12.6 Projectile Motion

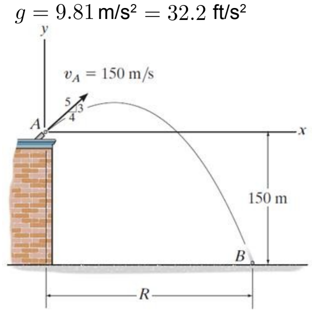
- 2D motion with constant acceleration $\vec{a} = -g\,\hat{j}$

  $g = 9.81$ m/s² $= 32.2$ ft/s²
- integrate the kinematic equations to obtain

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + t \begin{bmatrix} v_{0x} \\ v_{0y} \end{bmatrix} + \tfrac{1}{2}t^2 \begin{bmatrix} 0 \\ -g \end{bmatrix}$$

$$\begin{bmatrix} v_x(t) \\ v_y(t) \end{bmatrix} = \begin{bmatrix} v_{0x} \\ v_{0y} \end{bmatrix} + t \begin{bmatrix} 0 \\ -g \end{bmatrix}$$

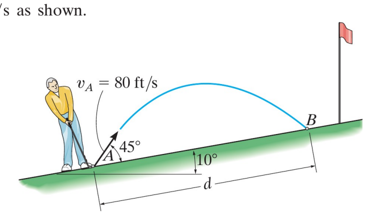
$$v_y^2 = v_{0y}^2 - 2g(y - y_0)$$



12.6 Projectile Motion

- establish a coordinate axis (note positive direction)
- identify initial conditions
- solve the kinematic equations

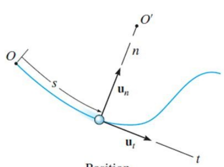
A golf ball is struck with a velocity of 80 ft/s as shown. Determine the distance *d* to where it will land.



12.7 *n-t* Components

- motion can be analyzed using n-t axes (*normal* and *tangential*)
- origin of this coordinate system is always *at the particle*

  $\mathbf{u}_t$ points in direction of motion
  $\mathbf{u}_n$ points toward center of curvature

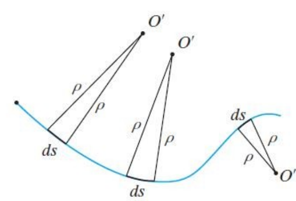


Position

12.7 *n-t* Components

- radius of curvature ρ
- can be found from the path $y(x)$:

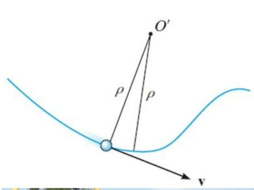
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

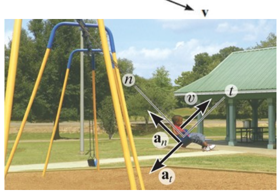


12.7 *n-t* Components

- velocity is always tangential

$$\vec{v} = v\hat{u}_t$$
$$v = \dot{s}$$





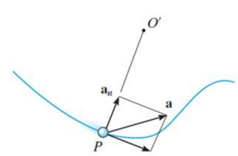
12.7 *n-t* Components

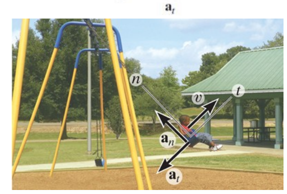
- acceleration has both components

$$\vec{a} = a_t\hat{u}_t + a_n\hat{u}_n$$

$a_t = \dot{v}$ speeding up (+) or slowing down (–)

$a_n = \dfrac{v^2}{\rho}$ turning (centripetal) acceleration (always positive)





12.7 *n-t* Components

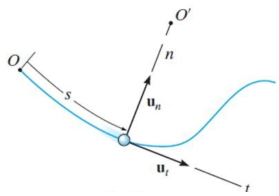
1D motion kinematics apply along the path (in tangential direction)

$v\ dt = ds \qquad a_t\ dt = dv$

$a_t\ ds = v\ dv$

If you're given $v(s)$, you might want to use

$$a_t = v\frac{dv}{ds}$$



12.7 *n-t* Components

- for a "space curve" (ie, non-planar), n-t axes are defined to lie in the plane of the curve ("osculating plane")
- same kinematics apply:

  $\vec{v} = v\hat{u}_t$

  $\vec{a} = a_t\hat{u}_t + a_n\hat{u}_n$
- the "binormal" direction is defined by

  $\hat{u}_t \times \hat{u}_n = \hat{u}_b$
- $\vec{v}$ and $\vec{a}$ never have any binormal component (it may be useful for a force sum: $\sum F_b = 0$)