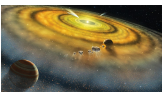
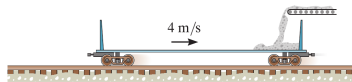
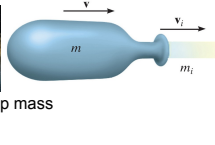


## 15.9 Variable Mass Systems

Use the impulse-momentum principle to study the motion of a system whose mass is changing.

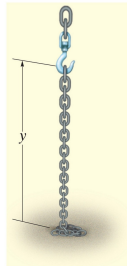


sweeping up mass



collecting mass

1



dynamically changing mass



expelling mass

## 15.9 Variable Mass Systems

What reaction forces ( $F_x, F_y$ ) are required to support the conveyor?

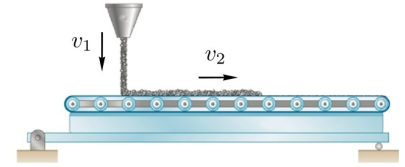
Newton's 2<sup>nd</sup> Law:

$$\vec{F}_{net} = \frac{d(m\vec{v})}{dt}$$

Horizontally:

$$F_x = \frac{d}{dt}(mv_2) = v_2 \frac{dm}{dt}$$

2

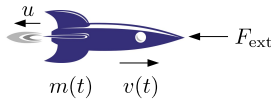


Vertically (with conveyor mass  $M$ ):

$$F_y = [M + m(t)]g + \frac{d}{dt}(mv_1) \\ = [M + m(t)]g + v_1 \frac{dm}{dt}$$

## 15.9 Variable Mass Systems

One can analyze (see text) momentum conservation in time  $dt$  for a rocket.



force due to momentum transfer

net external force (eg. air resistance)

mass is time-dependent

$$F_{ext} + \left| u \frac{dm}{dt} \right| = ma$$

3

## 15.9 Variable Mass Systems

Result:

- Treat the thrust as an external force of magnitude  $\left| u \frac{dm}{dt} \right|$
- Make sure the mass is time-dependent:  $m(t)a$
- Then proceed with normal dynamics.



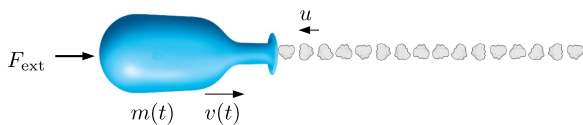
$$F_{ext} + \left| u \frac{dm}{dt} \right| = ma$$

4

## 15.9 Variable Mass Systems

Similar analysis for a *mass-gathering* device.

Mass enters at a rate of  $dm/dt$  with speed  $u$  (relative to device).



force due to momentum transfer

net external force

mass is time-dependent

$$F_{ext} - \left| u \frac{dm}{dt} \right| = ma$$

5

## 15.9 Variable Mass Systems

Result:

- Treat the momentum transfer as an external force of magnitude  $\left| u \frac{dm}{dt} \right|$
- Make sure the mass is time-dependent:  $m(t)a$
- Then proceed with normal dynamics.



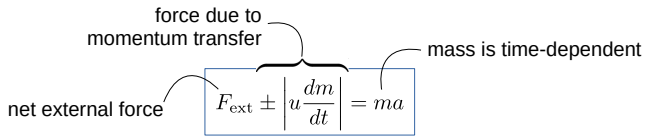
$$F_{ext} - \left| u \frac{dm}{dt} \right| = ma$$

6

## 15.9 Variable Mass Systems

**Summary:**

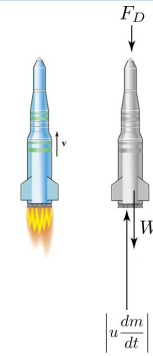
For systems that change mass, draw the FBD with an extra force of  $\left| u \frac{dm}{dt} \right|$  to account for the momentum transfer.



7

## 15.9 Variable Mass Systems

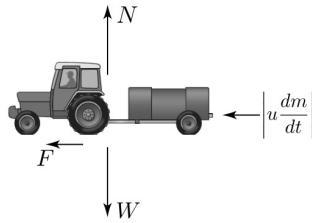
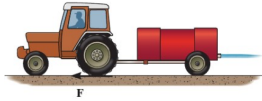
$$F_{\text{ext}} \pm \left| u \frac{dm}{dt} \right| = ma$$



8

## 15.9 Variable Mass Systems

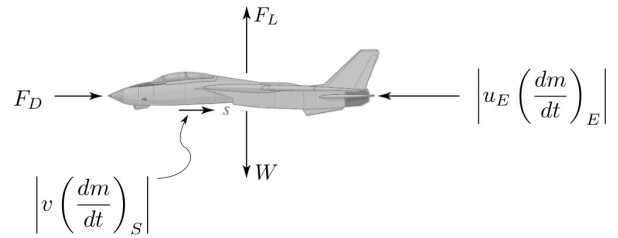
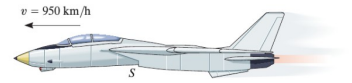
$$F_{\text{ext}} \pm \left| u \frac{dm}{dt} \right| = ma$$



9

## 15.9 Variable Mass Systems

$$F_{\text{ext}} \pm \left| u \frac{dm}{dt} \right| = ma$$



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