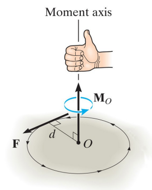
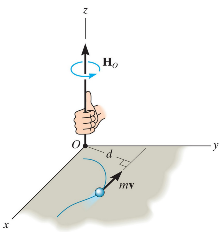
15.5-6 Angular Momentum

- The moment of a force about O: $\vec{M}_O = \vec{r} \times \vec{F}$



- The moment of a momentum about O:



$$\vec{H}_O = \vec{r} \times m\vec{v}$$

Called angular momentum.

Units: kg·m²/s

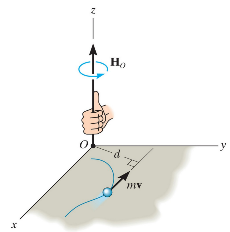
Value depends on choice of point O.

15.5-6 Angular Momentum

Angular momentum about O:

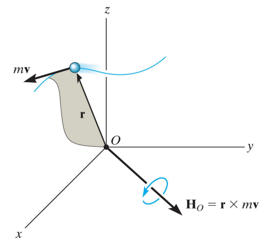
scalar formulation

$$(H_O)_z = dmv$$



vector formulation

$$\vec{H}_O = \vec{r} \times m\vec{v}$$



15.5-6 Angular Momentum

How does $\vec{M}_O$ relate to $\vec{H}_O$?
Take the moment of Newton's 2nd Law...

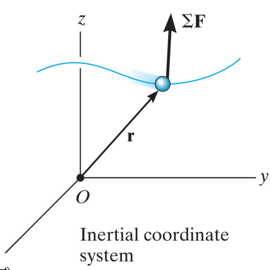
$$\vec{r} \times \left(\sum \vec{F} = \frac{d}{dt}(m\vec{v}) \right)$$

$$\sum \vec{r} \times \vec{F} = \frac{d}{dt}(\vec{r} \times m\vec{v})$$

$$\sum \vec{M}_O = \dot{\vec{H}}_O$$

A bit of a trick here.

$$\frac{d}{dt}(\vec{r} \times m\vec{v}) = \overbrace{\vec{v} \times m\vec{v}}^{=0} + \vec{r} \times \frac{d}{dt}(m\vec{v})$$



15.5-6 Angular Momentum

$$\sum \vec{M}_O = \dot{\vec{H}}_O$$

- this is the "rotational" version of Newton's 2nd Law $\sum \vec{F} = \frac{d}{dt}(m\vec{v})$
- For a system of particles, internal forces cancel. Then $\vec{H}_O$ is the total angular momentum of the system about O.