UNIVERSITY PHYSICS 1

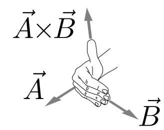
MADISON AREA TECHNICAL COLLEGE

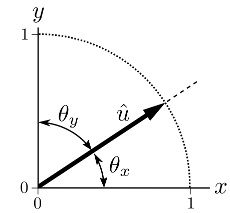
Vectors

$$\vec{A} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} = A\hat{u} \qquad \hat{u} = \frac{\vec{A}}{A} \qquad c\vec{A} + d\vec{B} = \begin{bmatrix} cA_x + dB_x \\ cA_y + dB_y \\ cA_z + dB_z \end{bmatrix}$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z = AB\cos\theta \qquad \vec{A}\times\vec{B} = (AB\sin\theta)\,\hat{u}_\perp$$

$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\,\hat{\imath} + (A_zB_x - A_xB_z)\,\hat{\jmath} + (A_xB_y - A_yB_x)\,\hat{k}$$





$$\sin\theta_x = \cos\theta_y$$
$$\hat{u} = \hat{\imath}\cos\theta_x + \hat{\jmath}\sin\theta_x$$

Kinematics

$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k} \qquad \vec{v} = \frac{d}{dt}\vec{r}(t) \qquad \vec{a} = \frac{d}{dt}\vec{v}(t)$$

$$\vec{r}(t) = \vec{r}_0 + \int_{t_0}^{t}\vec{v}(t')\,dt' \qquad \vec{v}(t) = \vec{v}_0 + \int_{t_0}^{t}\vec{a}(t')\,dt' \qquad \text{where } \begin{cases} \vec{v}_0 = \vec{v}(t_0) \\ \vec{r}_0 = \vec{r}(t_0) \end{cases}$$

relativity (Galilean)

$\vec{r}_{AB}$ and $\vec{v}_{AB}$ represent the location and velocity of A relative B

$$\vec{r}_{BA} = -\vec{r}_{AB} \qquad \vec{r}_{AB} = \vec{r}_{AC} + \vec{r}_{CB}$$
$$\vec{v}_{AB} = -\vec{v}_{BA} \qquad \vec{v}_{AB} = \vec{v}_{AC} + \vec{v}_{CB}$$

constant acceleration

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2 \qquad \vec{v}(t) = \vec{v}_0 + \vec{a}t \qquad v_s^2(t) = v_{0s}^2 + 2a_s\Delta s$$

projectile motion

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + t\begin{bmatrix} v_{0x} \\ v_{0y} \end{bmatrix} + \tfrac{1}{2}t^2\begin{bmatrix} 0 \\ -g \end{bmatrix} \qquad \begin{bmatrix} v_x(t) \\ v_y(t) \end{bmatrix} = \begin{bmatrix} v_{0x} \\ v_{0y} \end{bmatrix} + t\begin{bmatrix} 0 \\ -g \end{bmatrix}$$

circular motion

$$\vec{r}(t) = r\left[\hat{\imath}\cos\theta(t) + \hat{\jmath}\sin\theta(t)\right]$$

$$\omega = \frac{d\theta}{dt} = \frac{v}{r} \qquad \alpha = \frac{d\omega}{dt} = \frac{a_t}{r}$$

$$a_c = \frac{v^2}{r} = \omega^2 r \qquad a_t = \frac{d\,|\vec{v}|}{dt}$$

$$\theta(t) = \theta_0 + \int_{t_0}^{t}\omega(t')dt' \qquad \omega(t) = \omega_0 + \int_{t_0}^{t}\alpha(t')dt' \qquad \text{where } \begin{cases} \theta_0 = \theta(t_0) \\ \omega_0 = \omega(t_0) \end{cases} \qquad \vec{a} = \vec{a}_c + \vec{a}_t \qquad v = \frac{2\pi r}{T} \qquad \theta = \frac{s}{r}$$

Dynamics

$$\vec{F}_{\text{net}} = m\vec{a} \qquad \vec{F}_{\text{net}} = \textstyle\sum_i \vec{F}_i$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_c = m\vec{a}_c$$

conservative forces

$$\vec{F}_G = m\vec{g} \qquad F_{\text{Sp}} = -k(x - x_0)$$

nonconservative forces

$$f_s \le f_{s,\max} = \mu_s n \qquad f_k = \mu_k n \qquad F_D = \tfrac{1}{2}C\rho A v^2$$

1. identify system boundary
2. draw FBD w/ only external forces (direct contact or field force)
3. draw $\vec{F}_{\text{net}}$ vector below the FBD
4. construct equation $F_{\text{net}} = ma$ for each coordinate direction
5. repeat as needed for another system

Work & Energy

$$W = \int_A^B \vec{F}\cdot d\vec{s} = F\Delta s\cos\theta$$

(if $\vec{F}$ and $d\vec{s}$ are constant)

$$E_{\text{mech}} = K + U \qquad K = \tfrac{1}{2}mv^2 \qquad \Delta U = -W_{\text{int}} = -\int_A^B \vec{F}_{\text{int}}\cdot d\vec{s}$$

$$F_s = -\frac{dU}{ds} \qquad U_G = mgy \qquad U_{\text{Sp}} = \tfrac{1}{2}ks^2 \qquad P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$$

$$W_{\text{ext}} = \Delta E_{\text{sys}} = \Delta(K + U + E_{\text{th}})$$

if $W_{\text{ext}} = 0$ and no friction loss then $\Delta K + \Delta U = 0$

Constants

gravity field on Earth	g	9.81 N/kg
universal gravity const.	G	6.67×10^{-11} N·m^2/kg^2
intensity threshold	I_0	10^{-12} W/m^2
standard atm pressure	p_0	1.01325×10^{5} Pa

Center of Mass

$$x_{\text{cm}} = \frac{\sum_i m_i x_i}{M} = \frac{1}{M}\int x\,dm \qquad M = \textstyle\sum_i m_i$$

$$\vec{r}_{\text{cm}} = \frac{1}{M}\int \vec{r}\,dm \qquad \vec{v}_{\text{cm}} = \frac{d}{dt}\vec{r}_{\text{cm}}$$

Momentum

$\vec{p} = m\vec{v}$ $\quad \vec{P} = \sum_i \vec{p}_i$ $\quad P = M\vec{v}_{\text{cm}}$ $\quad \vec{F}_{\text{net}} = \frac{d\vec{P}}{dt}$ $\quad K = \frac{p^2}{2m}$

$\Delta\vec{P} = \vec{J} = \int \vec{F}_{\text{ext}}\,dt = \vec{F}_{\text{ave}}\Delta t$ $\quad$ if $\vec{F}_{\text{ext}} = 0$ then $\Delta\vec{P} = 0$

elastic 1D collisions, with $v_{2i} = 0$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i}$$

rockets

$$F_{\text{thrust}} = Ru$$

$$\Delta v = u\ln\left(\frac{m_i}{m_f}\right)$$

Rotational Dynamics

$\vec{\tau} = \vec{r}\times\vec{F}$ $\quad \tau = Fr\sin\phi = \pm Fd$ $\quad I_{\text{particle}} = mr^2$ $\quad I_{\text{disk}} = \frac{1}{2}mr^2$

$\vec{L} = \vec{r}\times\vec{p}$ $\quad \vec{L} = I\vec{\omega}$ $\quad \vec{\tau} = I\vec{\alpha}$ $\quad \vec{\tau} = \frac{d\vec{L}}{dt}$ $\quad \Omega = \frac{mgd}{I\omega}$

$K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}}$ $\quad K_{\text{rot}} = \frac{1}{2}I\omega^2$ $\quad$ (+) (−)

Statics & Elasticity

in static equilibrium $\begin{cases}\vec{F}_{\text{net}} = 0\\ \vec{\tau}_{\text{net}} = 0\end{cases}$ $\quad \frac{F}{A} = Y\frac{\Delta L}{L}$ $\quad p = -B\frac{\Delta V}{V}$

Gravitation

$F_G = G\frac{m_1 m_2}{r^2}$ $\quad g = \frac{GM}{r^2}$

$E_{\text{mech}} = K + U_G$

bound orbit: $E_{\text{mech}} < 0$

unbound orbit: $E_{\text{mech}} > 0$

$U_G = -G\frac{m_1 m_2}{r}$

$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$

circular orbits

$\frac{GM}{r^2} = \frac{v^2}{r}$ $\quad T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$ $\quad E_{\text{mech}} = \frac{1}{2}U_G$

Fluid Mechanics

$p = \frac{F}{A}$ $\quad \rho = \frac{m}{V}$ $\quad p = p_0 + \rho g d$ $\quad F_B = \rho V_{\text{disp}} g$

floating: $\frac{V_{\text{disp}}}{V_o} = \frac{\rho_o}{\rho_f}$

$Q = \frac{dV}{dt} = A\overline{v}$ $\quad p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$

Oscillations

$s''(t) = -\omega^2 s(t)$ $\quad f = \frac{1}{T}$ $\quad \omega = 2\pi f = \frac{2\pi}{T}$

$x(t) = A\cos(\omega t + \phi)$ $\quad \omega = \sqrt{\frac{k}{m}}$ $\quad \omega = \sqrt{\frac{g}{L}}$ $\quad \omega = \sqrt{\frac{mgL}{I}}$

$v(t) = -\omega A\sin(\omega t + \phi)$

$a(t) = -\omega^2 A\cos(\omega t + \phi)$ $\quad E = K + U = \frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2$

damped: $E(t) = E_0 e^{-bt/m} = E_0 e^{-t/\tau}$

Waves & Sound

$v^2\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{\partial^2 y(x,t)}{\partial t^2}$ $\quad y(x,t) = A\sin(kx \mp \omega t + \phi)$ $\quad k = \frac{2\pi}{\lambda}$

$v = f\lambda = \frac{\omega}{k}$ $\quad v = \sqrt{F_T/\mu} = \sqrt{B/\rho}$ $\quad v_{\text{sound}} = (331\,\frac{\text{m}}{\text{s}})\sqrt{\frac{T}{273\text{ K}}}$

$f_o = f_s\left(\frac{v \pm v_o}{v \pm v_s}\right)$ $\quad I = \frac{P}{A}$ $\quad \beta\,(\text{dB}) = 10\log_{10}\left(\frac{I}{I_0}\right)$

Symbols and Units

acceleration	a	m/s²
area	A	m²
amplitude	A	m
bulk modulus	B	N/m²
damping constant	b	kg/s
drag coefficient	C	(none)
moment arm	d	m
depth	d	m
energy	E	J
force	f, F	N
frequency	f	Hz, s^{-1}
gravity field strength	g	N/kg
moment of inertia	I	kg·m²
intensity	I	W/m²
impulse	J	N·s
kinetic energy	K	J
spring constant	k	N/m
wave number	k	rad/m
angular momentum	L	kg·m²/s
length	L	m
mass	m, M	kg
normal force	n	N
pressure	p	Pa
power	P	W, J/s
momentum	p, P	kg·m/s
radius, distance	r, R	m
fuel burn rate	R	kg/s
flow rate	Q	m³/s
path length	s	m
time	t	s
period	T	s
temperature	T	K
unit vector	$\hat{u}$	(none)
rocket exhaust speed	u	m/s
potential energy	U	J
velocity	v	m/s
volume	V	m³
work	W	J
x-position	x	m
y-position	y	m
Young's modulus	Y	N/m²
z-position	z	m
angular acceleration	α	rad/s²
angle	θ	radians
wavelength	λ	m
kinetic friction coeff.	μ_k	(none)
static friction coeff.	μ_s	(none)
density	ρ	kg/m³
torque	τ	N·m
time constant	τ	s
angle	ϕ	radians
angular velocity	ω	rad/s
angular frequency	ω	rad/s
gyro precession freq.	Ω	rad/s