16.5 Relative Motion Analysis

With multiple moving parts, consider using ***relative motion analysis***.

For points A and B located on a single rigid body:

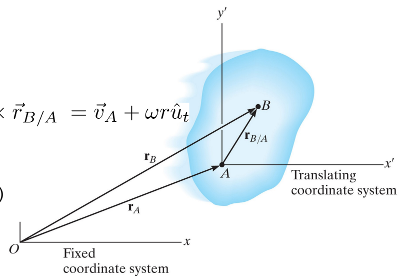
$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \;= \vec{v}_A + \vec{\omega} \times \vec{r}_{B/A} \;= \vec{v}_A + \omega r \hat{u}_t$

rotation of B about A (assuming A, B and O are in the plane of motion)

absolute velocity of A

absolute velocity of B



16.5 Relative Motion Analysis

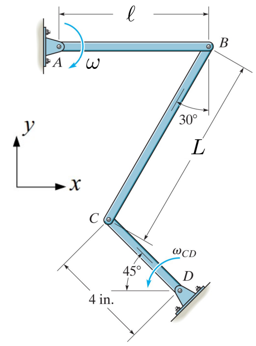
Example:
Find ω_{CD} in terms of ω.

Approach:

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

$\vec{v}_C = \vec{v}_B + \vec{v}_{C/B}$

$\vec{v}_D = \vec{v}_C + \vec{v}_{D/C}$



16.5 Relative Motion Analysis

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

$= \vec{v}_A + \omega r \hat{u}_t$

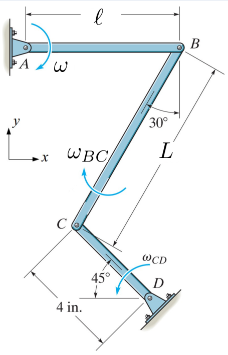
$= \vec{0} + \ell\omega\,(-\hat{\jmath}) = -\ell\omega\,\hat{\jmath}$

$\vec{v}_C = \vec{v}_B + \vec{v}_{C/B}$

$= -\ell\omega\,\hat{\jmath} + L\omega_{BC}\hat{u}_t$

$\hat{u}_t = -\hat{\imath}\cos 30° + \hat{\jmath}\sin 30°$

the positive direction of ω_{BC} is defined in the diagram



16.5 Relative Motion Analysis

The angular velocity is the same between any two points on a rigid body.

$\vec{v}_{B/A} = -\ell\omega\,\hat{\jmath}$

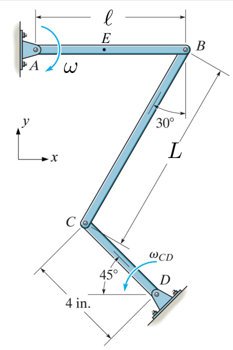
and

$\vec{v}_{A/B} = \ell\omega\,\hat{\jmath}$

and

$\vec{v}_{A/E} = \frac{1}{2}\ell\omega\,\hat{\jmath}$

all with the ***same*** value for ω



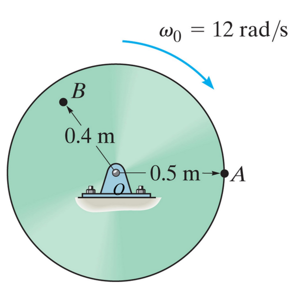
16.5 Relative Motion Analysis

A orbits O at a rate of 12 rad/s.

B orbits O at a rate of 12 rad/s.

A orbits B at a rate of 12 rad/s.

B orbits A at a rate of 12 rad/s.



16.5 Relative Motion Analysis

A velocity relative to a fixed point is the same as the velocity relative to any other fixed point.

$\vec{v}_{B/A} = \vec{v}_{B/C}$

$\vec{v}_{D/C} = \vec{v}_{D/A}$

$\vec{v}_{P/A} = \vec{v}_{P/C}$

$\vec{v}_{A/C} = \vec{0}$