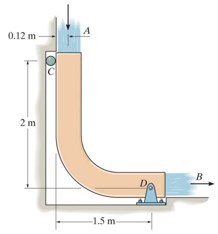
15.8 Steady Fluid Flow

The ***Principle of Impulse and Momentum*** is a convenient way to study reactions to fluid flow.

For example: As water flows through this fixture, how much force and moment do the supports have to provide?

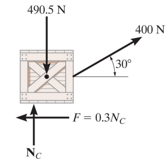


Answer:
The force required is given by
(momentum of the flow out) – (momentum of the flow in)

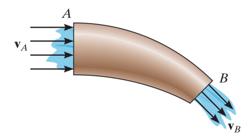
The moment required is given by
(angular momentum of the flow out) – (angular momentum of the flow in)

15.8 Steady Fluid Flow

- Typical dynamics analysis:
  - identify a “body”
  - draw a free body diagram
  - determine the resulting motion ($F=ma$)

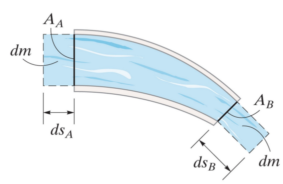


- We now consider ***steady flow*** – mass moves into and out of a ***control volume*** but total mass does not change anywhere.
  - the control volume serves as the “body”
  - instead of acceleration of the body, we consider momentum change of the fluid passing through it



15.8 Steady Fluid Flow

- Consider a mass m inside the pipe, plus a small mass dm that is just about to enter.
- One dt later, a mass dm leaves the other side but the rest is unchanged.

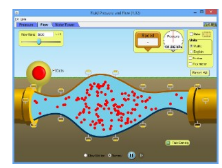


15.8 Steady Fluid Flow

Mass flow (kg/s) is the rate at which mass enters and leaves the volume:

$$\frac{dm}{dt} = \rho v A = \rho Q$$

$$Q = vA$$

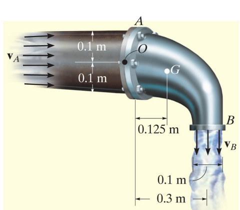


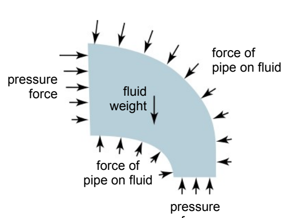
https://phet.colorado.edu/en/simulation/fluid-pressure-and-flow

- Q is volume flow rate (m^3/s)
- it is constant across all flow cross-sections

15.8 Steady Fluid Flow

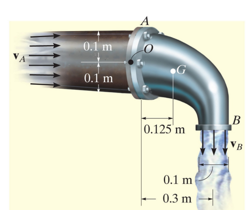
Consider the FDB of fluid in the control volume.

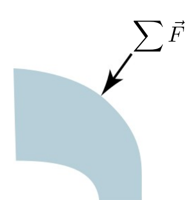




15.8 Steady Fluid Flow

Add all external forces on fluid.

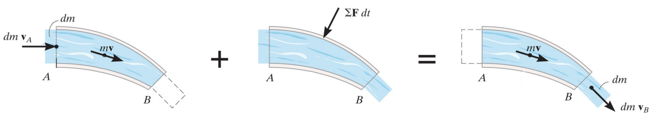




15.8 Steady Fluid Flow

The impulse delivered by these forces changes the fluid momentum.

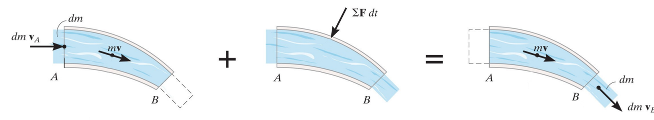
initial momentum + impulse = final momentum



15.8 Steady Fluid Flow

The impulse delivered by these forces changes the fluid momentum.

$$dm\,\vec{v}_A + m\vec{v} \quad + \quad \sum \vec{F}\,dt \quad = \quad m\vec{v} + dm\,\vec{v}_B$$



15.8 Steady Fluid Flow

The impulse delivered by these forces changes the fluid momentum.

$$dm\,\vec{v}_A + m\vec{v} \quad + \quad \sum \vec{F}\,dt \quad = \quad m\vec{v} + dm\,\vec{v}_B$$

cancels for steady flow

$$\sum \vec{F} = \frac{dm}{dt}(\vec{v}_B - \vec{v}_A) \qquad \frac{dm}{dt} = \rho Q$$

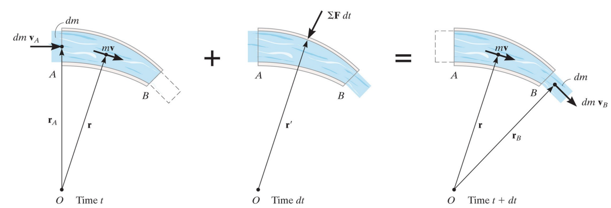
$$(\rho Q\,\vec{v})_{\text{out}} - (\rho Q\,\vec{v})_{\text{in}}$$

rate of change of momentum of the mass in the control volume

15.8 Steady Fluid Flow

The angular impulse creates an angular momentum change.

$$dm\,\vec{r}_A \times \vec{v}_A + m\vec{r} \times \vec{v} \quad + \quad \sum \vec{r}' \times \vec{F}\,dt \quad = \quad m\vec{r} \times \vec{v} + dm\,\vec{r}_B \times \vec{v}_B$$



15.8 Steady Fluid Flow

The angular impulse creates an angular momentum change.

$$dm\,\vec{r}_A \times \vec{v}_A + m\vec{r} \times \vec{v} \quad + \quad \sum \vec{r}' \times \vec{F}\,dt \quad = \quad m\vec{r} \times \vec{v} + dm\,\vec{r}_B \times \vec{v}_B$$

$$\sum \vec{M}_O = \frac{dm}{dt}(\vec{r}_B \times \vec{v}_B - \vec{r}_A \times \vec{v}_A) \qquad \frac{dm}{dt} = \rho Q$$

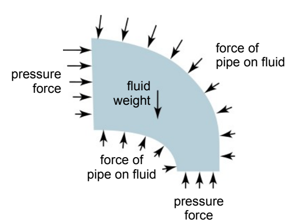
$$(\vec{r} \times \rho Q\,\vec{v})_{\text{out}} - (\vec{r} \times \rho Q\,\vec{v})_{\text{in}}$$

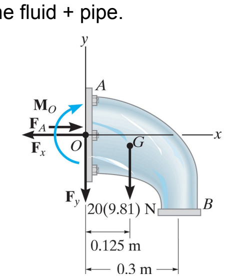
rate of angular momentum transfer to the mass in the control volume

15.8 Steady Fluid Flow

- Net force creates momentum change
- Net moment creates angular momentum change

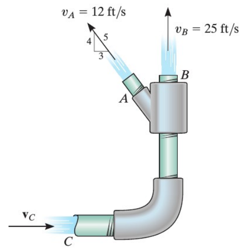
This is true for the fluid volume and also for the fluid + pipe.





15.8 Steady Fluid Flow

If the flow splits up, include all terms for the *out* and *in* flow.

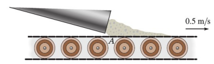


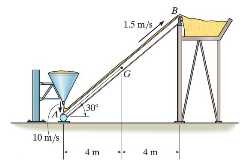
$$\sum \vec{F}_{\text{ext}} = (\rho Q_A \vec{v}_A + \rho Q_B \vec{v}_B) - \rho Q_C \vec{v}_C$$

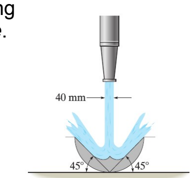
$$\sum \vec{M}_{P,\text{ext}} = \vec{r}_{PA} \times \rho Q_A\, \vec{v}_A + \vec{r}_{PB} \times \rho Q_B\, \vec{v}_B - \vec{r}_{PC} \times \rho Q_C\, \vec{v}_C$$

15.8 Steady Fluid Flow

Use this method whenever you can identify a volume having mass flow into and out of it, but with *zero net mass change*.

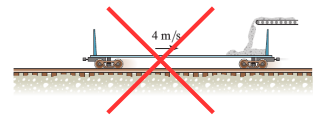






$$\sum \vec{F}_{\text{ext}} = (\rho Q\, \vec{v})_{\text{out}} - (\rho Q\, \vec{v})_{\text{in}}$$

$$\sum \vec{M}_{P,\text{ext}} = (\vec{r} \times \rho Q\, \vec{v})_{\text{out}} - (\vec{r} \times \rho Q\, \vec{v})_{\text{in}}$$



15.8 Steady Fluid Flow

Solution Procedure

- Determine if this procedure will work.
  - there should be no net mass change in the control volume
- Draw FBD of the control volume with all forces and moments on it.
  - support forces and moments
  - weight of solid and fluid
  - pressure forces at fluid entrance & exit
- Apply steady flow equations:

$$\sum \vec{F}_{\text{ext}} = (\rho Q\, \vec{v})_{\text{out}} - (\rho Q\, \vec{v})_{\text{in}}$$

$$\sum \vec{M}_{P,\text{ext}} = (\vec{r} \times \rho Q\, \vec{v})_{\text{out}} - (\vec{r} \times \rho Q\, \vec{v})_{\text{in}}$$

- ρ fluid density ($\mathrm{kg/m^3}$)
- Q volume flow rate ($\mathrm{m^3/s}$)
- $\vec{v}$ fluid velocity ($\mathrm{m/s}$)