained motion.

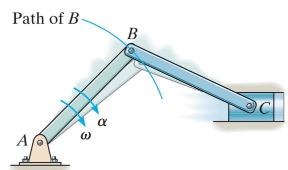


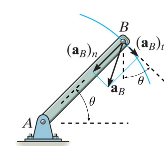
16.7 Relative Motion – Acceleration

- Start at fixed point A, then write expressions for $\mathbf{v}_B$ and $\mathbf{a}_B$

$$\vec{v}_B = \omega r\, \hat{u}_t \qquad = \omega \ell (\hat{\imath} \sin\theta - \hat{\jmath} \cos\theta)$$

$$\vec{a}_B = \alpha r\, \hat{u}_t + \omega^2 r\, \hat{u}_n \quad = \alpha \ell (\hat{\imath} \sin\theta - \hat{\jmath} \cos\theta) + \omega^2 \ell (-\hat{\imath} \cos\theta - \hat{\jmath} \sin\theta)$$



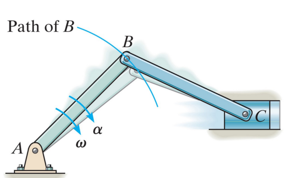


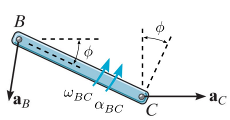
16.7 Relative Motion – Acceleration

- From point B, write expressions for $\mathbf{v}_C$ and $\mathbf{a}_C$

$$\vec{v}_C = \vec{v}_B + \omega_{BC} \ell_{BC} (\hat{\imath} \sin\phi + \hat{\jmath} \cos\phi)$$

$$\vec{a}_C = \vec{a}_B + \alpha_{BC} \ell_{BC} (\hat{\imath} \sin\phi + \hat{\jmath} \cos\phi) + \omega_{BC}^2 \ell_{BC} (-\hat{\imath} \cos\phi + \hat{\jmath} \sin\phi)$$





16.7 Relative Motion – Acceleration

- The final link will have constrained motion.

$$\vec{v}_C = v_C \hat{\imath}$$

$$\vec{a}_C = a_C \hat{\imath}$$

Now you can solve for unknowns.