14.1-2 Work and Energy

- We can change the ***energy*** of a body by doing ***work*** on it.
- Positive work done will increase energy, negative work done will decrease energy.
- Force in the direction of motion does positive work.
- Force opposite the direction of motion does negative work.

These crash barrels safely remove a car's energy of motion.

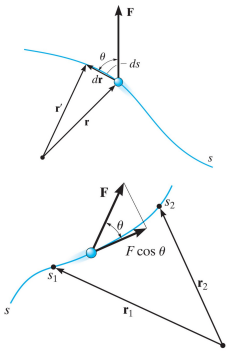
14.1 Work

- work is force (component in the direction of motion) times distance:

$$dU = \vec{F} \cdot d\vec{r}$$

- net work done in moving from 1 to 2 is

$$U_{1-2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = \int_{s_1}^{s_2} F \cos\theta \, ds$$



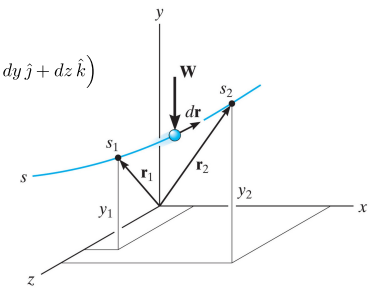
14.1 Work

Work done by gravity:

$$U_{1-2} = \int \vec{F} \cdot d\vec{r} = \int_{\vec{r}_1}^{\vec{r}_2} (-W\hat{j}) \cdot \left(dx\,\hat{\imath} + dy\,\hat{\jmath} + dz\,\hat{k}\right)$$

$$= \int_{y_1}^{y_2} -W\,dy$$

$$\boxed{U_{1-2} = -W\Delta y}$$

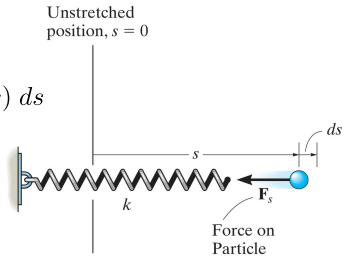


14.1 Work

Work done by spring
(or any linear restoring force)

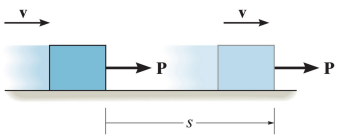
$$U_{1-2} = \int_{s_1}^{s_2} F_s\,ds = \int_{s_1}^{s_2} (-ks)\,ds$$

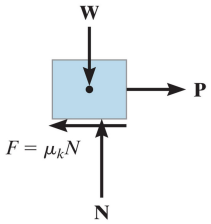
$$\boxed{U_{1-2} = -\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right)}$$



14.1 Work

In the motion shown:
- work done by friction is negative
- work done by *P* is positive
- work done by *W* and *N* are both zero





14.2 Work and Energy

Only the tangential component of force on a body does work.

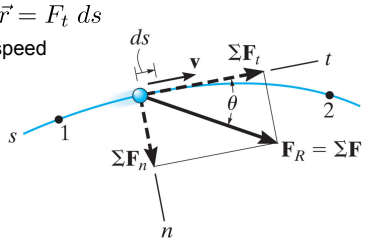
$$dU = \vec{F} \cdot d\vec{r} = F_t\,ds$$

Tangential acceleration is what will speed up or slow down the body.

$$a_t\,ds = v\,dv$$

$$\int ma_t\,ds = \int mv\,dv$$

$$\sum F_t = ma_t$$



14.2 Work and Energy

For total work done, consider the tangential component of the total force on a body.

The tangential acceleration is what will speed up or slow down the body.

$$a_t\,ds = v\,dv$$

$$\int ma_t\,ds = \int mv\,dv$$

$$\sum F_t = ma_t$$

$$\sum \int_{s_1}^{s_2} F_t\,ds = \int_{v_1}^{v_2} mv\,dv$$

$$= \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

$$\boxed{\sum U_{1-2} = \Delta T}$$

where T is the ***kinetic energy***:

$$T = \tfrac{1}{2}mv^2$$

14.2 Work and Energy

- The work–energy theorem: $\sum U_{1-2} = \Delta T$ $\quad T = \tfrac{1}{2}mv^2$
- Work has units of force times distance
  - $1\ \text{N·m} = 1\ \text{J}$ (Joule)
  - imperial (FPS) units: $1\ \text{ft·lb}$
- Work and energy have the same units.
- U can be positive or negative, but T is always positive
- U and T are scalars
- The work-energy theorem will give the same result as Newton's 2nd Law
  - but often with less effort since it's a scalar equation
  - use it when you need v and can calculate total U.