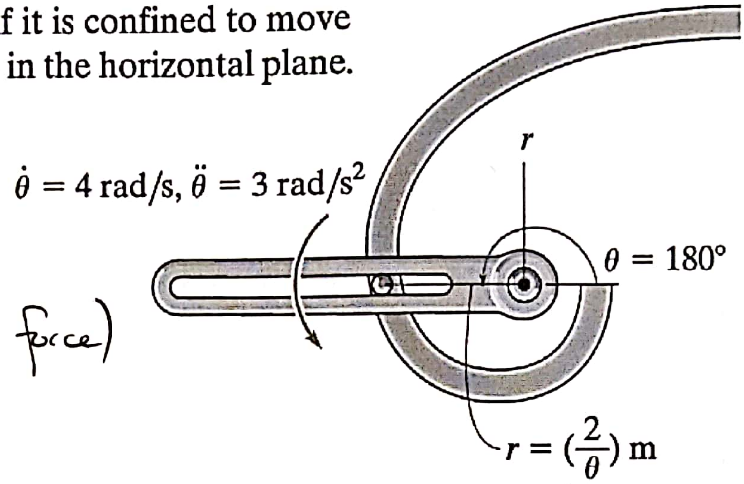
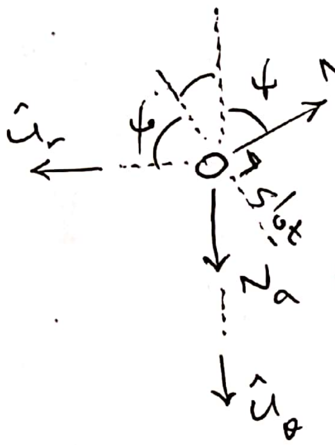
13.6 Cylindrical Coordinates

The arm is rotating at a rate of $\dot{\theta} = 4\ \mathrm{rad/s}$ when $\ddot{\theta} = 3\ \mathrm{rad/s^2}$ and $\theta = 180°$. Determine the force it must exert on the 0.5-kg smooth cylinder if it is confined to move along the slotted path. Motion occurs in the horizontal plane.



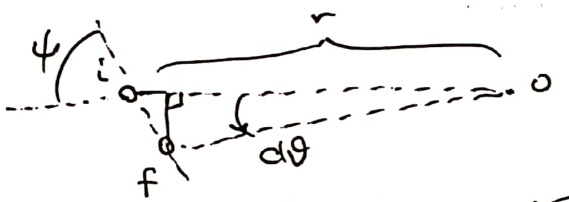
answer: ________________________

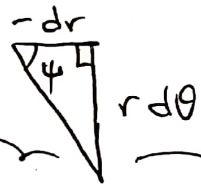


$$\sum F_r = m a_r = -N_s \sin\psi$$

$$\sum F_\theta = m a_\theta = N_a - N_s \cos\psi$$

find ψ:



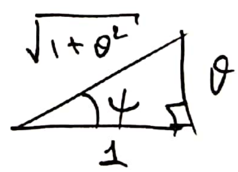


$$\tan\psi = \frac{r\,d\theta}{-dr} = -\frac{r}{dr/d\theta}$$

$$r(\theta) = \frac{A}{\theta} \quad A = 2\mathrm{m}$$

$$\frac{dr}{d\theta} = \frac{-A}{\theta^2}$$

$$\tan\psi = -\frac{r\theta^2}{-A} = \frac{A}{\theta} \cdot \frac{\theta^2}{A} = \theta$$



$$\sin\psi = \frac{\theta}{\sqrt{1+\theta^2}} \qquad \cos\psi = \frac{1}{\sqrt{1+\theta^2}}$$

$$ma_r = -N_s \cdot \frac{\theta}{\sqrt{1+\theta^2}} \quad (1)$$

$$ma_\theta = N_a - N_s \frac{1}{\sqrt{1+\theta^2}} \quad (2)$$

$a_r = \ddot{r} - r\dot{\theta}^2$ $\qquad$ $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$r(\theta) = \frac{A}{\theta}$ $\qquad$ $\dot{r} = -\frac{A}{\theta^2}\dot{\theta}$ $\qquad$ $\ddot{r} = 2\frac{A}{\theta^3}\dot{\theta}^2 - \frac{A}{\theta^2}\ddot{\theta}$

at $\theta = \pi$ $\quad$ $\dot{\theta} = 4 \frac{rad}{s}$ and $\ddot{\theta} = 3 \frac{rad}{s^2}$

Solve for N_s in (1)

Then plug into (2) and solve for N_a.

...

$$\boxed{N_a = -0.898\,N \quad (\text{acting upward})}$$