Ch 7: Electric Potential

1. A positive charge $q$ is fired through a small hole in the positive plate of a capacitor. Does $q$ speed up or slow down inside the capacitor? Answer this question twice:

   a. First using the concept of force.
   
      It speeds up. A positive charge inside the capacitor experiences an attractive force toward the negative plate and a repulsive force from the positive plate. Both forces are to the right.

   b. Second using the concept of energy.
   
      It speeds up. After entering the capacitor the positive charge maintains constant total energy. So as the charge heads toward the negative plate, it loses potential energy but gains kinetic energy.

2. Charge $q_1 = 3 \text{ nC}$ is distance $r$ from a positive point charge $Q$. Charge $q_2 = 1 \text{ nC}$ is distance $2r$ from $Q$. What is the ratio $U_1/U_2$ of their potential energies due to their interactions with $Q$?

   $$\frac{U_1}{U_2} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 Q}{r} = \frac{q_1 (2r)}{q_2 (r)} = \frac{3 \text{ nC}}{1 \text{ nC}} \frac{(2r)}{(r)} = 6$$
3. The figure shows the potential energy of a positively charged particle in a region of space.

a. What possible arrangement of source charges is responsible for this potential energy? Draw the source charges above the axis below.

![Potential Energy Graph]

b. With what kinetic energy should the charged particle be launched from \( x = 0 \) mm to have a turning point at \( x = 3 \) mm? Explain.

\[ K = 45 \text{ J initially (at } x = 0 \text{ mm)}. \] The total energy of the charge is constant. The graph shows an increase of 45 J of potential energy \( U \) as the charge moves from \( x = 0 \) mm to \( x = 3 \) mm. This gain in \( U \) must come from a loss in \( K \). \( K \) is zero at \( x = 3 \) mm (turning point).

\[ 15 \text{ J}. \] The graph shows \( U \) increases by 30 J so the loss in \( K \) must be 30 J.

4. An electron moves along the trajectory from \( i \) to \( f \).

a. Does the electric potential energy increase, decrease, or stay the same? Explain.

\[ U = \frac{1}{4 \pi} \frac{q_1 q_2}{r} \] where \( r \) = separation distance

The potential energy is proportional to \( \frac{1}{r} \), but since the charges have opposite sign the potential energy increases (becomes less negative) as \( r \) increases from \( i \) to \( f \).

b. Is the electron's speed at \( f \) greater than, less than, or equal to its speed at \( i \)? Explain.

Less than. As the potential energy increases the kinetic energy and speed decrease since the total energy must remain constant.
5. Inside a parallel-plate capacitor, two protons are launched with the same speed from point 1. One proton moves along the path from 1 to 2, the other from 1 to 3. Points 2 and 3 are the same distance from the negative plate.

a. Is \( \Delta U_{1 \to 2} \), the change in potential energy along the path 1 \( \to \) 2, larger than, smaller than, or equal to \( \Delta U_{1 \to 3} \)? Explain.

\[
\Delta U_{1 \to 2} = \Delta U_{1 \to 3}
\]

\( \Delta U = qE \Delta s \) where \( \Delta s \) is the change in vertical position from point 1 to point 2 or point 3, \( \Delta s_{1 \to 2} = \Delta s_{2 \to 3} \).

Only vertical displacement matters here since there is no horizontal component of \( \vec{E} \) or the electric force.

b. Is the proton's speed \( v_2 \) at point 2 larger than, smaller than, or equal to \( v_3 \)? Explain.

\[ v_2 = v_3 \]. Both paths have the same \( \Delta K \) since both have the same \( \Delta U \) and total energy must be constant.

6.

a. The graph on the left shows the electric potential along the x-axis. Use the axes on the right to draw a graph of the potential energy of a 0.1 C charged particle in this region of space. Provide a numerical scale on the energy axis.

b. If the charged particle is shot toward the right from \( x = 1 \) m with 1.0 J of kinetic energy, where is its turning point? Explain.

At \( x = 1 \) m, the total energy \( E = K_1 + U_1 = 1J + 1J = 2J \).

At turning point \( v = 0 \) so \( K_2 = 0 \) and

\[ E = K_2 + U_2 = 2J \] so \( U_2 = 2J \) which occurs at \( x = 3 \) m.

Turning point is at \( x = 3 \) m.

2. Will the charged particle of part b ever reach \( x = 0 \) m? If so, how much kinetic energy will it have at that point? If not, why not?

\[ \text{Yes. Since } E = 2J \text{ (constant) and } U = 0 \text{ J at } x = 0 \text{ m, we know } K = 2J. \]
7. The figure shows two points inside a capacitor. Let $V = 0$ V at the negative plate.
   a. What is the ratio $V_2/V_1$ of the electric potentials at these two points? Explain.

   $\frac{V_2}{V_1} = \frac{E_s_2}{E_s_1} = \frac{s_2}{s_1} = \frac{3}{1} = 3$

   b. What is the ratio $E_2/E_1$ of the electric field strengths at these two points? Explain.

   $\frac{E_2}{E_1} = 1$
   The E-field is constant between the plates.

8. The figure shows two capacitors, each with a 3 mm separation. A proton is released from rest in the center of each capacitor.

   a. Draw an arrow on each proton to show the direction it moves.
   b. Which proton reaches a capacitor plate first? Or are they simultaneous? Explain.

   They reach the plate simultaneously. The electric field strength is the same, so the size of acceleration is the same.

9. Each figure shows a contour map on the left and a set of graph axes on the right. Draw a graph of $V$ versus $x$. Your graph should be a straight line or a smooth curve.

   a. $\begin{array}{cccccc}
   0 \text{ V} & 10 \text{ V} & 20 \text{ V} & 30 \text{ V} & 40 \text{ V} \\
   \hline
   0 \text{ m} & 1 \text{ m} & 2 \text{ m} & 3 \text{ m} & 4 \text{ m}
   \end{array}$

   b. $\begin{array}{cccccc}
   0 \text{ V} & 10 \text{ V} & 20 \text{ V} & 30 \text{ V} & 40 \text{ V} \\
   \hline
   0 \text{ m} & 1 \text{ m} & 2 \text{ m} & 3 \text{ m} & 4 \text{ m}
   \end{array}$

   $\begin{array}{cccccc}
   V (V) & & & & & \\
   0 & 10 & 20 & 30 & 40 & \\
   \hline
   0 & 2 & 4 & & &
   \end{array}$

   $\begin{array}{cccccc}
   V (V) & & & & & \\
   0 & 10 & 20 & 30 & 40 & \\
   \hline
   0 & 2 & 4 & & &
   \end{array}$
10. Rank in order, from largest to smallest, the electric potentials $V_1$ to $V_5$ at points 1 to 5.

**Order:** $V_2 = V_3 = V_4 > V_1 = V_5$

**Explanation:**

$$V = \frac{1}{4\pi \varepsilon_0} \frac{q}{r}$$

As $r$ increases, $V$ decreases.

11. Rank in order, from least negative to most negative, the electric potentials $V_1$ to $V_5$ at points 1 to 5.

**Order:** $V_1 = V_5 > V_2 = V_3 = V_4$

**Explanation:**

$$V = \frac{1}{4\pi \varepsilon_0} \frac{q}{r}$$ where $q$ is negative.

As $r$ increases, $V$ increases (becomes less negative).

12. The figure shows two points near a positive point charge.

a. What is the ratio $V_1/V_2$ of the electric potentials at these two points? Explain.

$V$ is inversely proportional to $r$.

So since $r_1 = \frac{r_2}{3}$, then $V_1 = 3V_2$.

$$\frac{V_1}{V_2} = 3.$$ 

b. What is the ratio $E_1/E_2$ of the electric field strengths at these two points? Explain.

$E$ is inversely proportional to $r^2$.

So since $r_1 = \frac{r_2}{3}$, then $E_1 = (3)^2 E_2$.

$$\frac{E_1}{E_2} = 9.$$
13. A 1 nC positive point charge is located at point A. The electric potential at point B is
\[ \text{a.} \ 9 \text{ V} \quad \text{b.} \ 9 \cdot \sin 30^\circ \text{ V} \quad \text{c.} \ 9 \cdot \cos 30^\circ \text{ V} \quad \text{d.} \ 9 \cdot \tan 30^\circ \text{ V} \]
Explain the reason for your choice.
\[
V = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r} = \left(9.0 \times 10^9 \text{ Nm}^2/\text{C}^2\right)\left(1.0 \times 10^{-9} \text{ C}\right) = 9 \text{ V}
\]

14. An inflatable metal balloon of radius \( R \) is charged to a potential of 1000 V. After all wires and batteries are disconnected, the balloon is inflated to a new radius \( 2R \).

a. Does the potential of the balloon change as it is inflated? If so, by what factor? If not, why not?

Yes. The potential decreases by a factor of 2.
Charge \( Q \) is constant.

\[
V_1 = \frac{1}{4\pi \varepsilon_0} \frac{Q}{R} \quad \text{and} \quad V_2 = \frac{1}{4\pi \varepsilon_0} \frac{Q}{2R}.
\]

b. Does the potential at a point at distance \( r = 4R \) change as the balloon is inflated? If so, by what factor? If not, why not?

No. Outside the sphere, the potential is the same as that of a point charge \( Q \) located at the center of the sphere. The distance \( r = 4R \) is always outside the balloon as it is inflated, and \( Q \) is unchanged.
15. Each figure below shows three points in the vicinity of two point charges. The charges have equal magnitudes. Rank in order, from largest to smallest, the potentials $V_1$, $V_2$, and $V_3$.

<table>
<thead>
<tr>
<th>a.</th>
<th>b.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
</tr>
<tr>
<td>$V_2 &gt; V_1 = V_2$</td>
<td>$V_1 &gt; V_2 &gt; V_3$</td>
</tr>
</tbody>
</table>

c. d.

<table>
<thead>
<tr>
<th>c.</th>
<th>d.</th>
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<tbody>
<tr>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
</tr>
<tr>
<td>$V_2 &gt; V_1 = V_3$</td>
<td>$V_1 = V_2 = V_3 = 0$</td>
</tr>
</tbody>
</table>

16. On the axes below, draw a graph of $V$ versus $x$ for the two point charges shown.

<table>
<thead>
<tr>
<th>a.</th>
<th>b.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
</tr>
</tbody>
</table>

17. For each pair of charges below, are there any points (other than at infinity) at which the electric potential is zero? If so, show them on the figure with a dot and a $V = 0$ label. If not, why not?

<table>
<thead>
<tr>
<th>a.</th>
<th>b.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image7" alt="Diagram" /></td>
<td><img src="image8" alt="Diagram" /></td>
</tr>
<tr>
<td>No. The potential is always positive in the given range. $V \to 0$ as $r \to \infty$.</td>
<td>$V = 0$</td>
</tr>
</tbody>
</table>
18. For each pair of charges below, at which grid point or points could a double-negative point charge \( q = -2 \) be placed so that the potential at the dot is 0 V? There may be more than one possible point. Draw the charge on the figure at all points that work.

a.

b.

19. The graph shows the electric potential along the \( x \)-axis due to point charges on the \( x \)-axis.

a. Draw the charges on the axis of the figure. Note that the charges may have different magnitudes.

b. An electron is placed at \( x = 2 \) cm. Is its potential energy positive, negative, or zero? Explain.

The electron’s potential energy is negative.

\[ U = qV \] where \( q \) is negative and \( V \) is positive at \( x = 2 \) cm.

c. If the electron is released from rest at \( x = 2 \) cm, will it move right, move left, or remain at \( x = 2 \) cm? Base your explanation on energy concepts.

The electron will move left. A charge will move in the direction of decreasing potential energy. The electron’s potential energy becomes more negative as it moves left.

20. A ring has radius \( R \) and charge \( Q \). The ring is shrunk to a new radius \( \frac{1}{2}R \) with no change in its charge. By what factor does the on-axis potential at \( z = R \) increase?

\[
\frac{V}{V_i} = \frac{\sqrt{R_1^2 + z^2}}{\sqrt{R_2^2 + z^2}} = \frac{\sqrt{R^2 + \frac{R^2}{4} + R^2}}{\sqrt{\frac{R^2}{4} + R^2}} = \frac{\sqrt{2}}{\sqrt{\frac{5}{4}}} \approx 1.26
\]

\( V \) increases by a factor of \( 1.26 \).
21.

a. \( E_x (V/m) \)

b. \( E_x (V/m) \)

\[
\Delta V = -\int_{x_1}^{x_2} E_y \, dx
\]

\[
\text{Slope} = -100 \text{ V/m}
\]

c. \( E_x (V/m) \)

d. \( E_y (V/m) \)

\[
V(V) \quad 200 \quad x (m)
\]

\[
V(V) \quad 200 \quad x (m)
\]
22. The top graph shows the electric potential as a function of \( x \). On the axes below the graph, draw the graph of \( E_x \) versus \( x \) in this same region of space. Add an appropriate scale on the vertical axis.

\[
E_x = -\frac{\partial V}{\partial x}
\]

23. For each contour map:

i. Estimate the electric fields \( \vec{E}_a \) and \( \vec{E}_b \) at points a and b. Don't forget that \( \vec{E} \) is a vector. Show how you made your estimate.

ii. On the contour map, draw the electric field vectors at points a and b.

\[
\vec{E}_a = \frac{\Delta V}{\Delta x} = \frac{-10 \text{ V}}{1 \text{ m}} = -10 \frac{\text{V}}{\text{m}}
\]

\[
\vec{E}_b = -10 \frac{\text{V}}{\text{m}}
\]

The minus sign means the electric field vector points "downhill" on the contour map.
24. Draw the electric field vectors at the dots on this contour map. The length of each vector should be proportional to the field strength at that point.

25. Draw the electric field vectors at the dots on this contour map. The length of each vector should be proportional to the field strength at that point.

26. The figure shows an electric field diagram. Dashed lines 1 and 2 are two surfaces in space, not physical objects.
   a. Is the electric potential at point a higher than, lower than, or equal to the electric potential at point b? Explain.

   \[ V_a > V_b \text{ since } E \text{ points toward decreasing potential.} \]

   b. Rank in order, from largest to smallest, the magnitudes of potential differences \( \Delta V_{ab}, \Delta V_{cd}, \text{ and } \Delta V_{ef} \).

   \[ \Delta V_{ef} > \Delta V_{cd} > \Delta V_{ab} \]

   Order: In magnitudes, \( \Delta V_{ef} > \Delta V_{cd} > \Delta V_{ab} \)

   Explanation:

   \[ |\Delta V| = |-E\Delta s| \text{ where } E \text{ is uniform everywhere and where } \Delta s_{ef} > \Delta s_{cd} > \Delta s_{ab}. \]

   c. Is surface 1 an equipotential surface? What about surface 2? Explain why or why not.

   Surface 1 is an equipotential surface, but not surface 2. An equipotential surface must be perpendicular to \( E \)-field lines.
27.

Two metal spheres are connected by a metal wire that has a switch in the middle. Initially the switch is open. Sphere 1, with the larger radius, is given a positive charge. Sphere 2, with the smaller radius, is neutral. Then the switch is closed. Afterward, sphere 1 has charge $Q_1$, is at potential $V_1$, and the electric field strength at its surface is $E_1$. The values for sphere 2 are $Q_2$, $V_2$, and $E_2$.

a. Is $V_1$ larger than, smaller than, or equal to $V_2$? Explain.

$$V_1 = V_2.$$

Both spheres and the wire become one conductor all at the same potential.

b. Is $Q_1$ larger than, smaller than, or equal to $Q_2$? Explain.

$$Q_1 > Q_2.$$ Since $V_1 = V_2$ and $V \propto \frac{Q}{r}$, the sphere with the larger radius must hold more charge.

c. Is $E_1$ larger than, smaller than, or equal to $E_2$? Explain.

$$E_1 < E_2.$$ Since $V_1 = V_2$ and $E = \frac{V}{r}$, the sphere with the larger radius must have a weaker electric field at the surface.
28.
The figure shows two flat metal electrodes that are held at potentials of 100 V and 0 V.
a. Used dashed lines to sketch a reasonable approximation of the 20 V, 40 V, 60 V, and 80 V equipotential lines.
b. Draw enough electric field lines to indicate the shape of the electric field.

- Field lines begin and end perpendicular to electrodes and are perpendicular to the equipotentials.