12.4 General (Particle) Kinematics

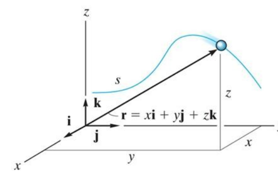
- “curvilinear” is still one-dimensional, but not necessarily straight
- we now need vector quantities:

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$
$$\vec{v} = \frac{d\vec{r}}{dt}$$
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

12.5 Rectangular Components

- note the book’s vector notation

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$
$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

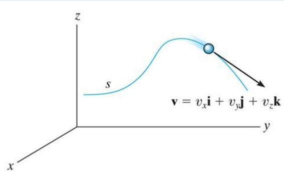


Position

12.5 Rectangular Components

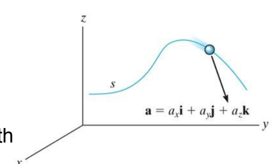
- unit vectors are constants

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$
$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$$
$$= \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$$



- velocity always tangent to path

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$
$$= \dot{v}_x\hat{\imath} + \dot{v}_y\hat{\jmath} + \dot{v}_z\hat{k}$$
$$= \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$$

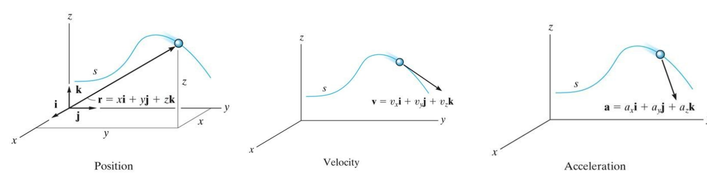


- acceleration usually not tangent to the path

12.5 Rectangular Components

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt}$$

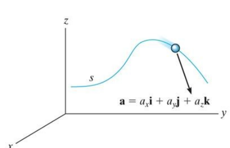
- $\vec{v}$ points in the direction of changing $\vec{r}$
- $\vec{a}$ points in the direction of changing $\vec{v}$



12.5 Rectangular Components

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt}$$

- $\vec{v}$ points in the direction of changing $\vec{r}$
- $\vec{a}$ points in the direction of changing $\vec{v}$
- acceleration tangent to the path represents speeding up (+) & slowing down (–)
- acceleration perpendicular to the path points in the direction it’s turning



Acceleration

12.5 Rectangular Components

The 1D kinematic equations may be used individually for each component:

$$v_x = \frac{dx}{dt} \qquad a_x = \frac{dv_x}{dt} \qquad v_x\,dt = dx \qquad a_x\,dt = dv_x$$
$$a_x\,dx = v_x\,dv_x$$

1D kinematics also apply to distance along the curved path, s
(as long as the acceleration refers only to the tangential component):

$$v = \frac{ds}{dt} \qquad a_t = \frac{dv}{dt} \qquad v\,dt = ds \qquad a_t\,dt = dv$$
$$a_t\,ds = v\,dv$$

12.5 Rectangular Components

Many solutions will require that you're comfortable with the chain rule (see Appendix C). For example:

$$y = f(x) \qquad \dot{y} = \frac{dy}{dt} = \frac{df}{dx}\frac{dx}{dt} = \frac{df}{dx}\dot{x} \qquad \ddot{y} = \frac{d}{dt}\left(\frac{df}{dx}\frac{dx}{dt}\right) = \frac{df}{dx}\ddot{x} + \dot{x}\frac{d}{dt}\left(\frac{df}{dx}\right)$$