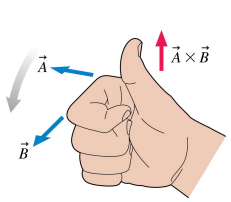
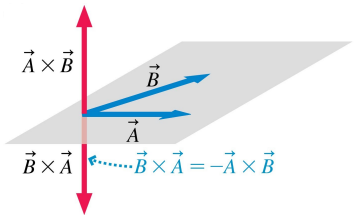
Aside: Cross Product Review

$$\vec{A} \times \vec{B} = \vec{C}$$



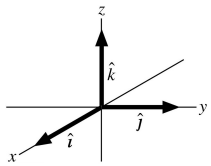


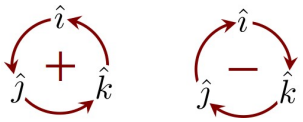
1

Aside: Cross Product Review

$$\vec{A} \times \vec{B} = AB \sin\theta \hat{u}$$

$$\begin{array}{lll} \hat{\imath} \times \hat{\imath} = \vec{0} & \hat{\imath} \times \hat{\jmath} = \hat{k} & \hat{\imath} \times \hat{k} = -\hat{\jmath} \\ \hat{\jmath} \times \hat{\imath} = -\hat{k} & \hat{\jmath} \times \hat{\jmath} = \vec{0} & \hat{\jmath} \times \hat{k} = \hat{\imath} \\ \hat{k} \times \hat{\imath} = \hat{\jmath} & \hat{k} \times \hat{\jmath} = -\hat{\imath} & \hat{k} \times \hat{k} = \vec{0} \end{array}$$





2

Aside: Cross Product Review

$$\vec{A} \times \vec{B} = AB \sin\theta \hat{u}$$

$$\begin{array}{lll} \hat{\imath} \times \hat{\imath} = \vec{0} & \hat{\imath} \times \hat{\jmath} = \hat{k} & \hat{\imath} \times \hat{k} = -\hat{\jmath} \\ \hat{\jmath} \times \hat{\imath} = -\hat{k} & \hat{\jmath} \times \hat{\jmath} = \vec{0} & \hat{\jmath} \times \hat{k} = \hat{\imath} \\ \hat{k} \times \hat{\imath} = \hat{\jmath} & \hat{k} \times \hat{\jmath} = -\hat{\imath} & \hat{k} \times \hat{k} = \vec{0} \end{array}$$

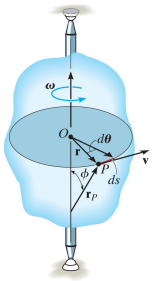
$$\begin{aligned} \vec{A} \times \vec{B} &= \left(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\right) \times \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right) \\ &= (A_x\hat{\imath} \times B_x\hat{\imath}) + (A_x\hat{\imath} \times B_y\hat{\jmath}) + (A_x\hat{\imath} \times B_z\hat{k}) + \\ &\quad (A_y\hat{\jmath} \times B_x\hat{\imath}) + (A_y\hat{\jmath} \times B_y\hat{\jmath}) + (A_y\hat{\jmath} \times B_z\hat{k}) + \\ &\quad (A_z\hat{k} \times B_x\hat{\imath}) + (A_z\hat{k} \times B_y\hat{\jmath}) + (A_z\hat{k} \times B_z\hat{k}) \end{aligned}$$

$$= (A_yB_z - A_zB_y)\,\hat{\imath} + (A_zB_x - A_xB_z)\,\hat{\jmath} + (A_xB_y - A_yB_x)\,\hat{k}$$

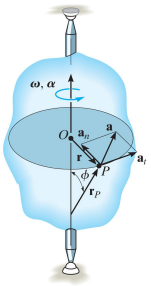
3

Aside: Cross Product Review

$$\vec{v} = \vec{\omega} \times \vec{r}_P$$



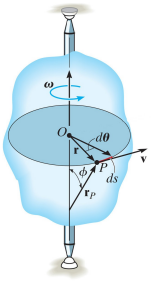
$$\vec{a} = \vec{\alpha} \times \vec{r}_P + \vec{\omega} \times (\vec{\omega} \times \vec{r}_P)$$



4

Aside: Cross Product Review

Note: $\vec{v} = \vec{\omega} \times \vec{r}_P$ is the same as $\vec{v} = \vec{\omega} \times \vec{r}$



Let $\vec{\omega} = \omega\hat{k}$ and $\vec{r}_P = h\hat{k} + \vec{r}$

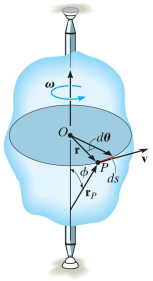
Then:

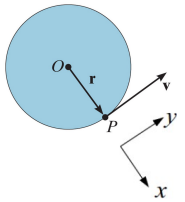
$$\begin{aligned} \vec{v} &= \vec{\omega} \times \vec{r}_P = \omega\left(\hat{k} \times h\hat{k}\right) + \vec{\omega} \times \vec{r} \\ &= \vec{\omega} \times \vec{r} \end{aligned}$$

5

Aside: Cross Product Review

$$\vec{v} = \vec{\omega} \times \vec{r}$$



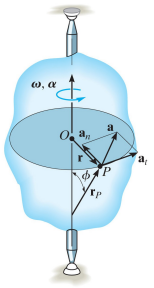


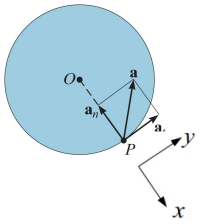
$$\vec{v} = \omega\hat{k} \times r\hat{\imath} = r\omega\hat{\jmath}$$

6

Aside: Cross Product Review

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$





$$\begin{aligned}\vec{a} &= \left[\alpha\hat{k} \times r\hat{\imath}\right] + \left[\omega\hat{k} \times (\omega\hat{k} \times r\hat{\imath})\right] \\ &= \alpha r\hat{\jmath} - r\omega^2\hat{\imath}\end{aligned}$$