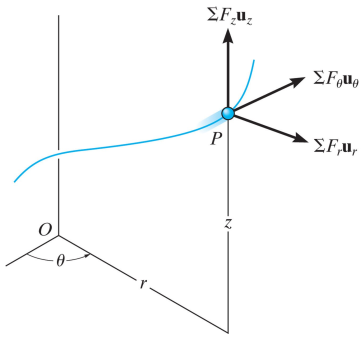
13.6 Cylindrical Coordinates

- r–θ–z coordinates
- convenient when moving along a path given by $r(\theta)$

$$\sum \vec{F} = m\vec{a}$$

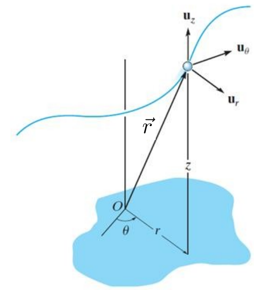
$$\sum F_r = ma_r$$
$$\sum F_\theta = ma_\theta$$
$$\sum F_z = ma_z$$



13.6 Cylindrical Coordinates

Recall from Section 12.8

$$\vec{r} = r\hat{u}_r + z\hat{u}_z$$
$$\vec{v} = \frac{d\vec{r}}{dt}$$
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$



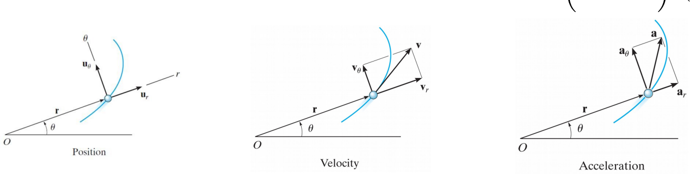
13.6 Cylindrical Coordinates

Recall from Section 12.8
for planar (2D) motion:

$$\vec{r} = r\hat{u}_r$$

$$\vec{v} = \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta$$

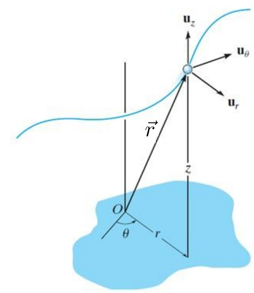
$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{u}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{u}_\theta$$



13.6 Cylindrical Coordinates

Recall from Section 12.8
for arbitrary 3D motion:

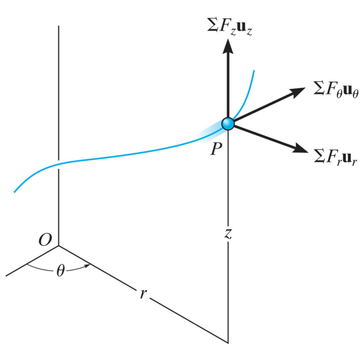
$$\vec{r} = r\hat{u}_r + z\hat{u}_z$$
$$\vec{v} = \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta + \dot{z}\hat{u}_z$$
$$\vec{a} = \underbrace{\left(\ddot{r} - r\dot{\theta}^2\right)}_{a_r}\hat{u}_r + \underbrace{\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)}_{a_\theta}\hat{u}_\theta + \underbrace{\ddot{z}}_{a_z}\hat{u}_z$$



13.6 Cylindrical Coordinates

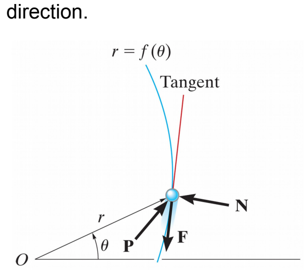
$$\sum \vec{F} = m\vec{a}$$

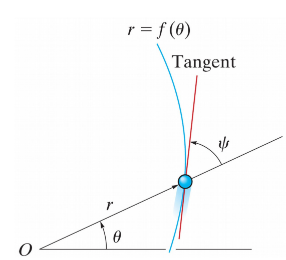
$$\sum F_r = ma_r = m\left(\ddot{r} - r\dot{\theta}^2\right)$$
$$\sum F_\theta = ma_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$
$$\sum F_z = ma_z = m\ddot{z}$$



13.6 Cylindrical Coordinates

Finding the cylindrical components of the ***normal*** and ***tangential*** forces often requires finding the angle (ψ) between the path and the radial direction.

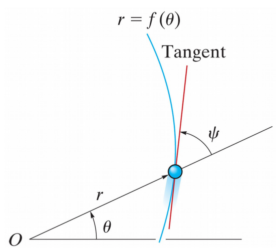


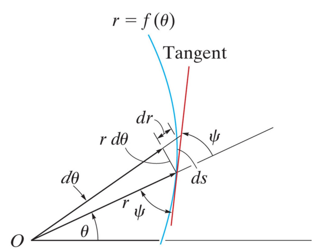


13.6 Cylindrical Coordinates

How to find ψ :

$$\tan\psi = \frac{r\,d\theta}{dr} = \frac{r}{\frac{dr}{d\theta}}$$





13.6 Cylindrical Coordinates

How to find ψ :

$$\tan\psi = \frac{r}{\frac{dr}{d\theta}}$$

- if r decreases as θ increases (for example, here), then $\frac{dr}{d\theta}$ will be negative
- if $\frac{dr}{d\theta}$ is negative then ψ will be negative.
- Negative angles are measured clockwise.

(alternatively, just keep track of the geometry – see example problem)