15.3 Momentum Conservation

- The Impulse-Momentum Principle: $\int_{t_1}^{t_2} \vec{F}_{\text{net}}\, dt = \Delta(m\vec{v})$
- If external forces add to zero, then linear momentum is conserved. $\Delta(m\vec{v}) = 0$

$$m\vec{v}_1 = m\vec{v}_2$$

initial ↑ final

- For a system of particles: $\sum (m_i\vec{v}_i)_1 = \sum (m_i\vec{v}_i)_2$

equivalently: $\Delta\vec{v}_G = 0$ ← CM velocity

- see simulation: https://phet.colorado.edu/en/simulation/legacy/collision-lab

15.3 Momentum Conservation

If the time interval is short, we may neglect some external forces. Call these "***nonimpulsive forces***".

Gravity is nonimpulsive during a firework explosion.

15.3 Momentum Conservation

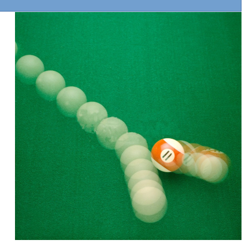
Example:

15–43.

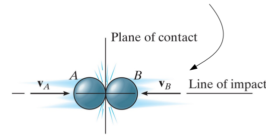
The 20-g bullet is traveling at 400 m/s when it becomes embedded in the 2-kg stationary block. Determine the distance the block will slide before it stops. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$.

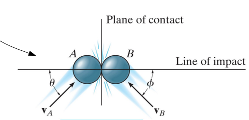
Friction is nonimpulsive during the collision.

15.4 Impact



- Momentum is conserved.
- Coefficient of Restitution, e, describes elasticity of the impact ($0 \le e \le 1$)
  - Kinetic energy is conserved for elastic collisions (e=1)
  - Energy is lost when $e < 1$
- We will consider central and oblique impacts.

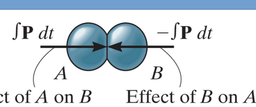




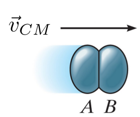
15.4 Central Impact

Newton's 3rd Law requires that

$$\vec{F}_{A\,\text{on}\,B} = -\vec{F}_{B\,\text{on}\,A}$$

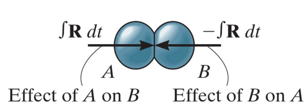


At instant of maximum deformation, both move at the CM velocity.



Momentum is conserved:

$$m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$$



15.4 Coefficient of Restitution, e

- e is the collision "elasticity"; the ratio of restitution impulse (outgoing) to deformation impulse (incoming) $\quad e = \dfrac{\int R\, dt}{\int P\, dt}$
- e is also the ratio of the particles' relative separation velocity to the particles' relative approach velocity $\quad e = -\dfrac{v_{B2} - v_{A2}}{v_{B1} - v_{A1}}$
- Plastic impact ($e = 0$): Relative separation velocity is zero; the particles stick together and move with a common velocity after the impact.
- Some typical values of e :

Steel on steel: 0.5 – 0.8 Wood on wood: 0.4 – 0.6

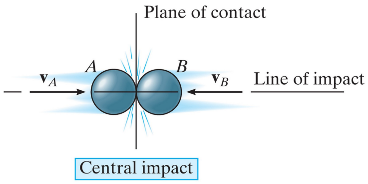
Lead on lead: 0.12 – 0.18 Glass on glass: 0.93 – 0.95

15.4 Central Impact

Solve with two equations and two unknowns.

$$m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}$$

$$e = -\frac{v_{B2} - v_{A2}}{v_{B1} - v_{A1}}$$



15.4 Oblique Impact

- Define x axis along line of impact, and y axis in the plane of contact. Assume smooth bodies.
- There is no impulse in the y direction.

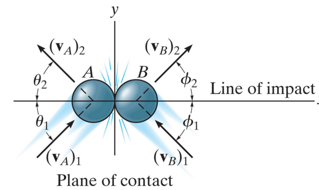
$$v_{A1y} = v_{A2y}$$

$$v_{B1y} = v_{B2y}$$

- In the x direction it's a central impact.

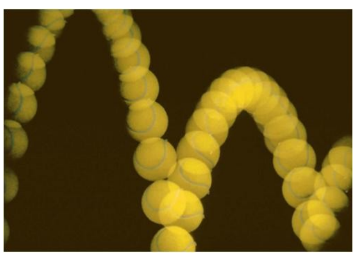
$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

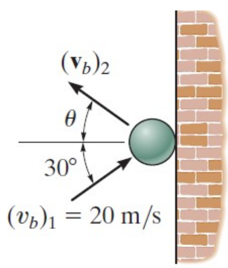
$$e = -\frac{v_{B2x} - v_{A2x}}{v_{B1x} - v_{A1x}}$$



15.4 Surface Bounce

If one of the masses is very large: $m_A \gg m_B$





15.4 Surface Bounce

If one of the masses is very large: $m_A \gg m_B$

- In the reference frame of the massive body

$$v_{A1} = v_{A2} = 0$$

- The line of impact (x axis) is normal to the surface.

$$\boxed{v_{B1y} = v_{B2y}}$$

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x} \quad \Rightarrow \quad v_{A1x} = v_{A2x}$$

$$e = -\frac{v_{B2x} - v_{A2x}}{v_{B1x} - v_{A1x}} \quad \Rightarrow \quad \boxed{e = -\frac{v_{B2x}}{v_{B1x}}}$$