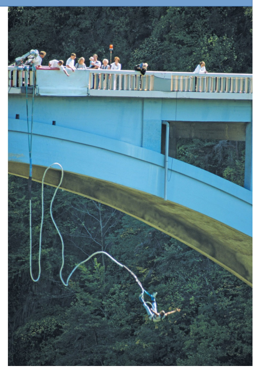
Section 14.5: Potential Energy

For a special class of forces (conservative forces), an equivalent way to analyze the work-energy relation is by defining a ***potential energy***.

- symbol $V(\mathbf{r})$
- units :

  J (Joule)

  or $\text{ft}\cdot\text{lb}$



Section 14.5: Potential Energy

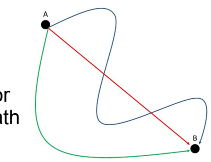
A conservative force...

- does work independent of the path of the motion (depends only on initial and final locations)

  U_{A-B} same for each path

- stores potential energy V (equal to the negative of the work done by the force)

  $$\Delta V = -U$$



The potential energy is a scalar quantity that depends only on position in space: $V(\mathbf{r})$

Section 14.5: Potential Energy

Conservative forces:

- gravity $F = mg$ $V_g = Wy$
- elastic force $F = -ks$ $V_e = \frac{1}{2}ks^2$

Nonconservative forces:

- friction
- applied forces (person, motor, etc)

(no potential energy exists)

Section 14.5: Potential Energy

Relationship between force (*F*) and potential energy (*V*)

- Work done (by a conservative force) in displacing a body is $U = -\Delta V$
- For a small incremental bit of work done: $dU = \mathbf{F}\cdot d\mathbf{r} = -dV$
- This *suggests* something like $\mathbf{F} = -\frac{dV}{d\mathbf{r}}$

  (but that makes no sense)

Section 14.5: Potential Energy

- Something like "$\mathbf{F} = -\frac{dV}{d\mathbf{r}}$" (a spatial derivative of a scalar field that returns a vector)
- Consider the Cartesian (*xyz*) components of $\mathbf{F}\cdot d\mathbf{r} = -dV$

$$F_x dx + F_y dy + F_z dz = -dV$$

$$F_x dx + F_y dy + F_z dz = -\frac{\partial V}{\partial x}dx - \frac{\partial V}{\partial y}dy - \frac{\partial V}{\partial z}dz$$

$$\mathbf{F} = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k} = -\nabla V$$

$$\boxed{\mathbf{F} = -\nabla V}$$

Section 14.6: Conservation of Energy

- When work is done on a system in going from position 1 to 2:

  $$U_{1-2} = \Delta T$$

- Separate the work done by conservative and nonconservative forces:

  $$\underbrace{U_{\text{cons}}}_{-\Delta V} + U_{\text{noncons}} = \Delta T \qquad \underbrace{U_{\text{noncons}}}_{\text{how to change the total energy}} = \Delta T + \Delta V = \Delta\underbrace{(T+V)}_{\text{total energy}}$$

- In the absence of nonconservative forces (for example, motion in a frictionless environment):

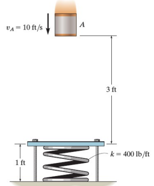
  $\Delta(T+V) = 0$ or $T_1 + V_1 = T_2 + V_2$

  Total energy is conserved.

Section 14.6: Conservation of Energy

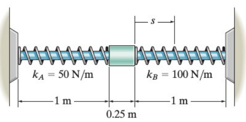
14–25.

The 5-lb cylinder is falling from A with a speed $v_A = 10$ ft/s onto the platform. Determine the maximum displacement of the platform, caused by the collision. The spring has an unstretched length of 1.75 ft and is originally kept in compression by the 1-ft long cables attached to the platform. Neglect the mass of the platform and spring and any energy lost during the collision.



14–29.

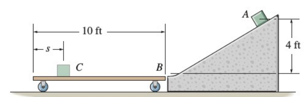
The collar has a mass of 20 kg and slides along the smooth rod. Two springs are attached to it and the ends of the rod as shown. If each spring has an uncompressed length of 1 m and the collar has a speed of 2 m/s when $s = 0$, determine the maximum compression of each spring due to the back-and-forth (oscillating) motion of the collar.



Section 14.6: Conservation of Energy

14–30.

The 30-lb box A is released from rest and slides down along the smooth ramp and onto the surface of a cart. If the cart is *prevented from moving* determine the distance s from the end of the cart to where the box stops. The coefficient of kinetic friction between the cart and the box is $\mu_k = 0.6$.



14–33.

The 10-lb block is pressed against the spring so as to compress it 2 ft when it is at A. If the plane is smooth, determine the distance d, measured from the wall, to where the block strikes the ground. Neglect the size of the block.